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# Development of a framework for coupled aerostructural optimisation of new generation wings

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PhD Candidate: Luca Scalia

Supervisors: prof. Andrea Cini  
prof. Rauno Cavallaro

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PhD program in Aerospace Engineering UC3M

**uc3m**  
PHD



# Motivation



The current trend in the aerospace industry is the achievement of sustainable aviation.

**Design of aerodynamically and structurally more efficient airframes: e.g., Large Aspect Ratio Wings (LARW) and Truss Braced Wings (TBW)**

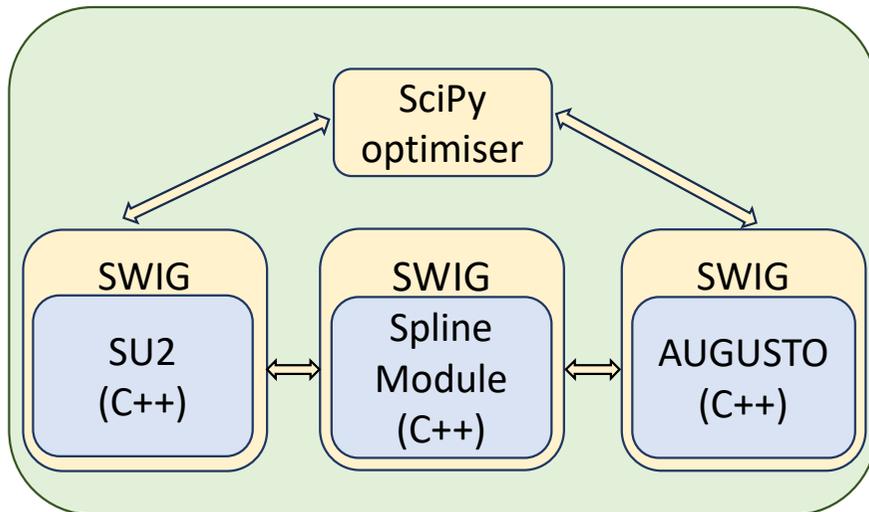


**New design concepts necessitate both appropriate and innovative strategies for design and optimisation**

*Innovative aircraft and propulsion technologies to improve air quality near airports.*  
<https://indigo-sustainableaviation.eu/>

# Objective

Devise a computational tool and procedure for efficient, high-fidelity design and optimisation of strongly coupled aerostructural systems



Main framework characteristics:

- ❑ High-fidelity modelling of a coupled aerostructural system
  - CFD
  - FEM (shell-based models) with geom. nonlinearities
- ❑ Speed & efficiency:
  - gradient-based optim. (aerodynamic & structural DVs)
  - adjoint method + algorithmic differentiation (AD) (CodiPack)
  - parallelisation for large-scale industrial problems (MPI)
- ❑ Modularity
  - easier code maintenance
  - python orchestrator for guiding analysis & optimisation phases

# State of the Art

**Haftka (1977). *Optimization of Flexible Wing Structures Subject to Strength and Induced Drag Constraints***

- Lifting-Line method for aerodynamics

**Maute et al. (2001). *Coupled analytical sensitivity analysis and optimization of three-dimensional nonlinear aeroelastic systems***

- Mesh deformation treated like an elastic solid for better handling of large structural displacements

**Kenway et al. (2014). *Aerostructural optimization of the Common Research Model configuration***

- RANS + shell-based high-fidelity FEM

**Bombardieri et al. (2021). *Aerostructural wing shape optimization assisted by algorithmic differentiation***

- Adjoint variables computed with fixed-point iterations + AD

**Adler et al. (2022). *Efficient Aerostructural Wing Optimization Considering Mission Analysis***

- Fuel burn objective computed through numerical integration of fuel flow across mission profile

**Wu et al. (2022). *Large-scale Multifidelity Aerostructural Optimization of a Transport Aircraft.***

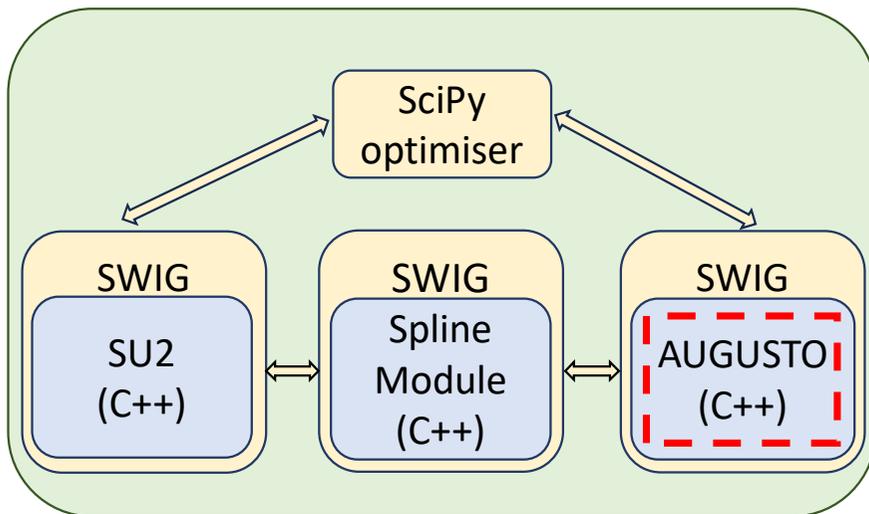
- Address large scale multifidelity aerostructural optimization + comparison with lower fidelity

# Contributions

- ❑ This PhD research builds upon and enhances an existing framework (\*)
- ❑ Greatest effort devoted to the development of the structural solver AUGUSTO

## Key contributions:

1. Development of an adjoint structural solver assisted by AD and parallel computing for size optimisation
2. Extensive study and evaluation of code performance
  - computational speed & scalability
  - memory footprint
  - sensitivity accuracy vs convergence level of primal problem
  - effect of partial AD recording



➔ Next slides focus on structural optimisation and the integration of eigenvalue problem sensitivity into the code

(\*) R. Bombardieri, R. Cavallaro, R. Sanchez, and N. R. Gauger. *Aerostructural wing shape optimization assisted by algorithmic differentiation*. *Structural and Multidisciplinary Optimization*, 64:739–760, 2021.

# The Structural Optimisation Problem

$$\min_{\alpha_s} m$$

$$\text{subject to } c_i \leq 0 \\ (i = 1, \dots, n_c)$$

Governing equations	$c_i \leq 0$
$\mathcal{S} = \mathbf{K}_{el} \mathbf{u}_s - \mathbf{f}_s = 0$	$J_\sigma = KS(g_i) = \frac{1}{\rho_{KS}} \log \sum_{i=1}^n e^{\rho_{KS} \cdot g_i} \leq 0$ (where $g_i = \frac{\sigma_i^{VM}}{\sigma_{ADM}} - 1$ )
$(\mathbf{K}_{el} - \omega_k^2 \mathbf{M}) \boldsymbol{\phi}_k = 0$	$J_\omega = \rho_\omega \frac{(\omega_j - \omega_i)_{init}}{\omega_j - \omega_i} - 1 \leq 0$
$(\mathbf{K}_{el} + \lambda_k^{cr} \mathbf{K}_g) \boldsymbol{\phi}_k^{cr} = 0$	$J_{\lambda_{cr}} = \rho_{\lambda_{cr}} \frac{(\lambda_1^{cr})_{init}}{\lambda_1^{cr}} - 1 \leq 0$

$m$ : structural mass  
 $\mathbf{u}_s$ : structural displacements  
 $\mathbf{K}_{el}$ : elastic part of the stiffness matrix  
 $\mathcal{S}$ : structural residual  
 $\mathbf{K}_g$ : geometric part of the stiffness matrix  
 $\mathbf{f}_s$ : applied external loads

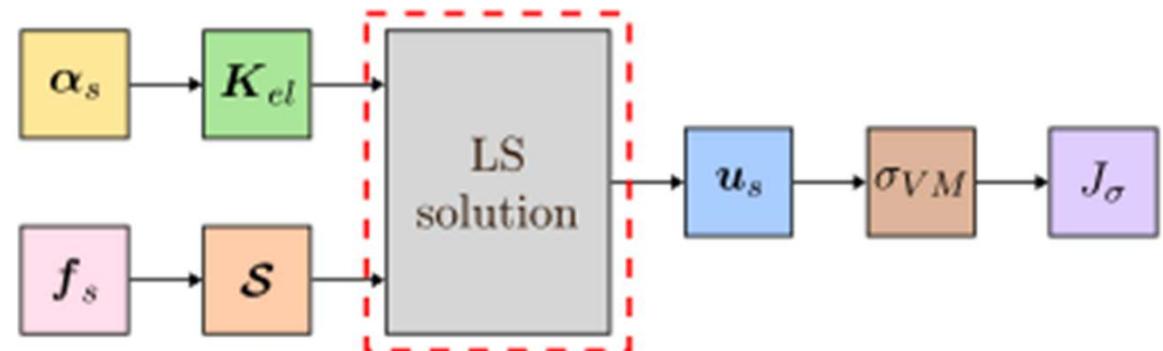
$\mathbf{M}$ : mass matrix  
 $\omega_k$ : k-th modal frequency  
 $\boldsymbol{\phi}_k$ : k-th modal eigenvector  
 $\lambda_k^{cr}$ : k-th critical eigenvalue  
 $\boldsymbol{\phi}_k^{cr}$ : k-th critical eigenvector

# Computation of gradients

After **recording with CodiPack** the computer functions that returns either  $m$ ,  $J_\sigma$ ,  $J_\omega$  or  $J_{\lambda_{cr}}$  (i.e., store the **computational graph**), compute the gradients in the following ways:

- **Mass gradient:** AD reverse mode (one run of reverse path)
- **Stress constraint gradient:** adjoint method + AD reverse mode

$$\mathbf{K}_{el} \bar{\mathbf{u}}_s = - \frac{\partial J_\sigma}{\partial \mathbf{u}_s}$$
$$\frac{dJ_\sigma}{d\alpha_s} = \frac{\partial J_\sigma}{\partial \alpha_s} + \bar{\mathbf{u}}_s^T \frac{d\mathcal{S}}{d\alpha_s}$$



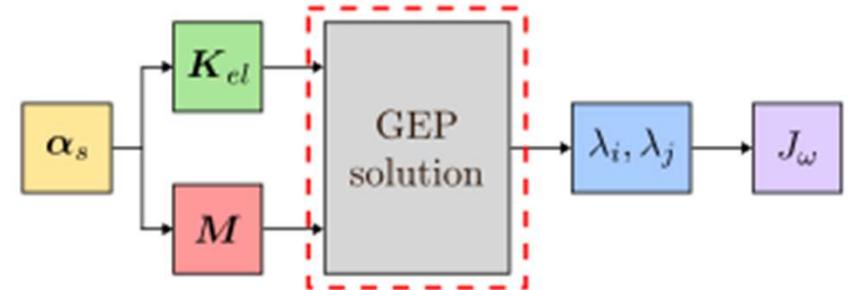
Two reverse path runs to solve:

- 1) get  $\bar{\mathbf{u}}_s$
- 2) get  $\frac{dJ_\sigma}{d\alpha_s}$

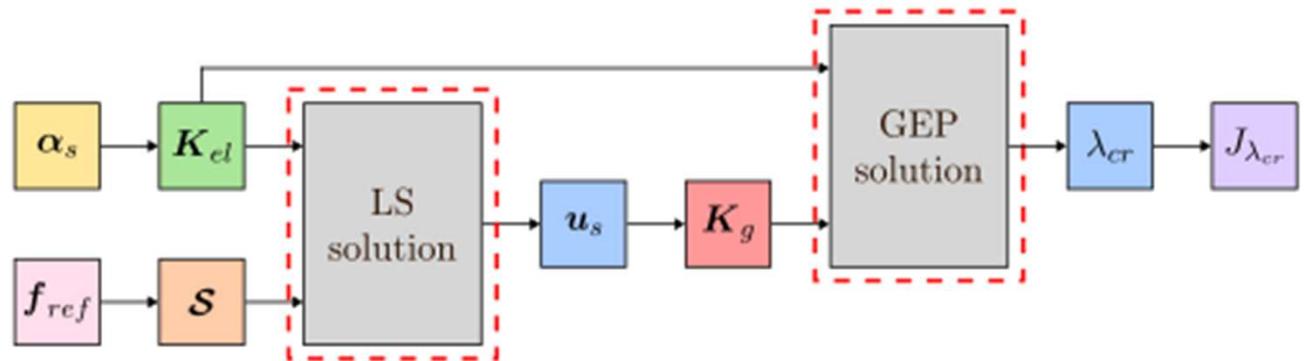
# Computation of gradients

- **Frequency & Buckling constraints:** hand-differentiated expressions of  $J_\omega$  and  $J_{\lambda_{cr}}$  + AD reverse mode

$$\frac{dJ_\omega}{d\alpha_s} = -\rho_\omega \frac{\omega_{j,init} - \omega_{i,init}}{(\omega_j - \omega_i)^2} \left( \frac{1}{2\omega_j} \frac{d\lambda_j}{d\alpha_s} - \frac{1}{2\omega_i} \frac{d\lambda_i}{d\alpha_s} \right)$$



$$\frac{dJ_{\lambda_{cr}}}{d\alpha_s} = -\rho_{\lambda_{cr}} \frac{\lambda_{cr,init}}{\lambda_{cr}^2} \frac{d\lambda_{cr}}{d\alpha_s}$$



Each term  $\frac{d\lambda_x}{d\alpha_s}$  requires one reverse path run

# Handling of Linear Systems and Eigenvalue Problems in AD

Differentiation of the solution of the linear system (LS) or generalized eigenvalue problem (GEP) is never performed:

- Efficiency issue (memory consumption)
- Impossible if solutions are handled by an external library

Procedure:

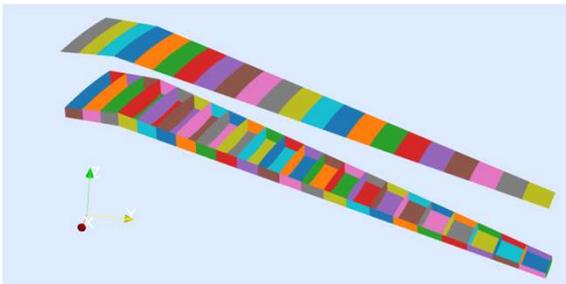
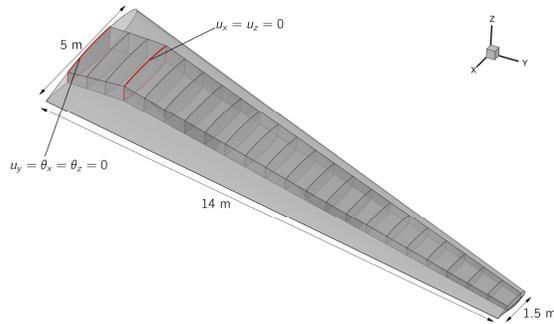
1. Interrupt computational graph recording in correspondence of LS and GEP solutions
2. Retrieve lost information by solving the adjoint statements related to the LS and GEP solutions



	Primal problem	Adjoint statement
Linear system	$K_{el} \mathbf{u}_s = \mathbf{f}_s$	$K_{el}^T \mathbf{s} = \frac{\partial J}{\partial \mathbf{u}_s}$ $\bar{K}_{el} += -\mathbf{s} \cdot \mathbf{u}_s$ $\bar{\mathbf{s}} += \mathbf{s}$
Generalised eigenvalue problem	$\mathbf{A} \boldsymbol{\phi}_k = \lambda_k \mathbf{B} \boldsymbol{\phi}_k$	$\bar{\mathbf{A}} += \boldsymbol{\phi}_k \boldsymbol{\phi}_k^T$ $\bar{\mathbf{B}} += -\lambda_k \bar{\mathbf{A}}$

# Structural optimisation – wing

## High-Fidelity Aeroelastic Optimisation Benchmark



- rib nodes loaded along the z direction; root & fus. intersection constr.
- **111 DVs** :  $\alpha_{S,ini} = 7.0$  mm;  $1.0$  mm  $\leq \alpha_{S,i} \leq 20.0$  mm
- **3 optimisation runs for 3 constraint sets:**

### 1. Stress constraint set: 5 aggregation areas

RIBS

FRONT SPAR

REAR SPAR

TOP SKIN

BOTTOM SKIN

$$\sigma_{ADM} = 320 \text{ Mpa}$$

Imposed simultaneously  
(5 constraints)

### 2. Frequency constraint

$$(\omega_2 - \omega_1) \geq \rho_\omega (\omega_2 - \omega_1)_{ini}$$

$$\rho_\omega = 1$$

$$(\omega_3 - \omega_2) \geq \rho_\omega (\omega_3 - \omega_2)_{ini}$$

$$\rho_\omega = 1$$

$$\omega_1 \geq \rho_\omega (\omega_1)_{ini}$$

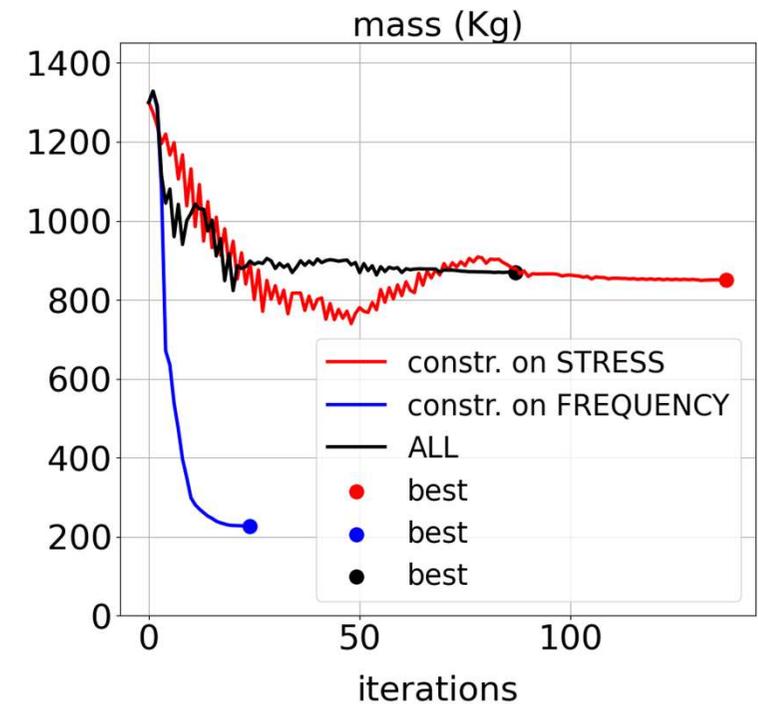
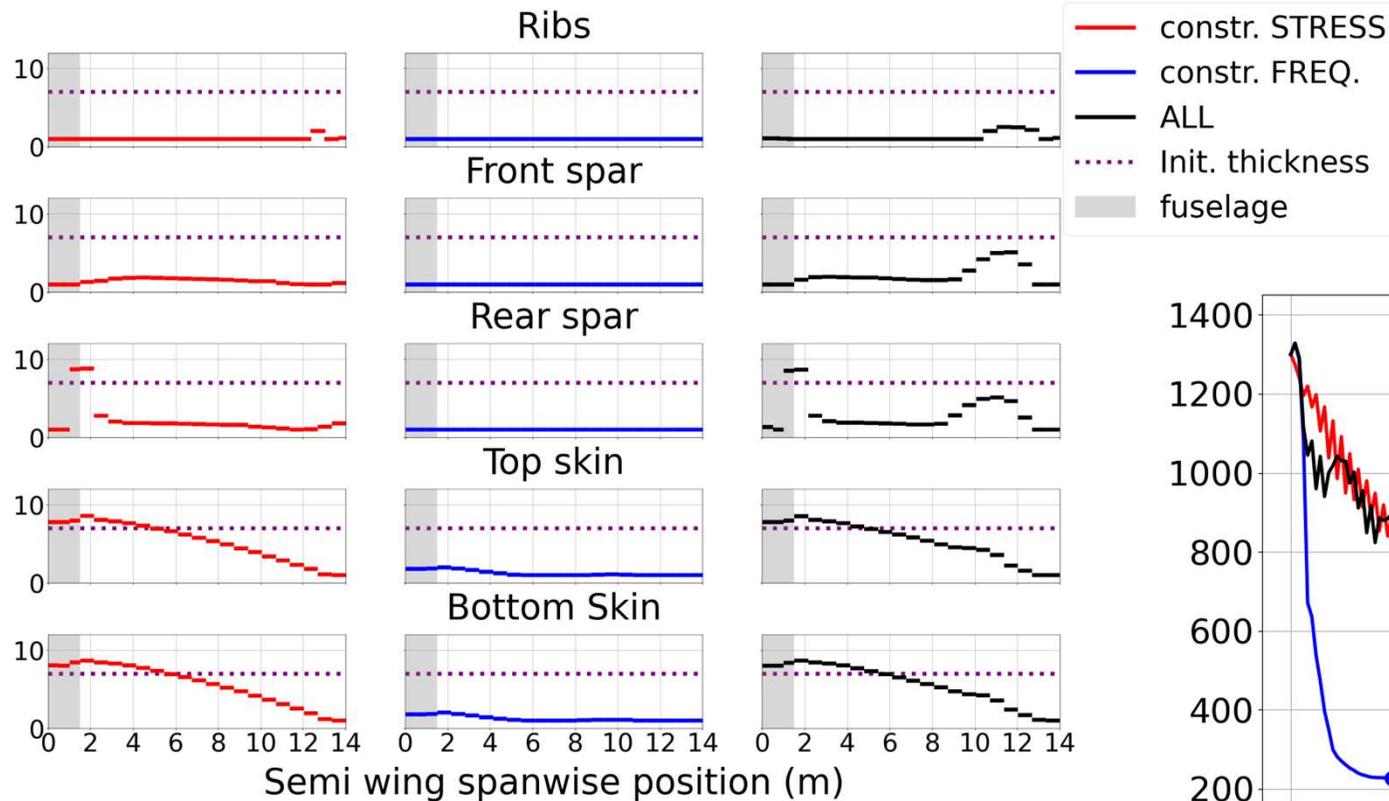
$$\rho_\omega = 1.15$$

Imposed simultaneously  
(3 constraints)

### 3. Mixed ("All") constraint set: stress + frequency constraint sets together (8 in total)

# Structural optimisation - wing - results

Optimal spanwise thickness distribution (mm)



Constraint	Stress	Frequency	all
M (Kg)	850.17	227.58	869.91
$\Delta m$ (%)	-34.58	-82.49	-33.06

# Next steps

- Publication of journal article: L. Scalia, R. Cavallaro, A. Cini. *Structural Sensitivity analysis assisted by Automatic Differentiation*.
- Coupled aerostructural optimisation with frequency and buckling constraints
- Introduction of a tracking method to detect frequencies swapping
- Introduction of composites

# Achievements

- L. Scalia, R. Cavallaro, A. Cini. *Development of a FE code for adjoint-based coupled aerostructural optimisation*. AeroBest 2023 - II ECCOMAS Thematic Conference on Multidisciplinary Design Optimization of Aerospace Systems. Instituto Superior Técnico - Lisbon. 19-21/07/2023. Portugal.
- L. Scalia, R. Cavallaro, A. Cini, N.R. Gauger, M. Sagebaum. *Efficient gradient-based structural optimisation with modal and buckling constraints assisted by algorithmic differentiation*. AeroBest 2025 - II ECCOMAS Thematic Conference on Multidisciplinary Design Optimization of Aerospace Systems. Instituto Superior Técnico - Lisbon. 22-24/04/2025. Portugal.
- Research stay at the Chair of Scientific Computing (SciComp) in University of Kaiserslautern-Landau (Germany). *Implementation of AD-based methods for sensitivity analysis of generalized eigenvalue problems. Application to modal and buckling optimisation constraints*.

Thank you for your attention!

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# References

1. R. Bombardieri, R. Cavallaro, R. Sanchez, and N. R. Gauger. *Aerostructural wing shape optimization assisted by algorithmic differentiation*. *Structural and Multidisciplinary Optimization*, 64:739–760, 2021.
2. M. Pini, S. Vitale, P. Colonna, G. Gori, A. Guardone, T. Economon, J. Alonso, and F. Palacios. Su2: the open-source software for non-ideal compressible flows. *Journal of Physics: Conference Series*, 821(1):012013, 2017.
3. R. Cavallaro, A. Iannelli, L. Demasi, and A. M. Razon. Phenomenology of nonlinear aeroelastic responses of highly deformable Joined Wings. *Advances in Aircraft and Spacecraft Science*, 2(2):125–168, April 2015.
4. T. Albring, M. Sagebaum, and N. R. Gauger. Development of a consistent discrete adjoint solver in an evolving aerodynamic design framework. 16th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference, 2015. ISBN 978-1-62410-368-1.
5. M. Schwalbach, T. Verstraete, N.R. Gauger. Discrete adjoint gradient evaluations for linear stress and vibration analysis. *Computing and Visualization in Science* (2019) 21:23-31
6. L. Scalia, A. Cini, R. Cavallaro. Development of a FE code for adjoint-based coupled aerostructural optimisation. In *AeroBest 2023 II ECCOMAS Thematic Conference on Multidisciplinary Design Optimisation of Aerospace Systems*. Lisbon. 19-21 July 2023, Portugal.

Spare slides

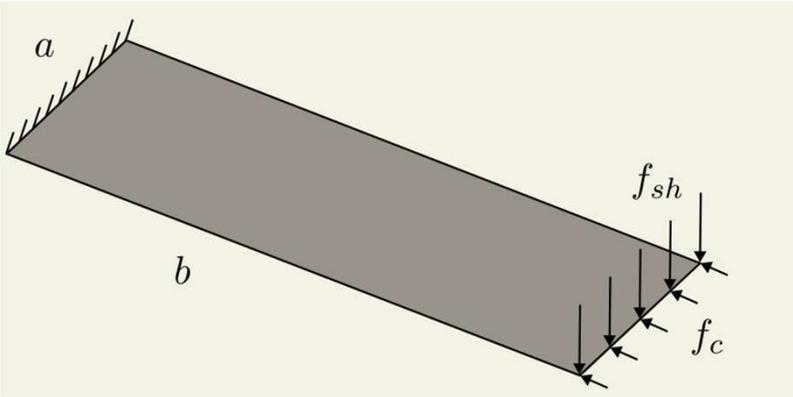
# Agenda

1. Motivation
2. Objective & overview of the framework
3. State of the Art
4. Contributions
5. The structural optimisation problem
6. Gradients computation
7. Optimisation results
8. Conclusions

# Conclusions

- extension of the FE-solver capability to tackle modal and buckling analysis
- extension of the FE-solver capability to compute gradients of natural frequencies and critical loads
- assessment of the capability in structural optimisation testcases considering a single constraint or a combination of them

# Structural optimisation – plate



- **8 DVs:**  $\alpha_{s,0} = 5.0 \text{ mm}$ ;  $1.0 \text{ mm} \leq \alpha_{s,i} \leq 10.0 \text{ mm}$

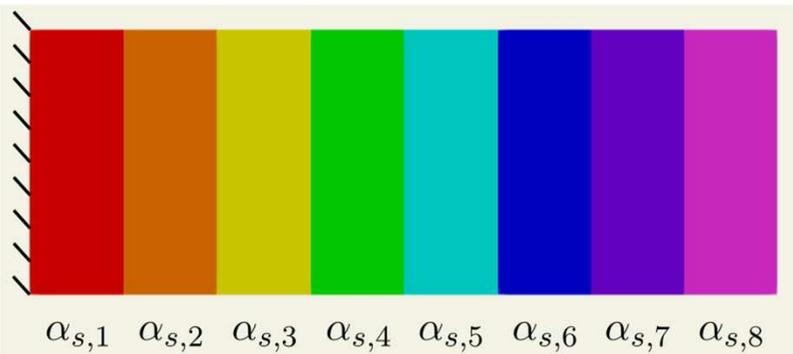
- **4 optimisation runs for 4 constraint sets:**

**1. Stress:**  $\sigma_{VM}$  aggregated on all elements;  $\sigma_{ADM} = 270 \text{ MPa}$

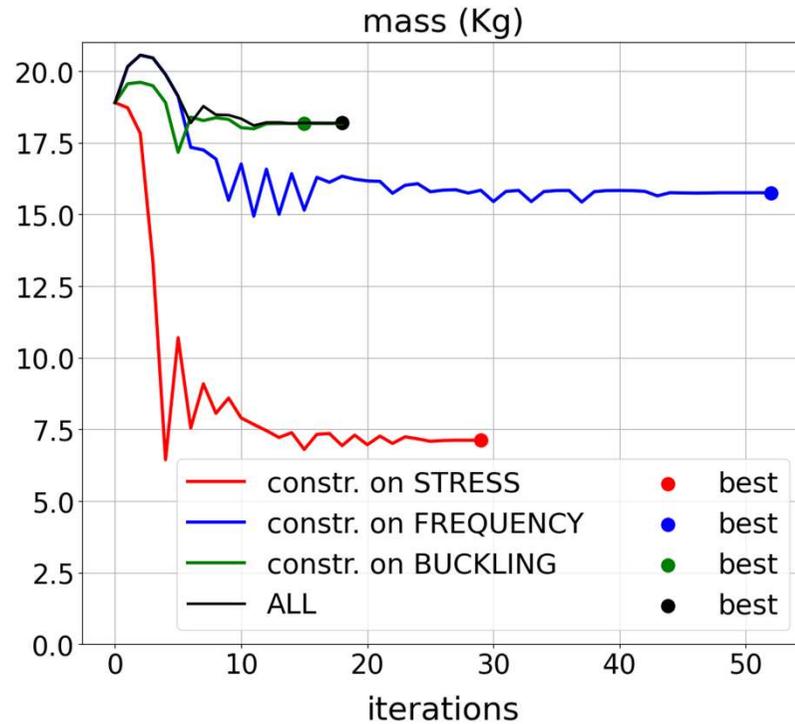
**2. Frequency:**  $(\omega_2 - \omega_1) \geq 1.2 * (\omega_2 - \omega_1)_{ini}$

**3. Buckling:**  $\lambda_1^{cr} \geq 1.2 * (\lambda_1^{cr})_{init}$

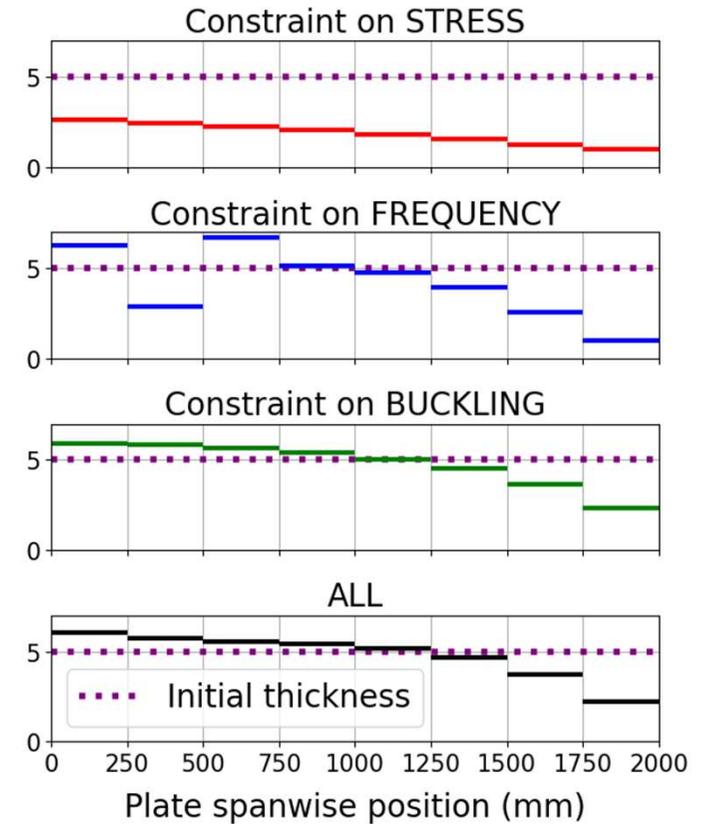
**4. All:** stress + frequency + buckling together



# Structural optimisation - plate- results



Optimal thickness distribution (mm)



Constraint	Stress	Frequency	Buckling	all
<b>M (Kg)</b>	7.12	15.76	18.17	18.19
<b><math>\Delta m</math> (%)</b>	-62.33	-16.61	-3.86	-3.76

# Structural optimisation - plate- constraints

Active constraints				
	Constr. on STRESS	Constr. on FREQUENCY	Constr. on BUCKLING	All
$J_{\sigma}$	Green	Grey	Grey	Red
$J_{\omega}$	Grey	Green	Grey	Green
$J_{\lambda_{cr}}$	Grey	Grey	Green	Green

# Structural optimisation – wing - constraints

Active constraints			
	Constr. on STRESS	Constr. on FREQUENCY	All
$J_{\sigma}^{RIBS}$	Green	Grey	Green
$J_{\sigma}^{RSPAR}$	Green	Grey	Green
$J_{\sigma}^{FSPAR}$	Green	Grey	Green
$J_{\sigma}^{TSKIN}$	Green	Grey	Green
$J_{\sigma}^{BSKIN}$	Green	Grey	Green
$J_{\omega}^{(\omega_1)}$	Grey	Green	Red
$J_{\omega}^{(\omega_2 - \omega_1)}$	Grey	Red	Red
$J_{\omega}^{(\omega_3 - \omega_2)}$	Grey	Green	Green

# State of the Art

**Haftka R. (1977).** *Optimization of Flexible Wing Structures Subject to Strength and Induced Drag Constraints.*

**Maute K. , Nikbay M. , Farhat C. (2001).** *Coupled analytical sensitivity analysis and optimization of three-dimensional nonlinear aeroelastic systems.*

**Kenway G., Martins J., Kennedy J. (2014).** *Aerostructural optimization of the Common Research Model configuration.*

**Sanchez R., Albring T., Palacios R., Gauger N. R. , Economon T. D., J. Alonso (2018).** *Coupled adjoint-based sensitivities in large-displacement fluid-structure interaction using algorithmic differentiation.*

**Bombardieri R., Cavallaro R., Sanchez R., N. R. Gauger (2021).** *Aerostructural wing shape optimization assisted by algorithmic differentiation.*

**Adler E., Martins J. (2022).** *Efficient Aerostructural Wing Optimization Considering Mission Analysis.*

**Wu N., C. Mader, J. Martins (2022).** *Large-scale Multifidelity Aerostructural Optimization of a Transport Aircraft.*

# The Finite Element solver - AUGUSTO

- ❑ Written in C++ and based on *object-oriented* programming
- ❑ Parallel implementation through MPI library
- ❑ Available finite elements:
  - 4-nodes bilinear isoparametric shell element (linear and non-linear)
  - 3-nodes CST
  - 2-nodes linear isoparametric Timoshenko beam element
  - Euler - Bernoulli beam with hermite interpolation
  - Rigid RBE2 element
- ❑ Differentiated with library CodiPack to compute gradients of responses
- ❑ Wrapped with SWIG for use in a python environment

# The Aerostructural optimisation problem

The aerostructural optimisation problem can be formulated as [4,5]:

$$\min_{\alpha} J(\mathbf{y}(\alpha), \alpha)$$

$$s. t: \quad \mathbf{G}(\mathbf{y}(\alpha), \alpha) = \mathbf{0} \quad \text{field equations}$$

$$\mathbf{g}(\mathbf{y}(\alpha), \alpha) \leq \mathbf{0} \quad \text{optimisation constraints}$$

$$\mathbf{G}(\mathbf{y}(\alpha), \alpha) = \begin{cases} \mathbf{F}(\mathbf{w}, \mathbf{z}) - \mathbf{w} = 0 \\ \mathbf{F}_f(\mathbf{w}, \mathbf{z}) - \mathbf{f}_f = 0 \\ \mathbf{M}(\mathbf{u}_{tot}) - \mathbf{z} = 0 \\ \mathbf{H}_{MLS}^T \mathbf{f}_f - \mathbf{f}_s = 0 \\ \mathbf{K}(\alpha_s) \mathbf{u}_s - \mathbf{f}_s = 0 \\ \mathbf{H}_{MLS} \mathbf{u}_s - \mathbf{u}_f = 0 \\ \mathbf{u}_{tot} - \mathbf{u}_f - \mathbf{u}_{F\alpha} = 0 \end{cases}$$

## State variables ( $\mathbf{y}$ )

$\mathbf{w}$ : aerodynamic state variables

$\mathbf{z}$ : fluid mesh nodes displacements

$\mathbf{u}_s$ : displacements defined on the structural mesh

$\mathbf{u}_f$ : displacements defined on the fluid mesh boundary

$\mathbf{f}_s$ : loads defined on the structural mesh

$\mathbf{f}_f$ : loads defined on the fluid mesh boundary

$\mathbf{u}_{tot}$ : cumulative displacement of the wing surface

## Design variables ( $\alpha$ )

$\alpha_s$ : structural (e. g. , wing panels thickness)

$\alpha_a$ : aerodynamic ( $\mathbf{u}_{F\alpha}$ : variation of the jig-shape)

# Discrete adjoint method (1)

The problem is reformulated by introducing the Lagrangian function  $\mathcal{L}$  and the adjoint state variables  $\bar{\mathbf{y}}$

$$\min_{\alpha} \mathcal{L}(\mathbf{y}, \bar{\mathbf{y}}, \alpha) = J(\mathbf{y}, \alpha) + \bar{\mathbf{y}}^T \mathbf{G}(\mathbf{y}, \alpha)$$

The Karush–Kuhn–Tucker (KKT) conditions are imposed by taking the first derivative of the Lagrangian with respect to  $\bar{\mathbf{y}}$ ,  $\mathbf{y}$ , and  $\alpha$ . The equivalence to zero holds true at the saddle point of the Lagrangian, which corresponds to the optimum of the optimisation problem

$$\frac{\partial \mathcal{L}}{\partial \bar{\mathbf{y}}} = \mathbf{0} \rightarrow \text{field equations}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{y}} = \mathbf{0} \rightarrow \text{adjoint equations}$$

$$\frac{\partial \mathcal{L}}{\partial \alpha} = \mathbf{0} \rightarrow \text{sensitivity equations}$$

# Discrete adjoint method (2)

## adjoint equations

$$\frac{\partial J}{\partial \mathbf{w}} + \bar{\mathbf{w}}_n^T \frac{\partial F}{\partial \mathbf{w}} + \bar{\mathbf{f}}_f^T \frac{\partial F_f}{\partial \mathbf{w}} = \bar{\mathbf{w}}_{n+1}^T$$

$$\frac{\partial J}{\partial \mathbf{z}} + \bar{\mathbf{w}}^T \frac{\partial F}{\partial \mathbf{z}} + \bar{\mathbf{f}}_f^T \frac{\partial F_f}{\partial \mathbf{z}} + \bar{\mathbf{z}}^T = \mathbf{0}^T$$

$$\frac{\partial J}{\partial \mathbf{u}_{tot}} + \bar{\mathbf{z}}^T \frac{\partial M}{\partial \mathbf{u}_{tot}} + \bar{\mathbf{u}}_{tot}^T = \mathbf{0}^T$$

$$\frac{\partial J}{\partial \mathbf{u}_f} - \bar{\mathbf{u}}_f^T - \bar{\mathbf{u}}_{tot}^T = \mathbf{0}^T$$

$$\mathbf{K}^T \bar{\mathbf{u}}_s = -\frac{\partial J}{\partial \mathbf{u}_s} - \mathbf{H}_{MLS}^T \bar{\mathbf{u}}_f$$

$$\frac{\partial J}{\partial \mathbf{f}_s} - \bar{\mathbf{u}}_s^T - \bar{\mathbf{f}}_s^T = \mathbf{0}^T$$

$$\frac{\partial J}{\partial \mathbf{f}_f} - \bar{\mathbf{f}}_f^T + \bar{\mathbf{f}}_s^T \mathbf{H}_{MLS}^T = \mathbf{0}^T$$

red: input from prev. block

green: output of current block

$iter_{adj} += 1$

- for any adjoint iteration, solve the adjoint equations in cascade
- first equation is solved in a fixed-point iteration style
- terms in the form  $\bar{\mathbf{y}} \frac{\partial A}{\partial x}$  are computed with the AD tool CoDiPack
- each module applies AD independently

## sensitivity equations

$$\frac{dJ}{d\mathbf{u}_{F\alpha}} = \frac{\partial J}{\partial \mathbf{u}_{F\alpha}} - \bar{\mathbf{u}}_{tot}^T$$

$$\frac{dJ}{d\alpha_s} = \frac{\partial J}{\partial \alpha_s} - \bar{\mathbf{u}}_s^T \frac{dS}{d\alpha_s}$$

# Discrete adjoint method (3)

The overall procedure is:

1. solve the primal problem to get the solution at aerostructural equilibrium ( $\mathbf{y}^*$ )
2. start CoDiPack taping and register the inputs
3. do an extra primal problem iteration allowing CoDiPack to register the computational path
4. stop CoDiPack taping and register outputs
5. solve the adjoint equations
6. evaluate sensitivities through the sensitivity equations

# Parallelisation strategy

## Primal problem

$$K\mathbf{u}_s = \mathbf{f}_s$$

- ❑ distribution of FEs and nodes among elements
- ❑ solution of linear system (MUMPS library)
- ❑ collective communication between nodes is needed for
  - assigning loads on shared nodes
  - checks on RBE2 elements

## Adjoint problem

$$K^T \bar{\mathbf{u}}_s = -\frac{\partial J}{\partial \mathbf{u}_s} - \cancel{H_{MLS}^T \bar{\mathbf{u}}_f}$$
$$\frac{dJ}{d\alpha_s} = \frac{\partial J}{\partial \alpha_s} - \bar{\mathbf{u}}_s^T \frac{d\mathcal{S}}{d\alpha_s}$$

- ❑ input registration
  - design variables  $\alpha_s$  registered by all processes
  - state variables: each process registers its allocated dofs
- ❑ output registration
  - response  $J$  : registered by one process only
  - Residual  $\mathcal{S}$  : each process registers its own components

# Framework overview

- ❑ CFD solver: SU2 [2]
  - C++ core, wrapped with SWIG
  - HPC capabilities
  - Automatic Differentiation through CoDiPack
- ❑ FE solver: AUGUSTO++
  - C++ core, wrapped with SWIG
  - parallel capability through MPI
  - Automatic Differentiation through CoDiPack
- ❑ Spline module [3]
  - C++ core, wrapped with SWIG
  - connects non-matching fluid/solid meshes
  - based on RBF and ANN library
- ❑ Python
  - orchestrator for guiding analysis & optimisation phases
  - optimisation algorithm (currently SciPy's SLSQP)

