# Plasma-wave interaction in Electrodeless Plasma Thrusters and its optimization


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## Electrodeless Plasma Thrusters (EPTs)

- In established electric propulsion technologies (Ion thrusters, Hall Effect Thrusters), **cathodes** are essential to **neutralize** the ion beam :
  - subject to **erosion**, limiting thruster's lifetime.
  - add **design complexity** and limit propellant options.
- **EPTs use electromagnetic (EM) waves** to ionize propellant and maintain quasineutral plasma ➡ **no cathodes required.**
  - **Helicon Plasma Thrusters (HPT)** (Fig.1) : an antenna launches RF waves, exciting helicon and TG modes.
  - **Electron Cyclotron Resonance (ECR)** : a waveguide injects microwaves tuned to the electron cyclotron frequency $\omega_{ce}$, heating electrons through **resonance** (Fig.2).

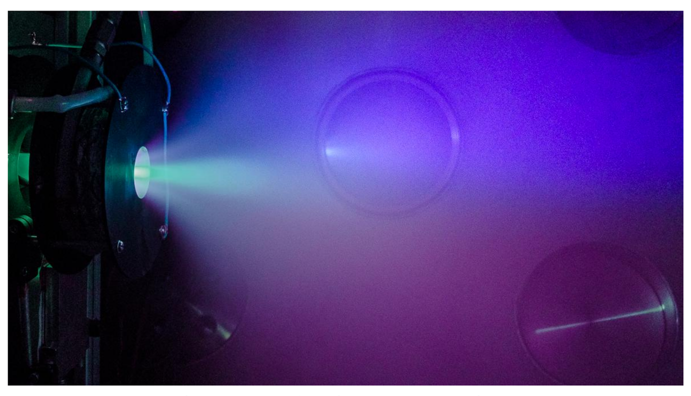

Fig.1 : HPT in operation.

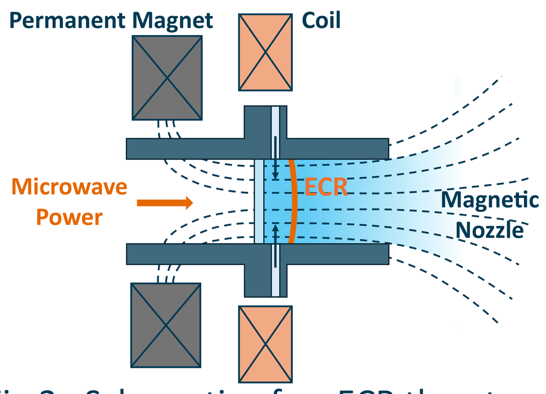


Fig.2 : Schematic of an ECR thruster.

## Ph.D. Motivation

- EPTs performance depends on **efficient wave-plasma coupling.**
  - Requires accurate modeling of wave-plasma interactions.
  - Previous EM waves simulations yield **spurious solutions** [1]

➡ **need for a more robust and reliable numerical approach**

- The research is structured around four main axes :
  1) **Develop an extended Finite Element (FE) wave solver,** based on the **cold-plasma model** and validated analytically.
  2) **Investigate numerical issues in wave-plasma simulations,** through in-depth analysis of spurious solutions.
  3) **Apply the wave solver in realistic EPT configurations** to optimize power deposition and study helicon mode transitions.
  4) **Assess the limits of the cold-plasma model.**

## Numerical simulation of Electromagnetic Waves in plasma

- A new full-wave solver based on the **Finite Element Method (FEM)** is developed to simulate EM wave propagation in plasma :
  - **advanced boundary conditions**, including a fully functional **waveguide port** compatible with plasma.
  - **analytical validation,** via **manufactured solutions**, and comparison with **generalized Fresnel solutions.**
  - **numerical analysis**, investigating sensitivity to mesh characteristics through **dispersion analysis.**
  - **integration with *HYPHEN*** (the in-house hybrid fluid-PIC code) for EPT simulations.
- The FEM is used (frequency domain) to solve the **wave equation** in plasma : $\nabla \times (\nabla \times \boldsymbol{E}) - k_0^2 \bar{\bar{\kappa}} \boldsymbol{E} = i k_0 Z_0 \boldsymbol{J}$
  - with **harmonic fields** : $\boldsymbol{E}(\boldsymbol{r}, t) = \tilde{\boldsymbol{E}} \exp\left[i(\boldsymbol{k} \cdot \boldsymbol{r} - \omega t)\right]$
- The **cold-plasma model** describes wave propagation in a magnetized, collisionless plasma.
  - Leads to dielectric tensor [2] : $\bar{\bar{\kappa}} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix}$
- The **CMA diagram** (Fig.3) maps wave propagation regimes as function of plasma & cyclotron frequencies.
  - It helps identify cutoffs and **resonances.**

- **Resonance** allows efficient energy transfer from the wave to the plasma
  - fundamental for **power deposition** (ECR heating)

$$\theta_c = \tan^{-1} \sqrt{-\frac{P}{S}}$$

- **Resonance cones** (Fig.4) define angular directions where resonance occurs.

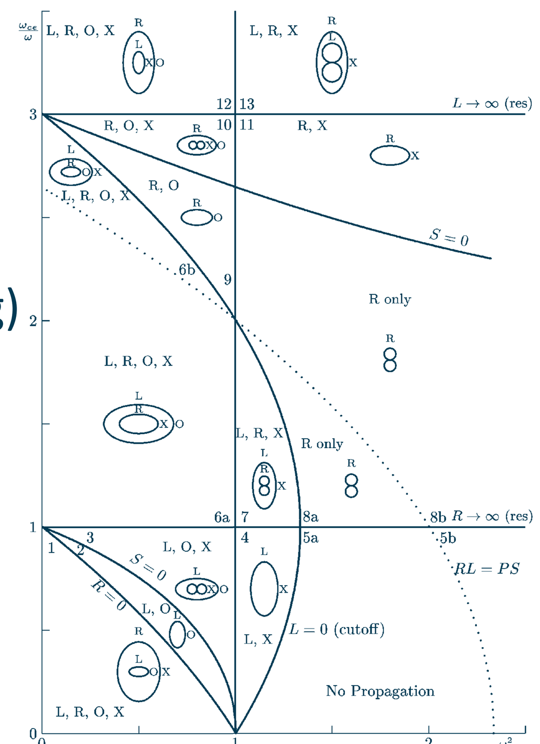

Fig.3 : The CMA diagram [2].

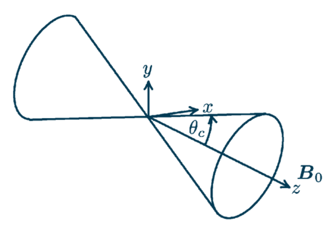

Fig.4 : Geometry of resonance cones [2].

- Inside : waves propagate and interact strongly with plasma.
- Outside: waves are reflected, refracted, or do not propagate.

- This model is based on **assumptions** :
  - **linearized response**
  - **frequency-domain formulation** (single $\omega$)
  - **neglected thermal effects**

and their validity will be assessed during this Ph.D.

## Wave solver applications

The wave solver is coupled with *HYPHEN* to

- **optimize power deposition** in EPTs :
  - explore and characterize **alternative thruster geometries**.
  - investigate **magnetic field topologies** (e.g. resonant cavities, birdcage antennas).
  - **optimization goals** : maximize power deposition near the center of the plasma and ensure efficient impedance matching.

➡ **Guide the design of more efficient EPT configurations through modeling.**

- **study mode transitions for helicon waves :**
  - often used for their **efficient ionization**, they exist in multiple radial and axial modes.
  - **mode transitions** : the dominant excited mode shifts ➡ abrupt changes in wave structure.
  - simulate helicon wave behavior under varying conditions ($B, \dot{m}$ and $P$) to analyze and **compare transitions with experimental data.**

➡ **Understand the physics behind helicon mode transitions and their impact on power coupling.**

## Generalized Fresnel problem : code validation & numerical analysis

- **Extension of the classical Fresnel problem to magnetized plasma**
  - paired waves can have distinct wave vectors and non-orthogonal polarizations (Fig.5)

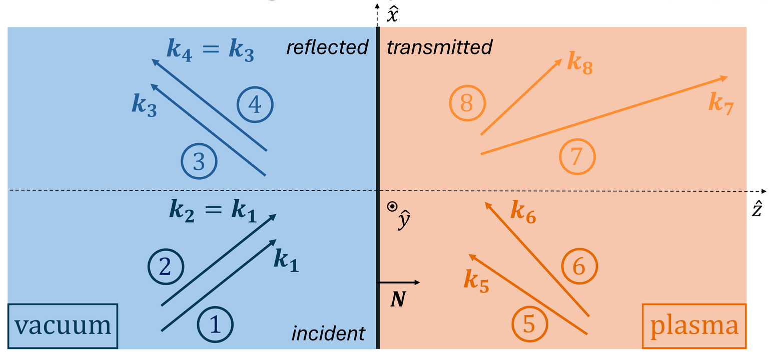


Fig.5 : Sketch of Fresnel problem generalized to anisotropic media; a planar wave travels from vacuum to a uniform magnetoplasma.

- **Continuity conditions** on fields at interface.
  - Equations for 8 amplitudes : solved for the 4 reflected/transmitted ones.
  - Incident waves amplitudes prescribed.

➡ **Analytical solution under cold-plasma assumptions : ideal verification test.**

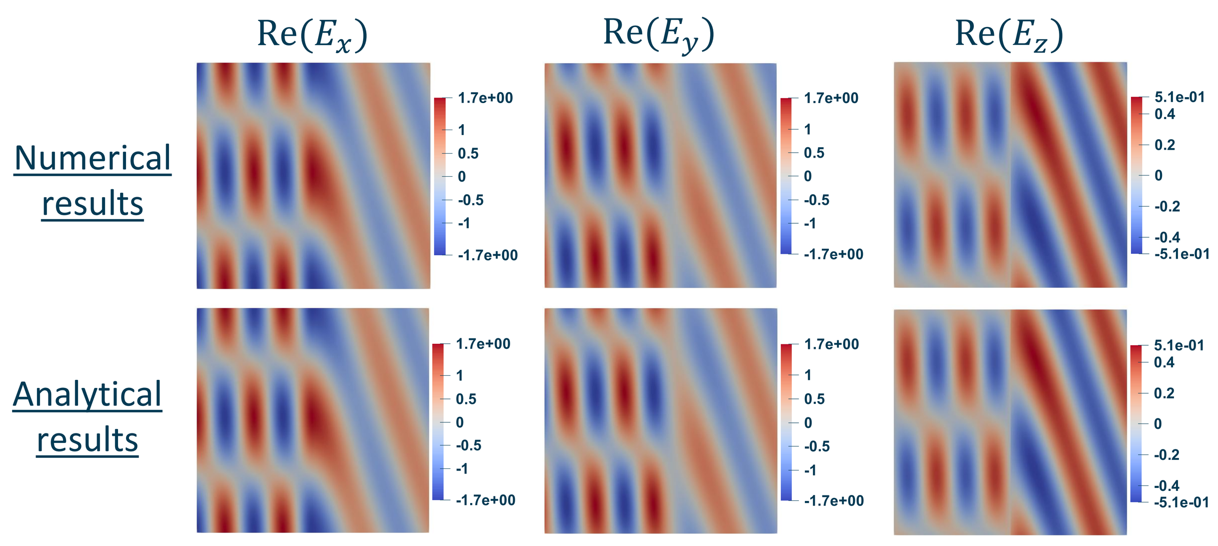


Fig.6 : Wave solver verification with analytical solution (Region II of CMA).

- **Preliminary results** confirm **model's accuracy** in handling wave interactions at vacuum-magnetoplasma interfaces.
- However, **spurious solutions** appear (Fig.7) in regions of the CMA diagram with resonance cones : **physical origin** ?

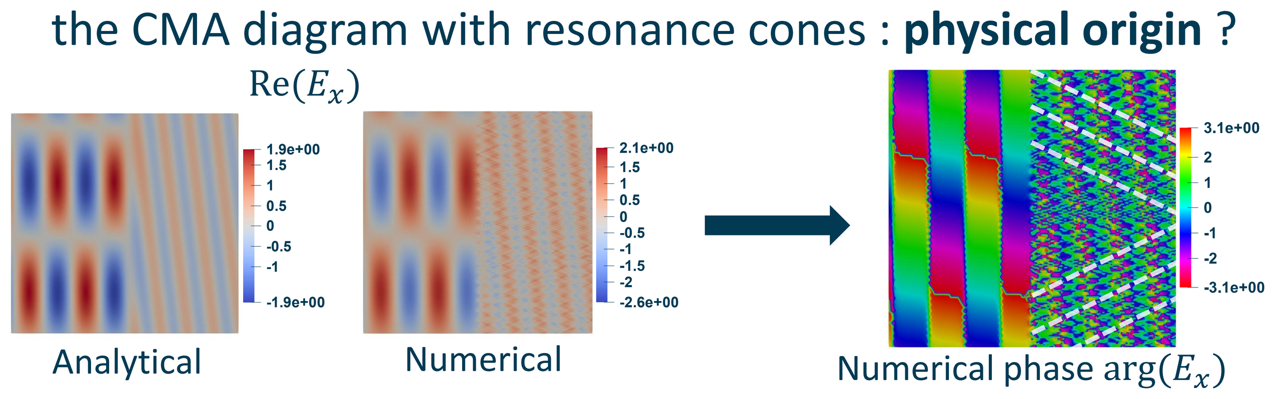


Fig.7 : Spurious solutions in Region VIII of CMA, alignment with iso-phase surfaces.

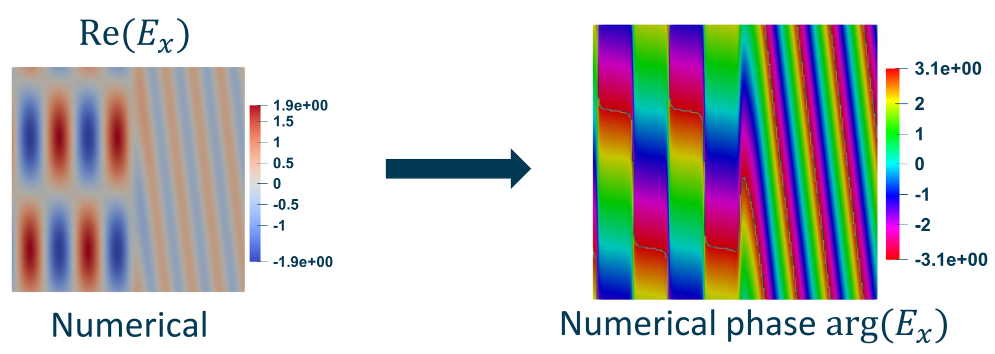


Fig.8 : Smooth solution in Region VIII of CMA, rectangular elements.

- No spurious solution (Fig.8) when using **rectangular finite elements** instead of triangular ones : **numerical artifact** ?
  - Further study requires **dispersion analysis.**
- **Convergence study** (Fig.9) using the $L_2$ error :

$$||\boldsymbol{E} - \boldsymbol{E}_h||_{L^2(\Omega)} = \int_\Omega |\boldsymbol{E}(\boldsymbol{r}) - \boldsymbol{E}_h(\boldsymbol{r})|^2 \,\mathrm{d}\boldsymbol{r}$$

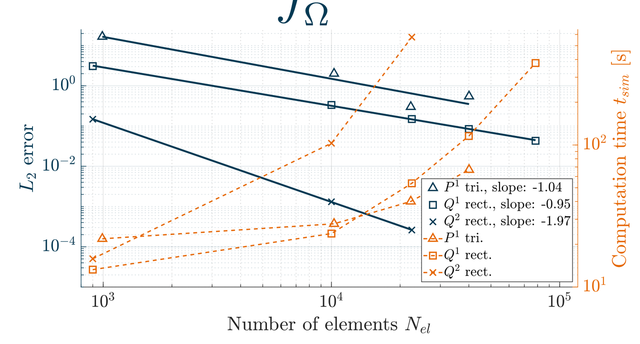


Fig.9 : Convergence study of meshes with 1st order triangular, rectangular, and 2nd order rectangular elements.


[1] Inchingolo, M. R., et al. "Simulation of the discharge and microwave-plasma coupling in a waveguide ECR thruster." (2024).
[2] Swanson, D. "Plasma Waves, 2nd edition". IOP Publishing, Bristol, UK, 2003.


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