PhD Aerospace Engineering UC3M
Doctoral Meetings 2026

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GPU-Accelerated Hybridizable Discontinuous Galerkin Solver

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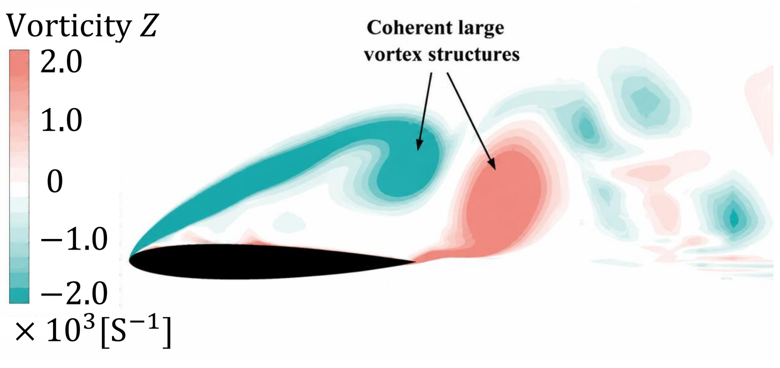
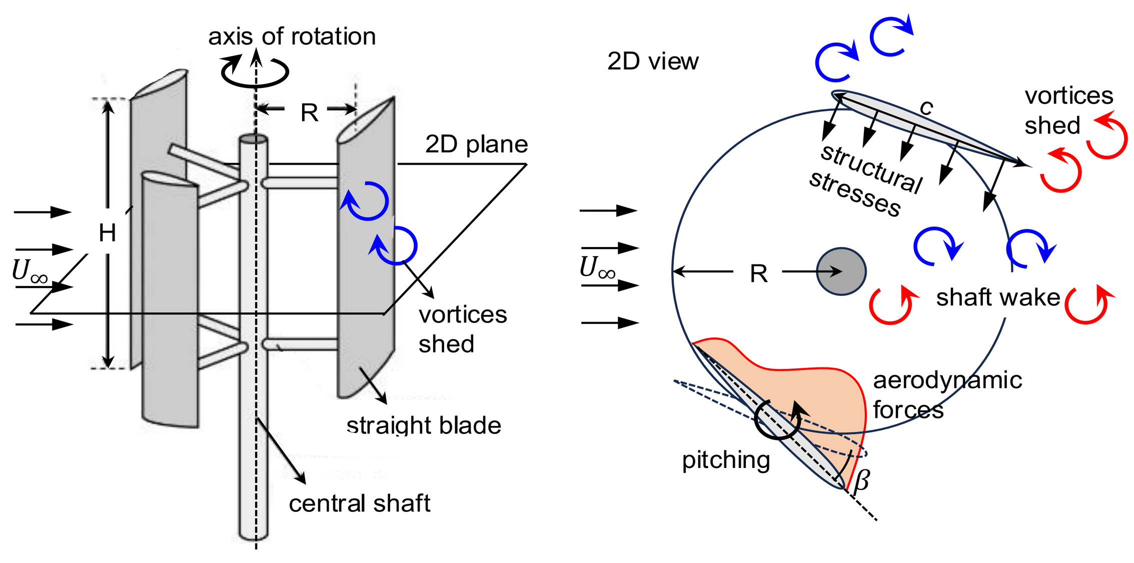
Motivation

To develop a GPU accelerated high order in-house fluid solver based on Hybridizable Discontinuous Galerkin (HDG) spatial discretization.

Use the solver to study the flow around **Vertical Axis Wind Turbine** (VAWT).

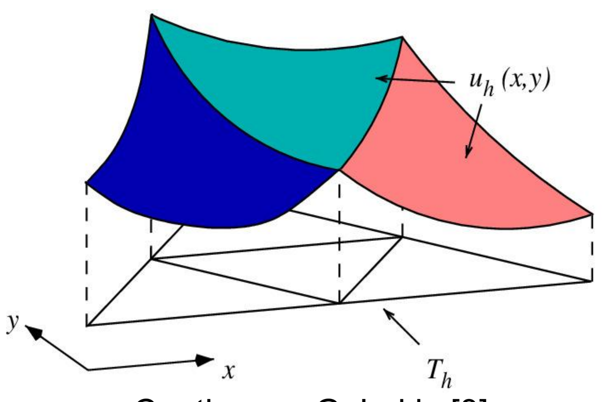


Spanwise vorticity field, adapted from [1]

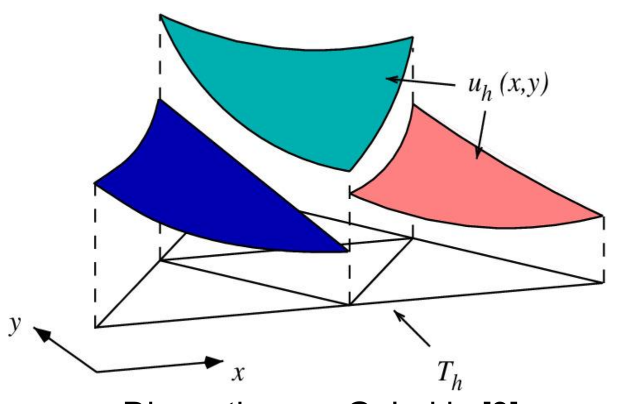


Schematic of an H-Darrieus VAWT, adapted from [2]

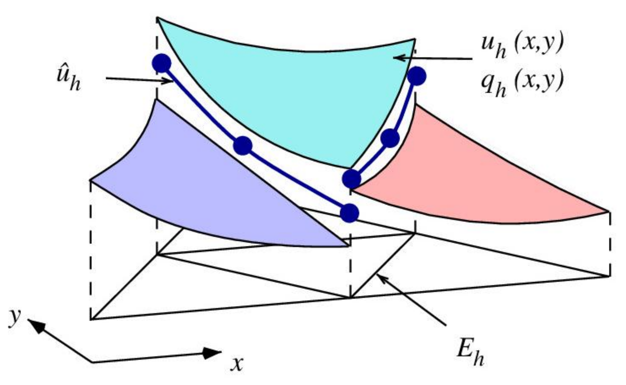
State of the Art



Continuous Galerkin [3]

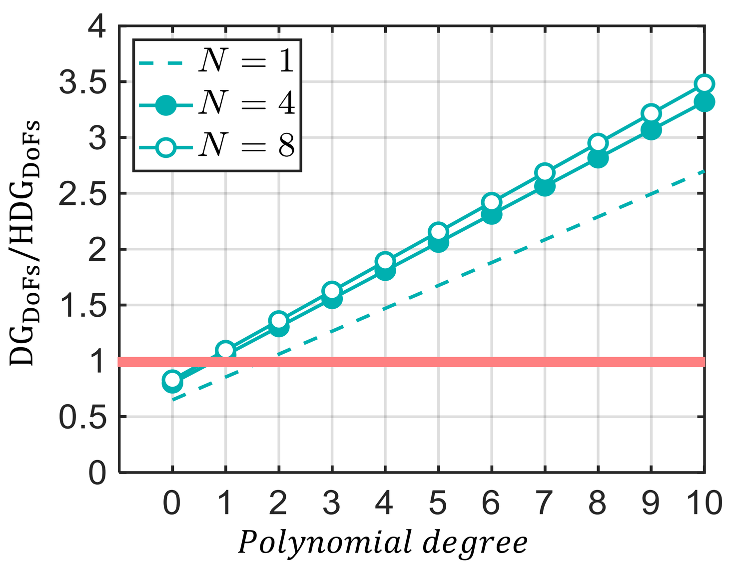


Discontinuous Galerkin [3]



Hybridized Discontinuous Galerkin [3]

For linear elasticity problem solved in a N×N×N cube domain, based on [4]



Solver	Method	
NGSolve [5]	DG/FEM	High-order finite elements
Exasim [6]	DG/HDG	High-performance PDE simulations
HORSES 3D [7]	DG	High-order compressible CFD
OpenFOAM [8]	FVM	Open-source CFD framework

Why HDG?

- High-order accuracy
- Discontinuous basis functions
- Reduced globally coupled DOFs
- Local heavy computations
- GPU suitability

Objectives

Develop a Python-based GPU HDG framework with progressively increasing complexity

Apr. 2026

1st year 2026	2nd year 2027	3rd year 2028
Poisson → Navier Stokes Steady → unsteady	Turbulence model (Spalart–Allmaras)	Apply the framework to VAWT aerodynamic simulations

Benchmark, validate, and compare against existing solvers

Methodology: HDG for Poisson

Governing PDE

$$-\nabla\cdot(\nu\nabla u) = f \quad \text{in } \Omega$$

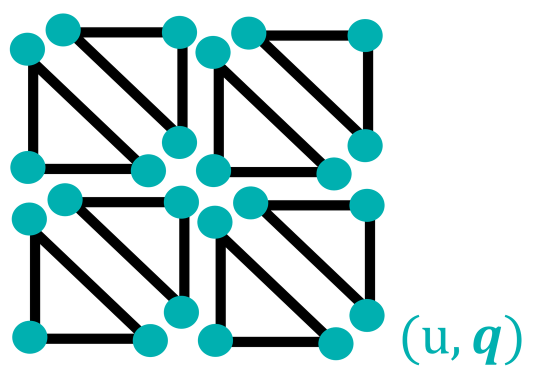
HDG reformulation:

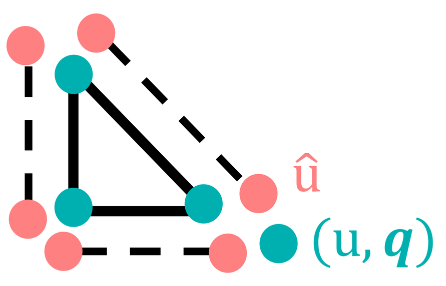
$$\mathbf{q} + \nu\nabla u = 0, \quad \nabla\cdot\mathbf{q} = f \quad in\ K_i$$

Local Weak problem on each element K_i:

$$\int_{K_i} v\,\nabla\cdot\mathbf{q}\,dV + \int_{\partial K_i} v\tau v\,(u-\hat{u})\,dS = \int_{K_i} v\,f\,dV$$

$$\int_{K_i} \mathbf{q}\cdot w\,dV - \int_{K_i} \nu u\,\nabla\cdot w\,dV + \int_{\partial K_i} \nu\hat{u}\; w\cdot n\,dS = 0$$





After discretization, the local problem becomes:

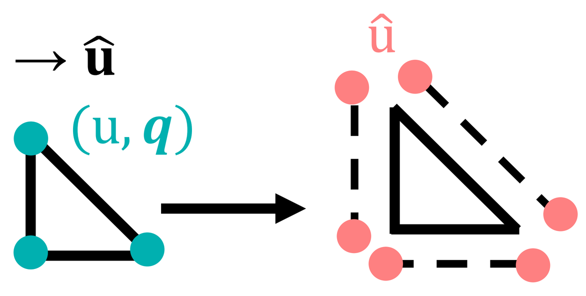
$$A_{uu}^{K_i} u^i + A_{uq}^{K_i}\mathbf{q}^i + A_{u\hat{u}}^{K_i}\hat{u} = f_u^{K_i}$$

$$A_{qu}^{K_i} u^i + A_{qq}^{K_i}\mathbf{q}^i + A_{q\hat{u}}^{K_i}\hat{u} = 0$$

Independent local element solvers: (u,q) → û

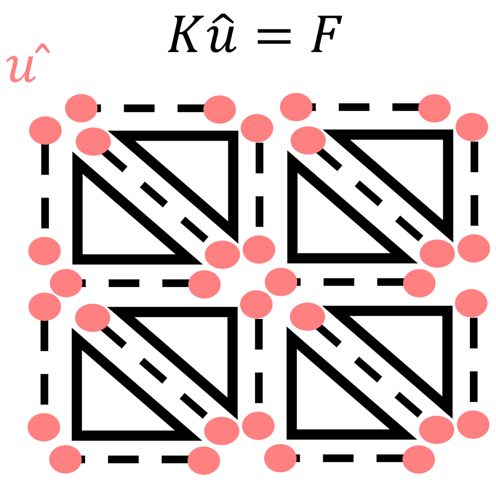
$$u^i = U^{K_i}\hat{u} + f_U^{K_i}$$

$$\mathbf{q}^i = Q^{K_i}\hat{u} + f_Q^{K_i}$$



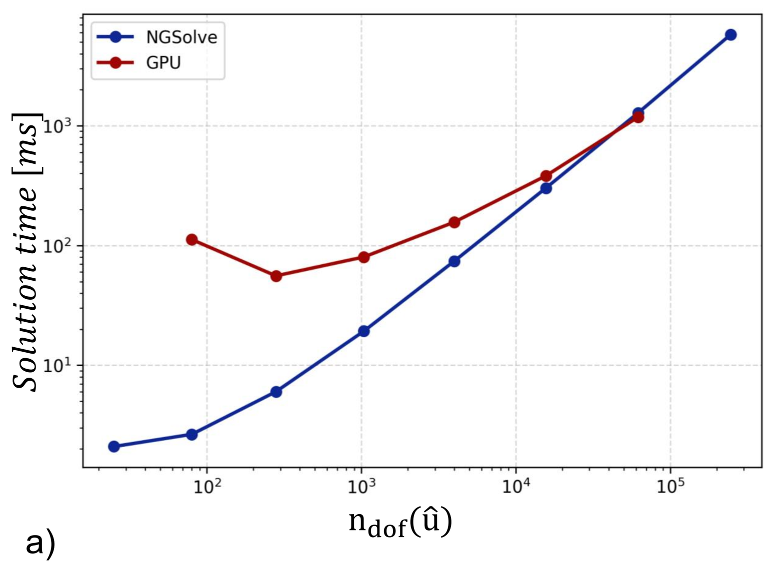
HDG Global solver: only trace system

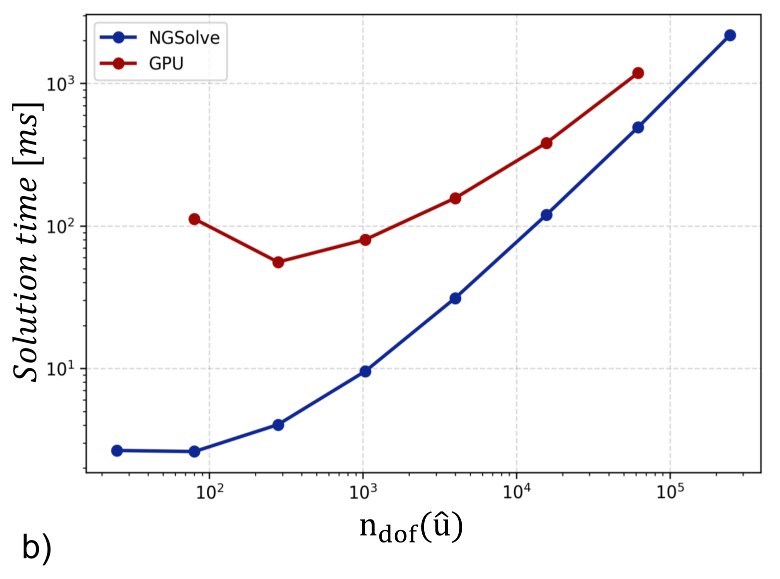
$$K\hat{u} = F$$



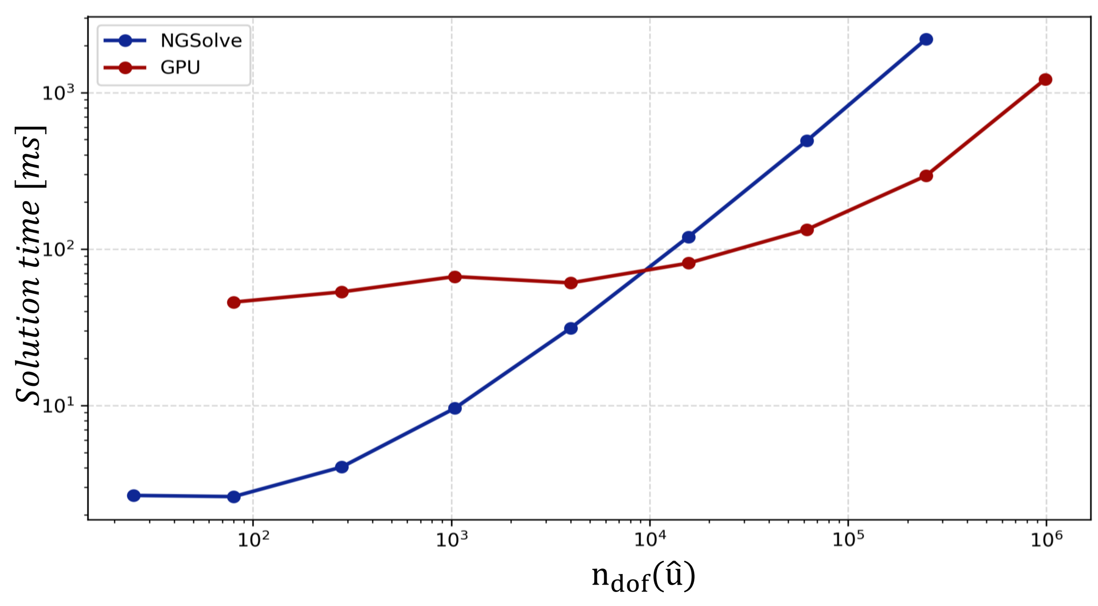
Compared to DG, HDG **reduces the globally coupled sparse system** to trace unknowns on the mesh skeleton. With **higher polynomial order** and **finer meshes**, computations become **increasingly local**, making HDG highly suitable for GPU architectures.

Preliminary Results





Initial GPU implementation compared with the NGSolve using, (a) 1 CPU core and (b) 4 CPU cores [9].



Improved GPU implementation compared with NGSolve using 4 CPU cores [9].

- **Increasing parallel CPU** cores showed **about 10% improvement** for NGSolve.
- Limited performance of the initial GPU implementation.

Improved by →

- Optimized of matrix assembly
- Reduced CPU-GPU data transfer
- Coalesced memory access

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