PhD Doctoral Meetings

PhD program in Aerospace Engineering UC3M
June 2025

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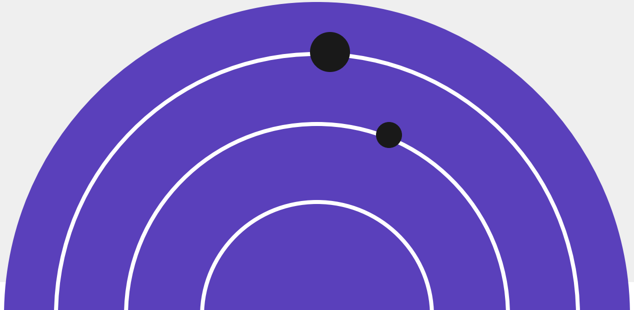
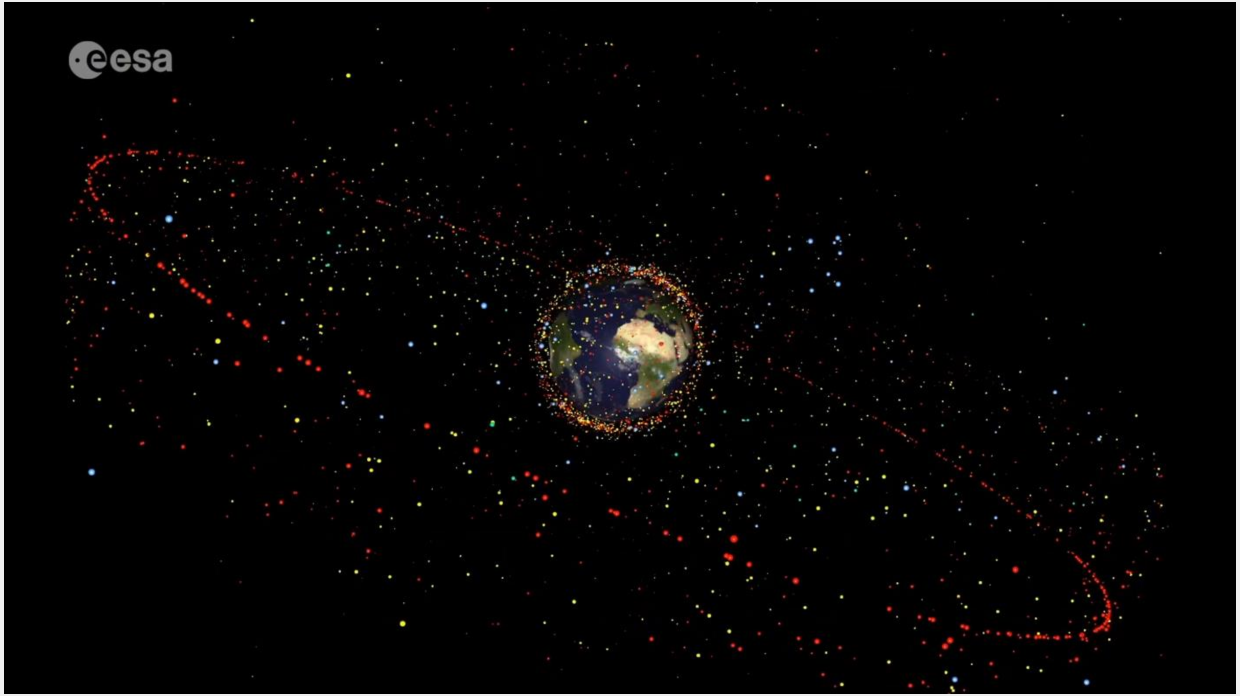
uc3m Universidad Carlos III de Madrid
Aerospace

38,000

Space objects in orbit as of March 2025.



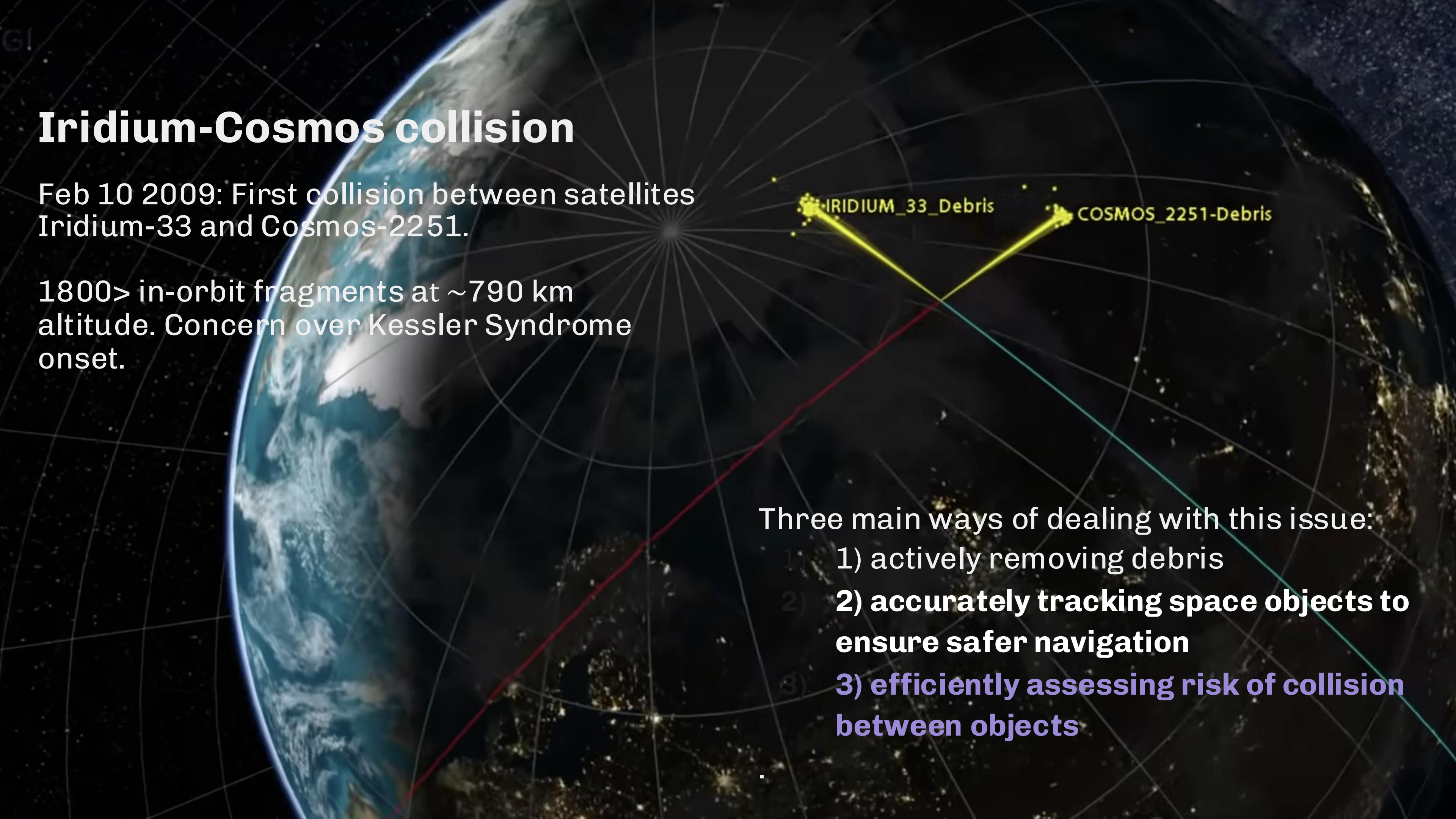
> 10 cm



Iridium-Cosmos collision

Feb 10 2009: First collision between satellites Iridium-33 and Cosmos-2251.

1800> in-orbit fragments at ~790 km altitude. Concern over Kessler Syndrome onset.



Three main ways of dealing with this issue:

1) actively removing debris

2) accurately tracking space objects to ensure safer navigation

3) efficiently assessing risk of collision between objects

PhD thesis contents

01
Computation of collision probability

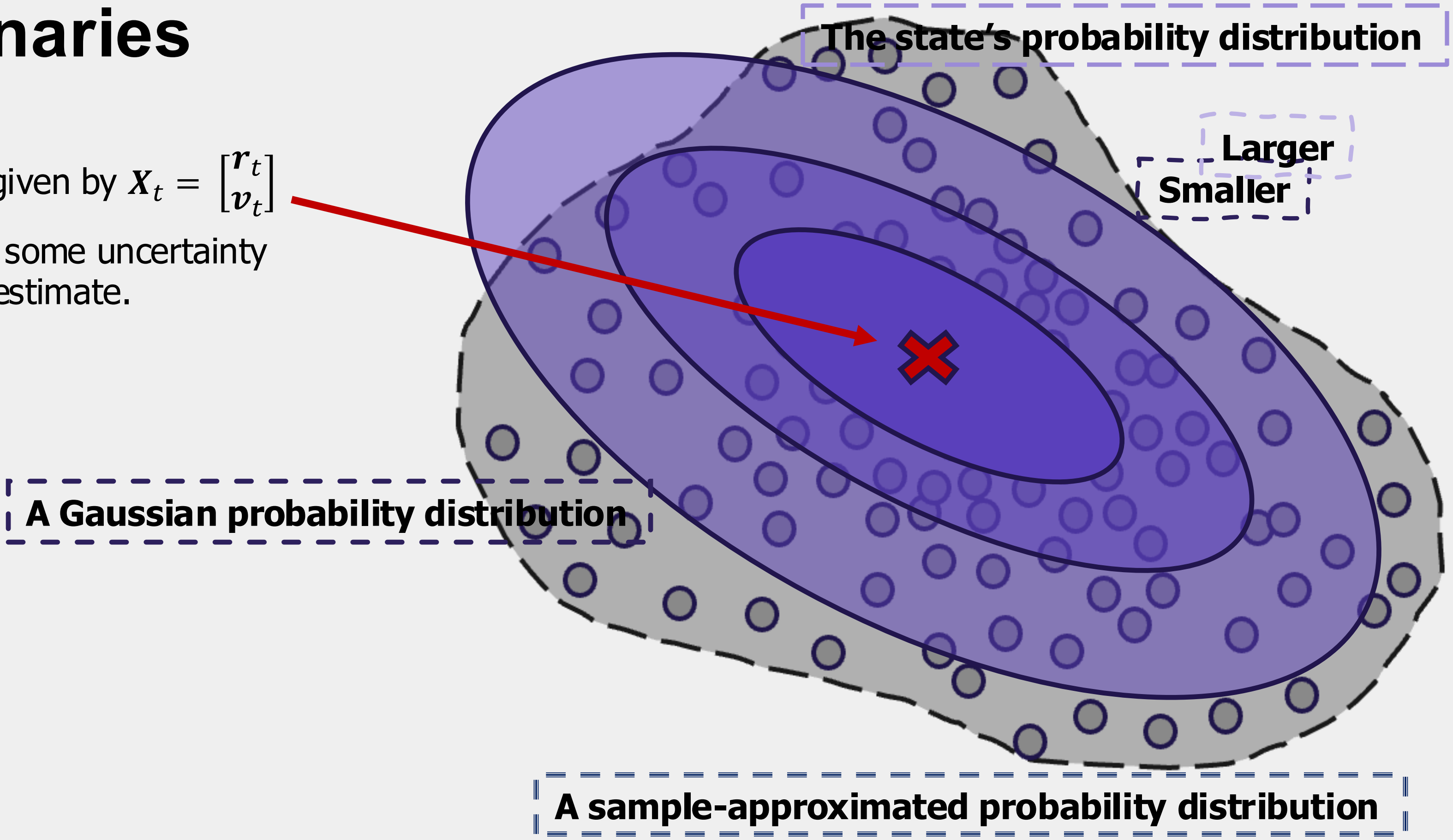
02
Efficient re-entry window prediction schemes

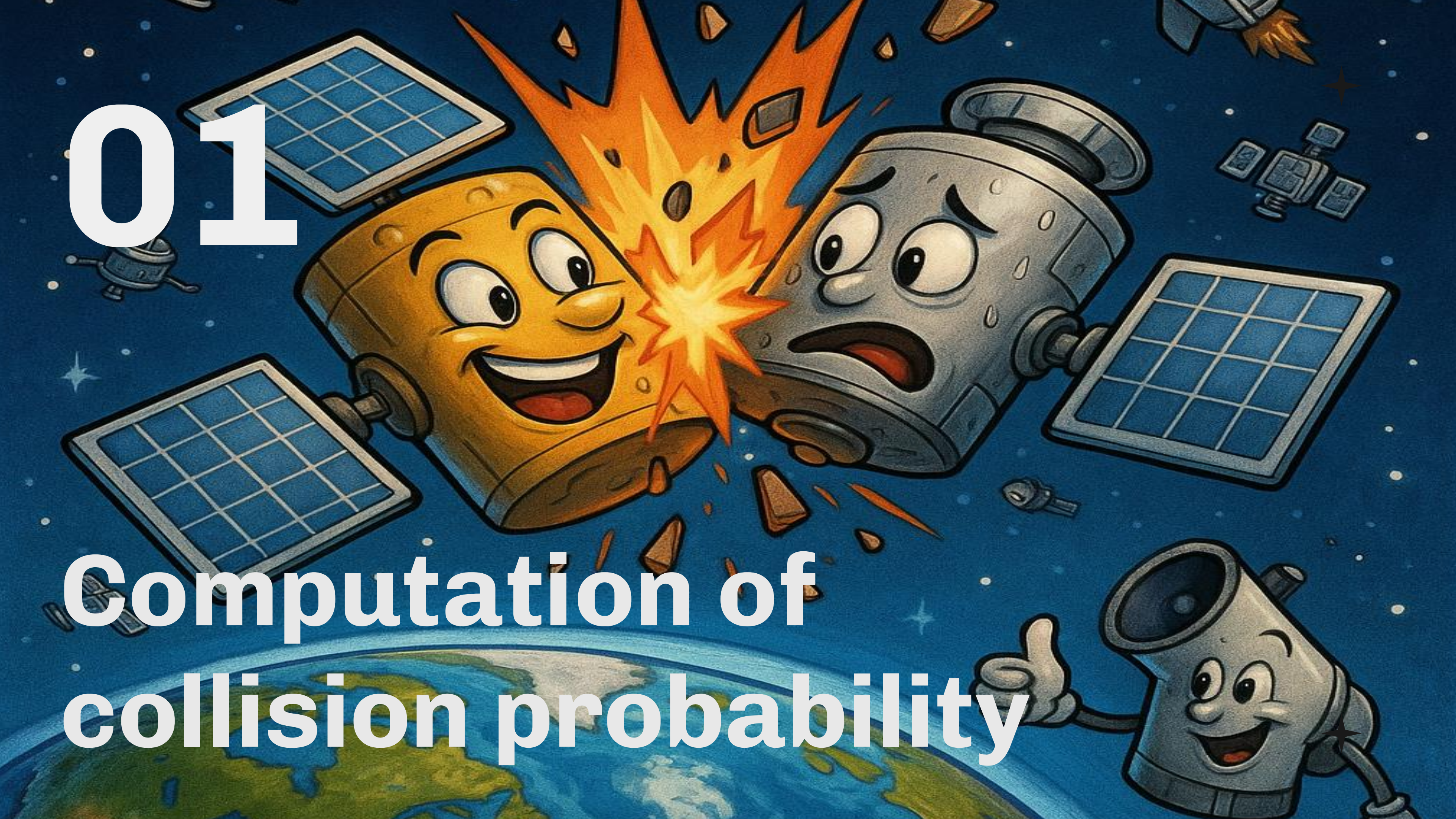
03
Spacecraft tracking

Preliminaries

State estimate given by $X_t = \begin{bmatrix} r_t \\ v_t \end{bmatrix}$

There is always some uncertainty around a state estimate.





01

Computation of collision probability

Challenges of computing probability of collision (PoC)

It is very hard to capture collisions with simple numerical integration in 12D, resolution must be super high.

→ **A smoother metric is needed.**

Classical ways of computing the PoC assume simplified simplified state distributions at time of collision. To avoid these, expensive methods are needed.

→ **Crude Monte Carlo (CMC)**

We use CMC variations to predict future collisions in an accurate, but efficient way.

→ **Importance sampling (IS)**

A convenient reduced-dimension metric

Conjunction function $\Upsilon(\boldsymbol{X}_0)$: $\mathbb{R}^{12} \rightarrow \mathbb{R}^{2}$

$$\boldsymbol{X}_0 \rightarrow \xi_c = [\Gamma_c \; \Delta_c]^T$$

$\boldsymbol{r}_{A,k}$ is the position vector of object A.

$\boldsymbol{U}_n$ is the line of nodes

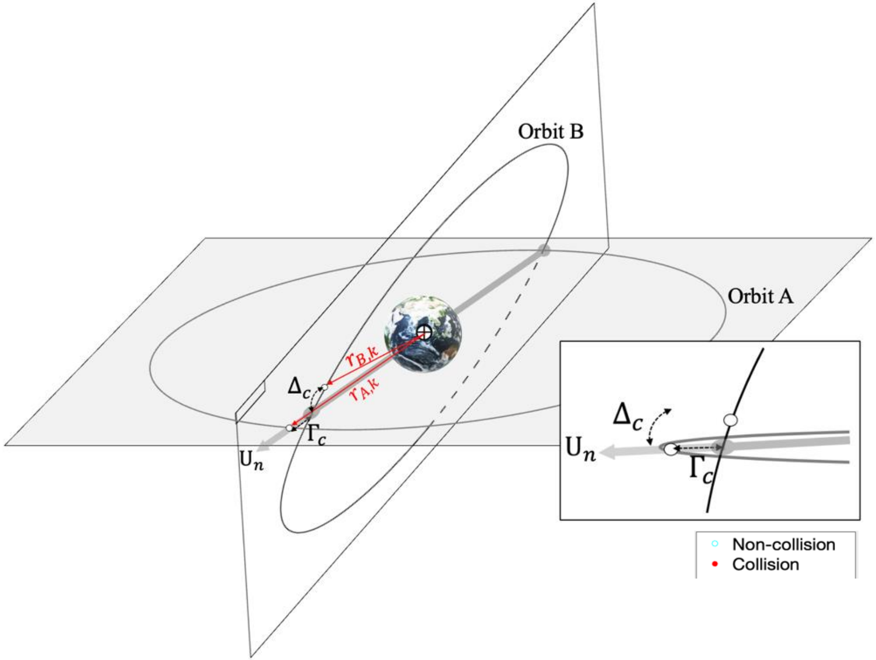
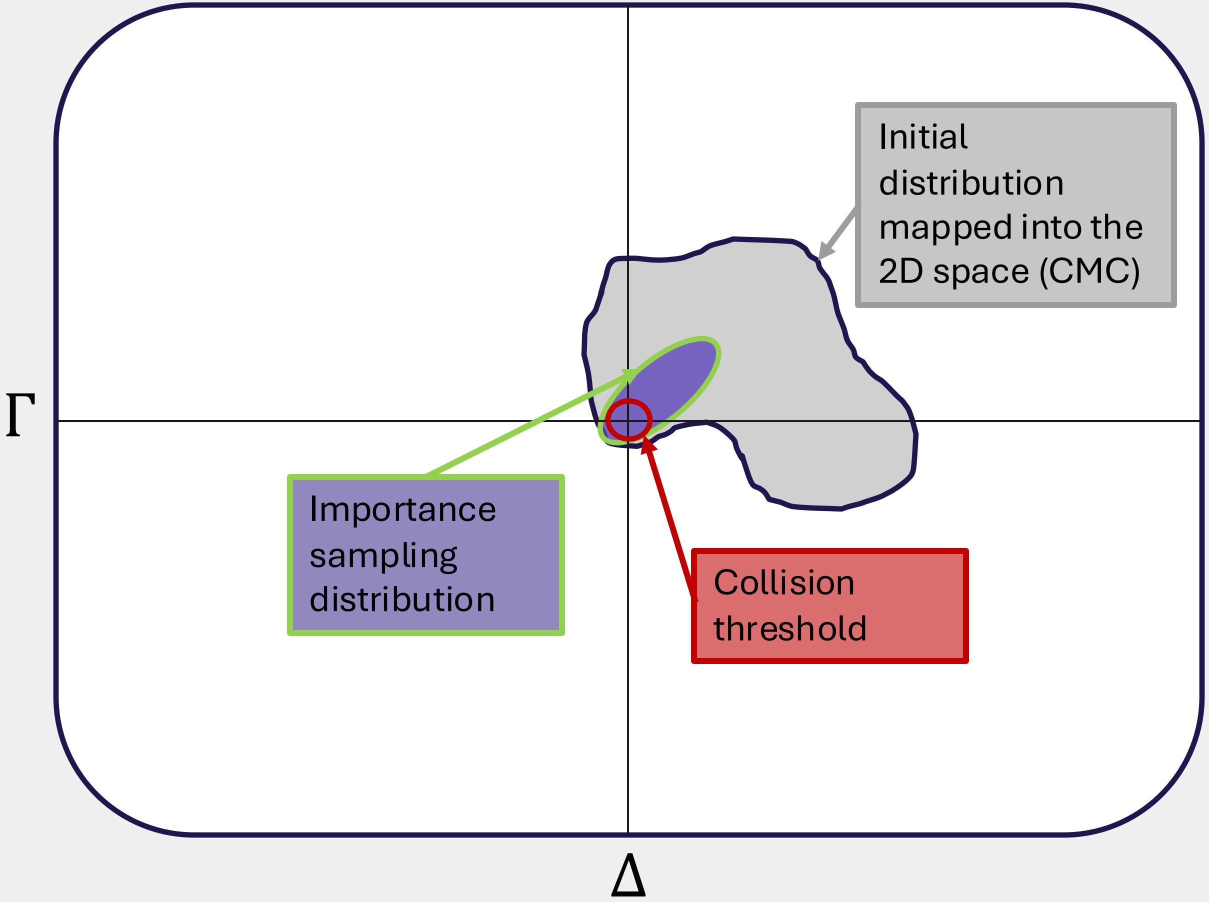


Figure 1: A (possible) conjunction geometry.

A convenient reduced-dimension metric



Results part 1: PoC and 2D map

Table 1:
The PoC estimates, along with their runtimes and empirical standard deviations, obtained with the proposed method (Algorithm 1) and with the CMC (benchmark).

	PoC estimate	Run-time (s)	σ
CMC simulation (HF)	1.3550×10^{-3}	2.0739×10^{6}	8.2254×10^{-5}
Algorithm 1	1.2832×10^{-3}	14,926	5.9902×10^{-5}

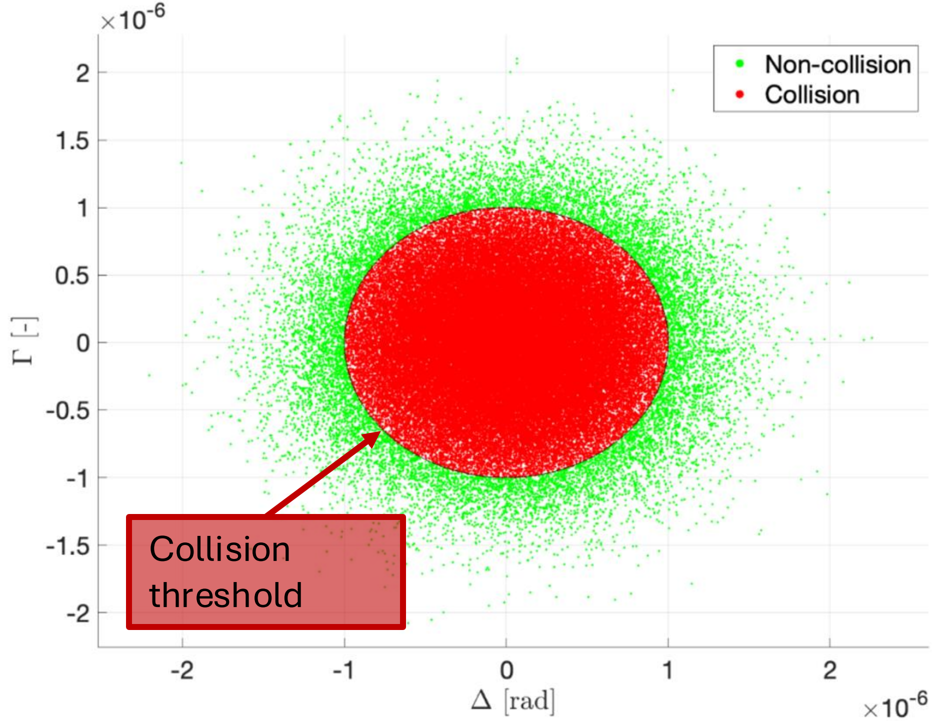
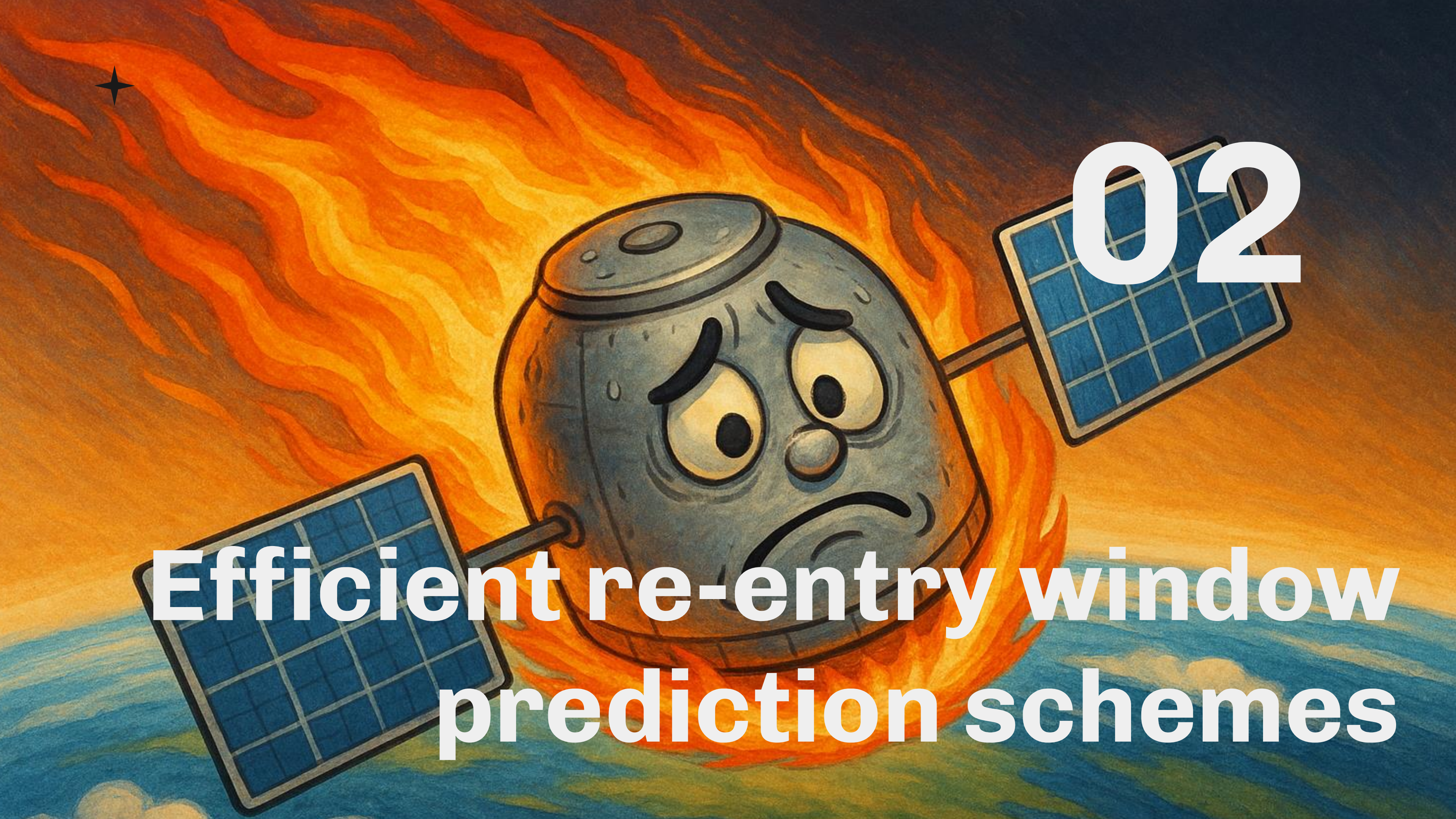


Figure 2:
The IS distribution mapped into $\mathbb{R}^2$.

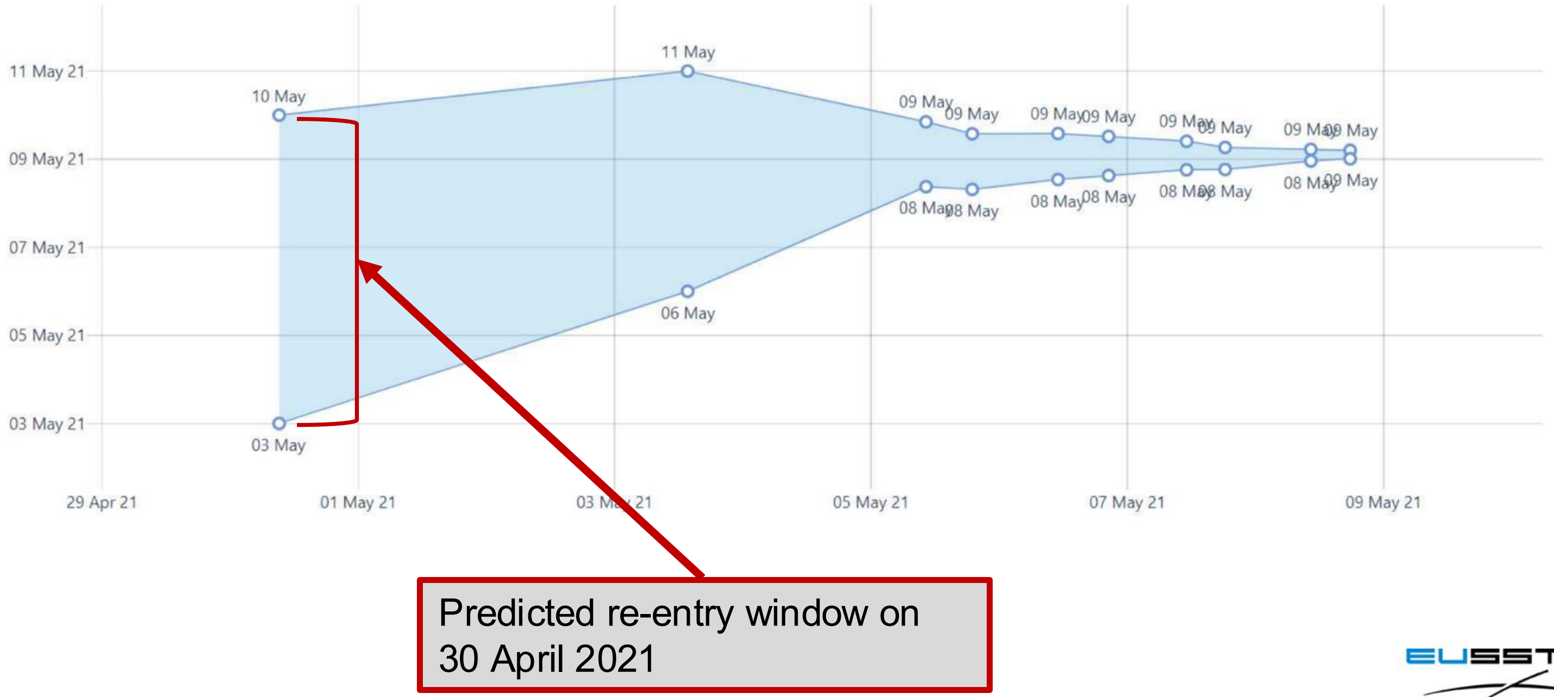


02

Efficient re-entry window prediction schemes

Figure 3:

Object CZ-5B R/B - Re-entry window evolution



Estimating re-entry windows

- Crude Monte Carlo simulations are accurate but costly.

These give us a histogram showing distribution of re-entry times of many samples

- We propose Multifidelity Monte Carlo, which optimally combines cheap dynamical models' efficiency and expensive high-fidelity models' accuracy.

MFMC produces unbiased* estimates and can be used to compute windows of re-entry time

Estimating re-entry windows

- Stochastic propagation schemes to properly reflect the level of assumed uncertainty: wider windows, less precise, but safer.

Assume orbital dynamics given by

$$d\boldsymbol{x}_t = \underbrace{f_t(\boldsymbol{x}_t)}_{\text{drift function (Eq. 1)}} dt + \underbrace{\sigma_w}_{\text{diffusion coeff.}}(\boldsymbol{x}_t) \underbrace{d\boldsymbol{W}_t}_{\text{Wiener process.}} \quad \text{(Eq. 2)}$$

$$\ddot{\boldsymbol{r}} = \underbrace{\frac{\mu \boldsymbol{r}}{r^3}}_{\text{gravitational pull}} + \mathbf{a}_{\underbrace{\mathrm{pert}}_{\text{acc. due to perturbations}}} \quad \text{(Eq. 1)}$$

Re-entry window size w.r.t. diffusion coefficient magnitude

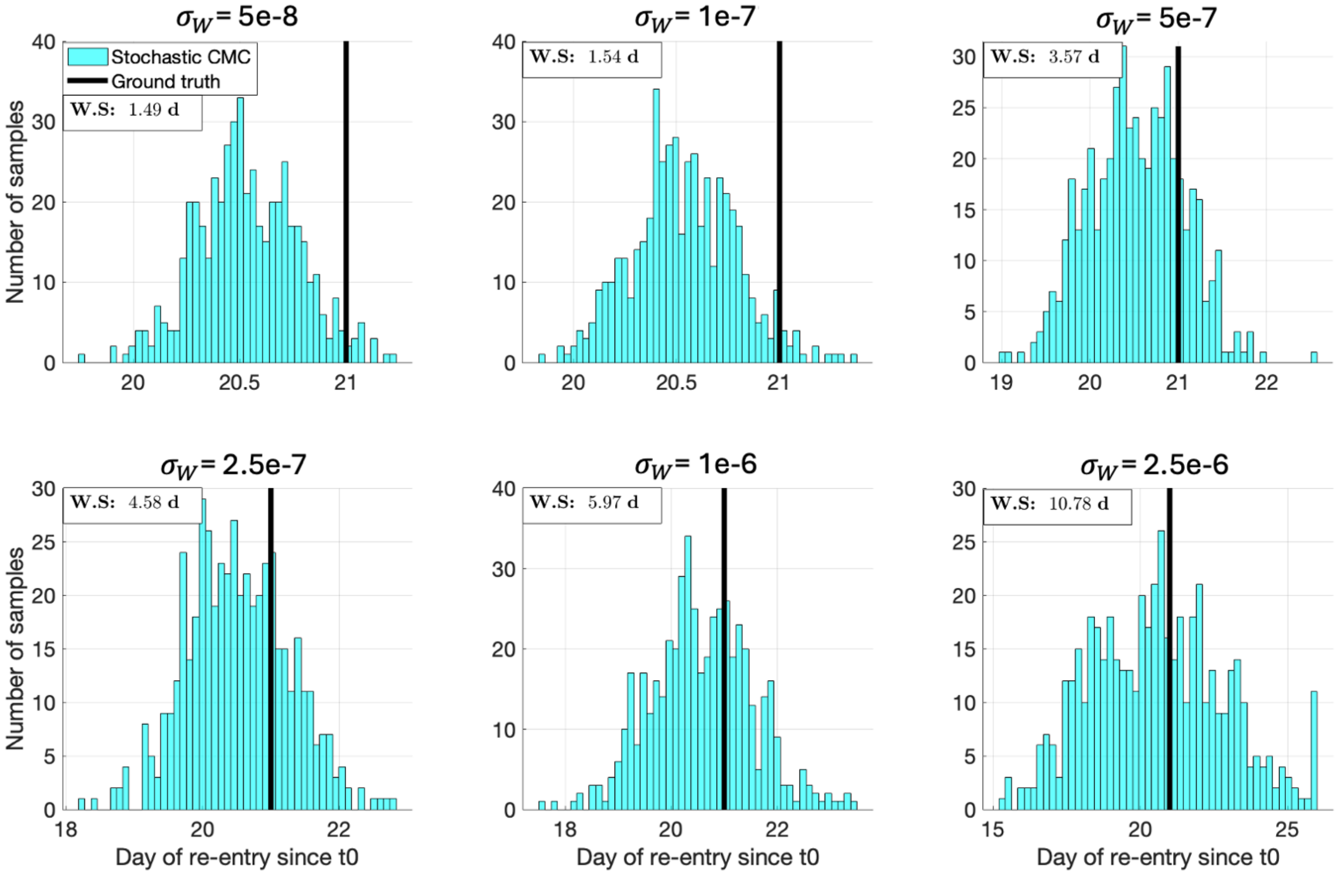


Figure 4: From top left to bottom right: the effect on re-entry time histogram spread when increasing σ_w.

Figure 5: Window size (in days) as σ_w is increased.

Results part 1: MFMC vs CMC

We compute:

- average estimate $\hat{s}^{avg}$
- window upper bound estimate $\hat{s}^{ub}$
- window lower bound estimate $\hat{s}^{lb}$.

The latter two are given by the 1% and 99% quantiles of a collection of sample re-entry times.

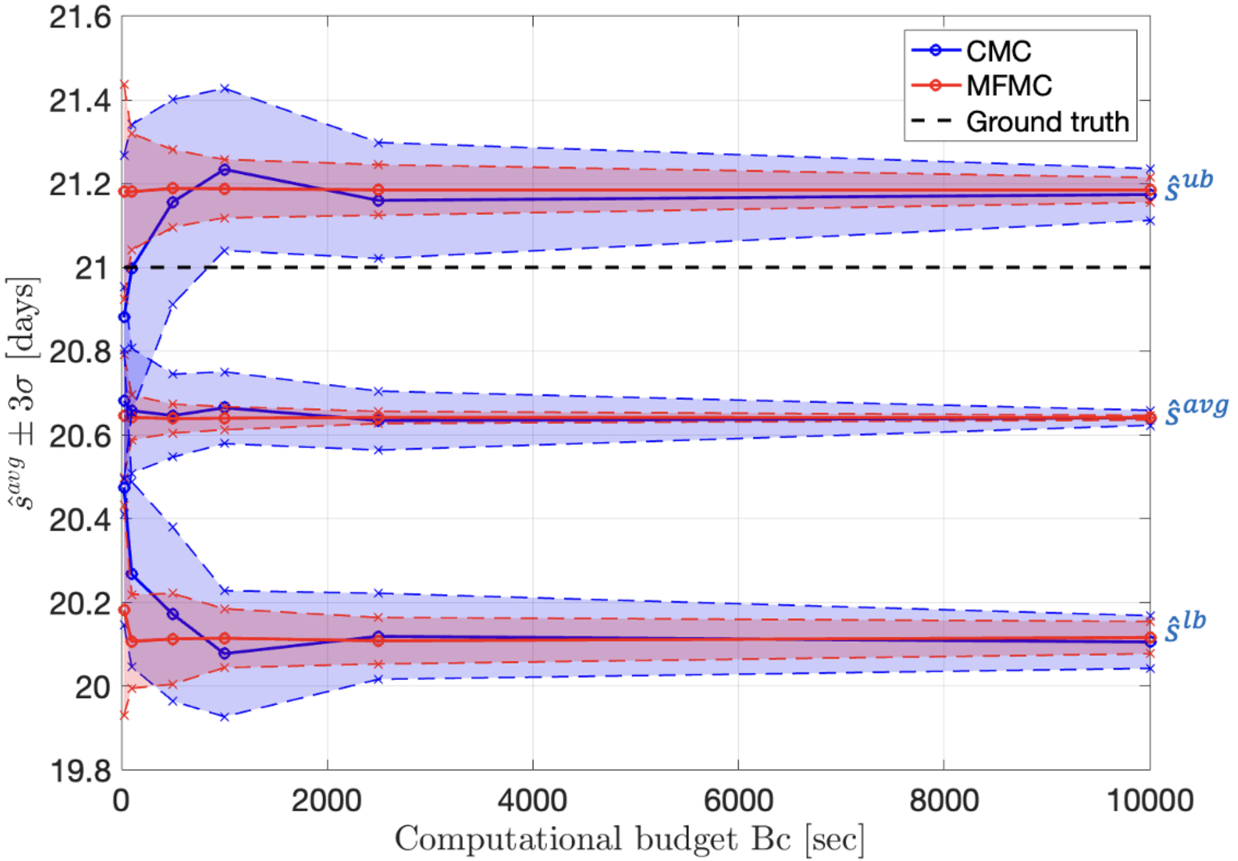
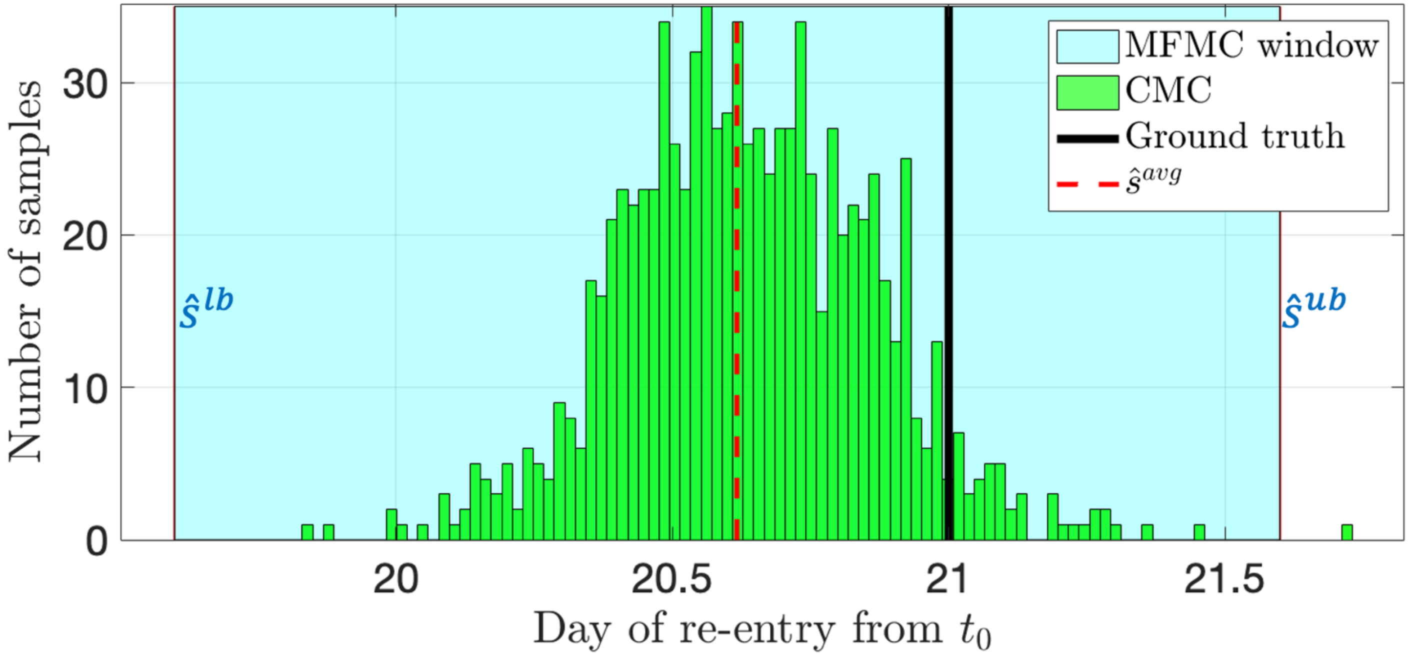


Figure 6: Empirical mean and standard deviations of the three estimators computed with MFMC (red) and CMC (blue).

Results part 2: window estimates



GOCE decay (2013)

Figure 7:

- MFMC window bounded by $\hat{s}^{lb}$ and $\hat{s}^{ub}$.
- Mean estimator $\hat{s}^{avg}$.
- CMC histogram computed with a higher computational budget (10,000sec vs 1,850sec).

Challenges in uncertainty quantification

- **PoC**: the probability computed is directly affected by the uncertainty in the initial conditions.
- Probability may be diluted, leading to lower PoC for worse given knowledge of the state distributions.

- **Re-entry**: the uncertainty assumed is largely ad-hoc or arbitrary, meaning that is it assumed fixed and decay onto lower atmosphere should increase the uncertainty.

The next chapter aims to provide proper uncertainty characterisation for state estimates.

03

Spacecraft Tracking

(Stay tuned for the defense)

Research things

Journal articles

1) "An approximate model for the computation of in-orbit collision probabilities using importance sampling", *Advances in Space Research*, vol. 75(4), pages 3791-3805. Manuscript published on 15 February 2025.

2) "Sequential filtering techniques for simultaneous tracking and parameter estimation". Manuscript submitted to *Journal of Astronautical Science* on 15 March 2025.

3) "Multifidelity Monte Carlo for the estimation of re-entry windows". Manuscript pending submission to *Advances in Space Research* this month.

Conferences

1) "Rare event sampling schemes for the efficient computation of in-orbit collision probabilities", at the *2nd NEO and Space Debris Conference* (ESA), January 2023.

2) "Novel method for the Computation of In-Orbit Collision Probability by Multilevel Splitting and Surrogate Modelling", at the *34th AIAA-SciTech Forum*, January 2024.

3) "Multifidelity Monte Carlo for the estimation of re-entry windows", at the *11th European Conference for Aerospace Sciences*, June 2025.

Research stays

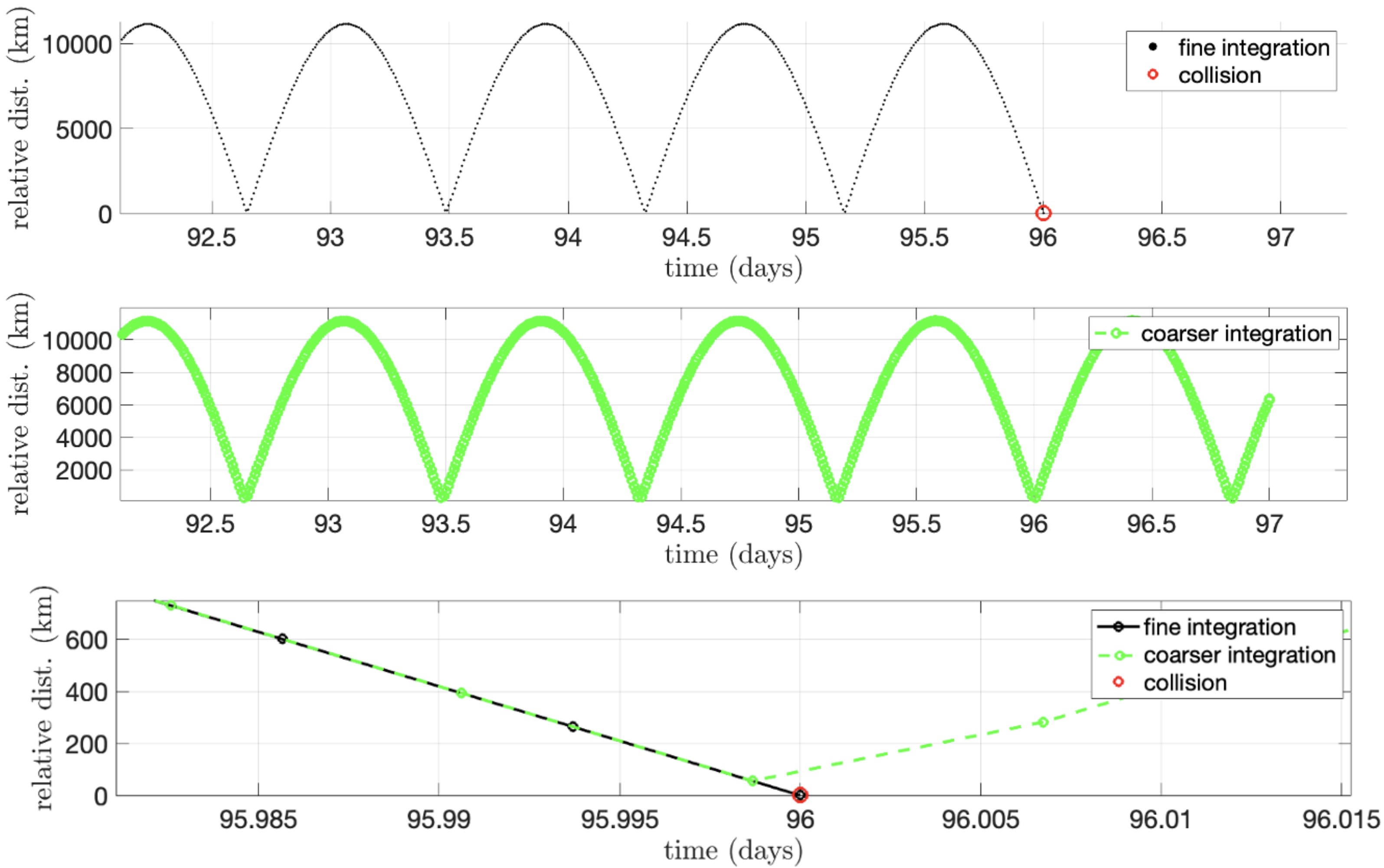
1) Research stay at the Jacobs School of Engineering, University of California San Diego, US.
Dates: 25 April 2024 – 12 September 2024. Supervision: Boris Kramer and Aaron J. Rosengren.

Other research collaborations

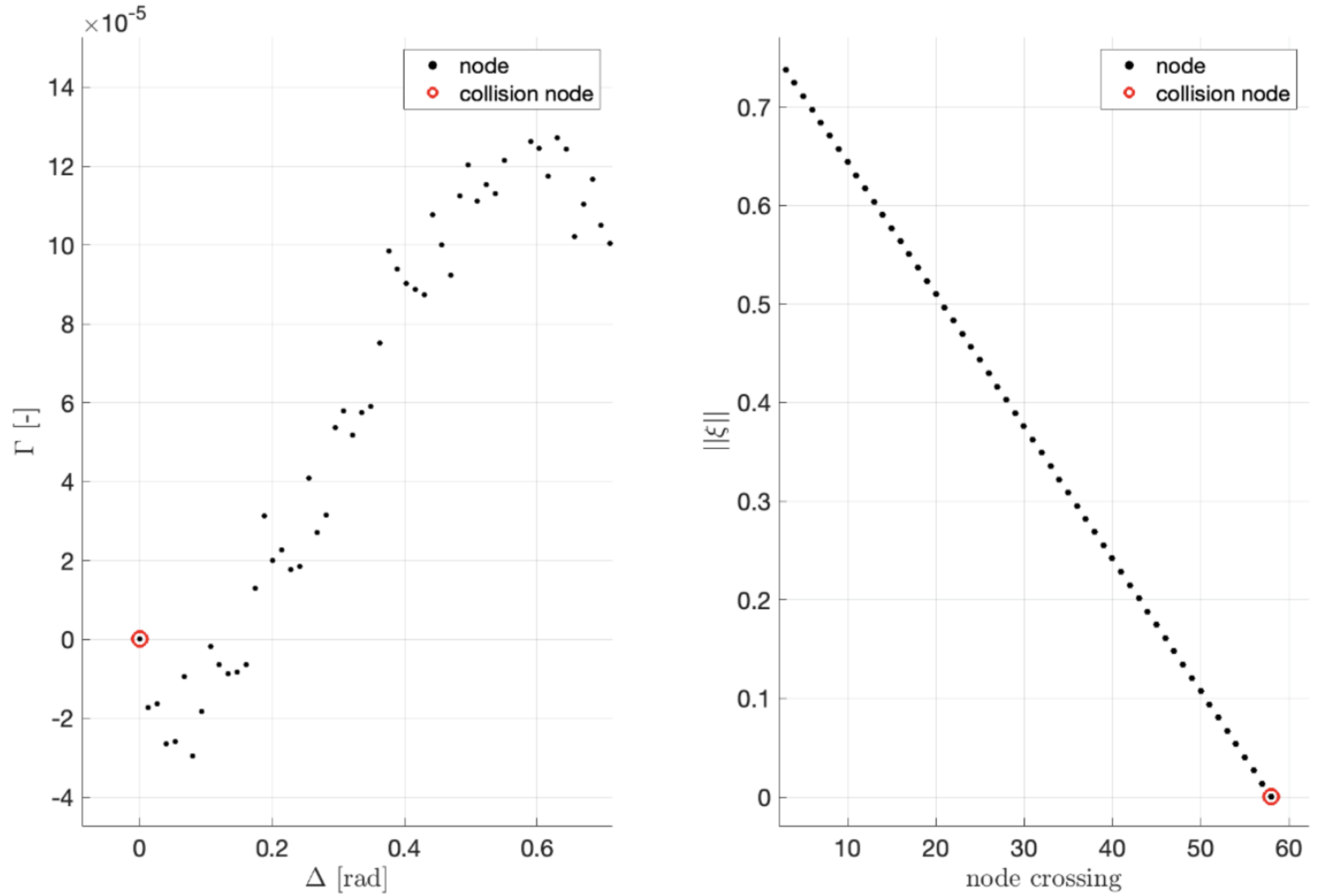
1) Research project on robust particle filter methodologies for spacecraft tracking.
Collaboration: ESA, GMV and the University of Liverpool.
Dates: November 2022 – May 2024.

Thank you for enduring me.

For any further very challenging questions please email my supervisor Manuel Sanjurjo =)



relative dist. (km)
10000
5000
0
92.5
93
93.5
94
94.5
95
95.5
96
96.5
97
time (days)
fine integration
collision
relative dist. (km)
10000
8000
6000
4000
2000
coarser integration
time (days)
relative dist. (km)
600
400
200
0
95.985
95.99
95.995
96
96.005
96.01
96.015
time (days)
fine integration
coarser integration
collision


×10^-5
Γ [-]
Δ [rad]
node
collision node
||ξ||
node crossing

Results part 2: HF validation

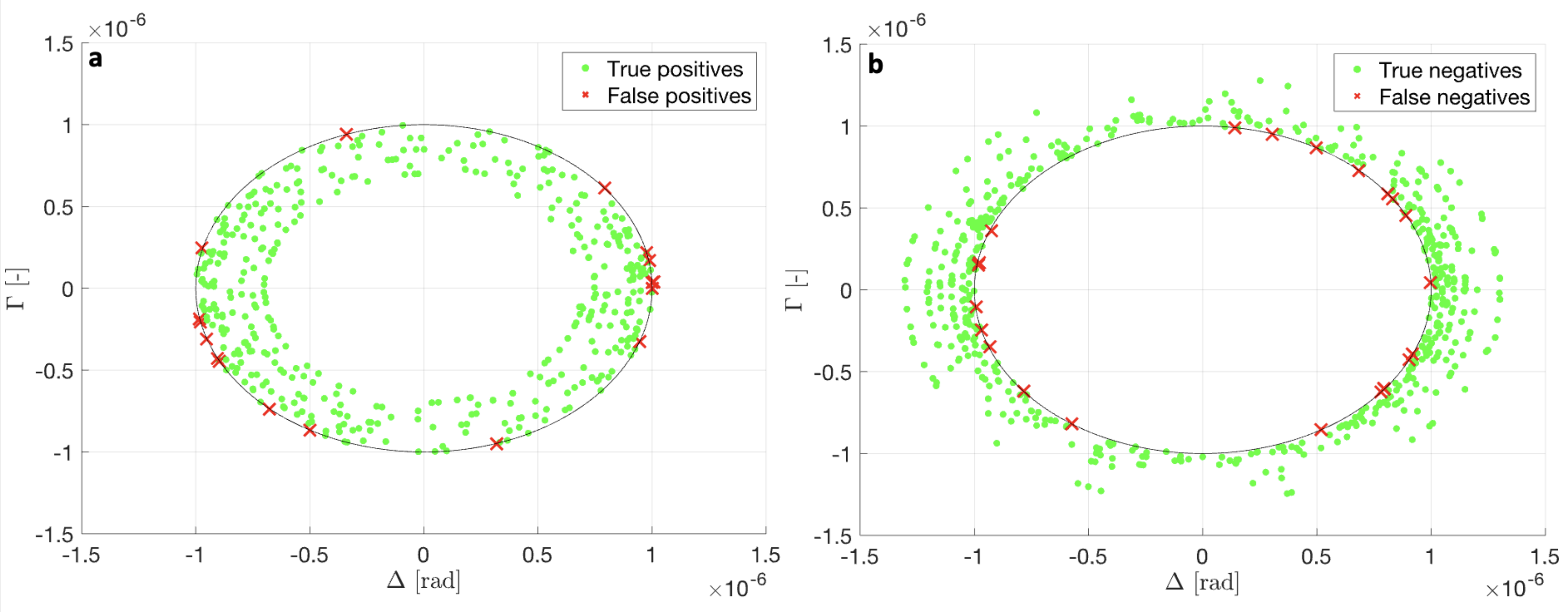


Figure 4 (above):
The IS proposal distribution in $\mathbb{R}^2$, which are evaluated in HF to detect false positives (left) and negatives (right).

Figure 5 (right):
Confusion matrix outlining the number of FP and FN.

~0.1% error

Predicted \ Actual	P	N
P	66831	17
N	22	33130