

Real-time Optimal Initialization and Maintenance of Spacecraft Formation Flying

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Motivation

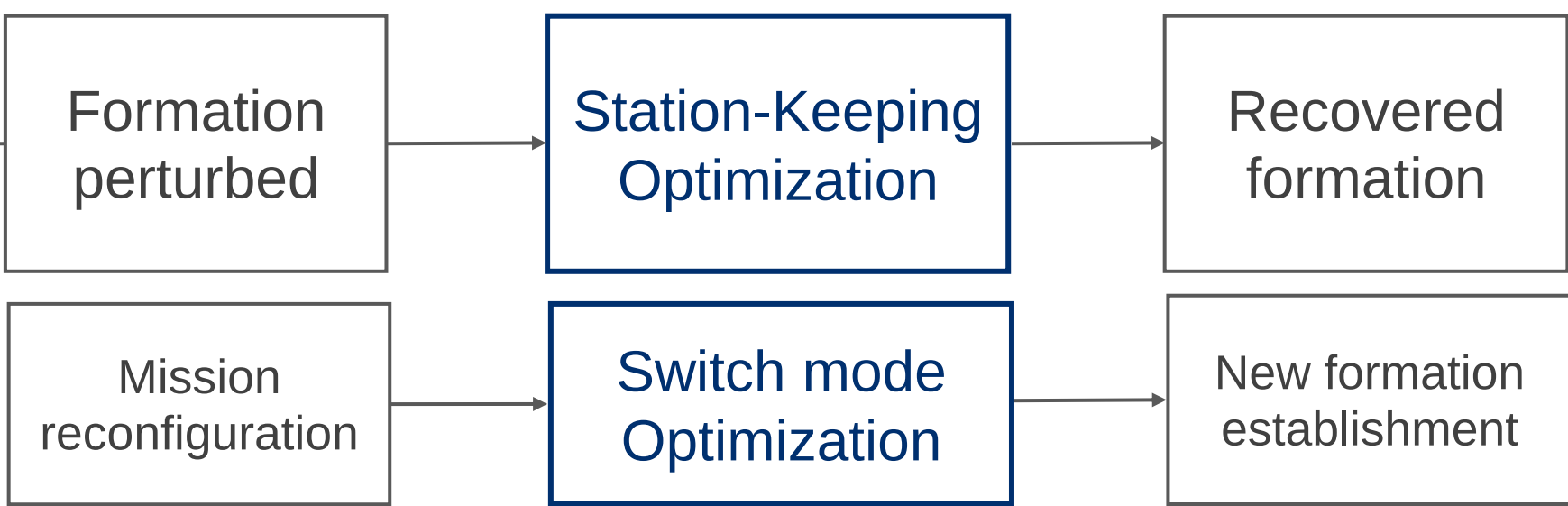
- The shift to distributed systems:** Emerging missions (interferometry, in-orbit servicing, Earth observation, asteroid formation flying) demand **multi-agent formations**, rendering traditional ground-in-the-loop control unscalable and cost-prohibitive.
- The need for autonomy:** Managing a distributed fleet under strict relative-motion constraints and dynamic perturbations drives system complexity, demanding **onboard autonomy** for safe close-proximity operations.
- Optimizing Guidance & Control:** We focus on guidance and control optimization for formation initialization and maintenance, enabling spacecraft to compute precise, **fuel(time)-optimal, collision-free trajectories** autonomously in real-time.

Formation Flying

Initialization

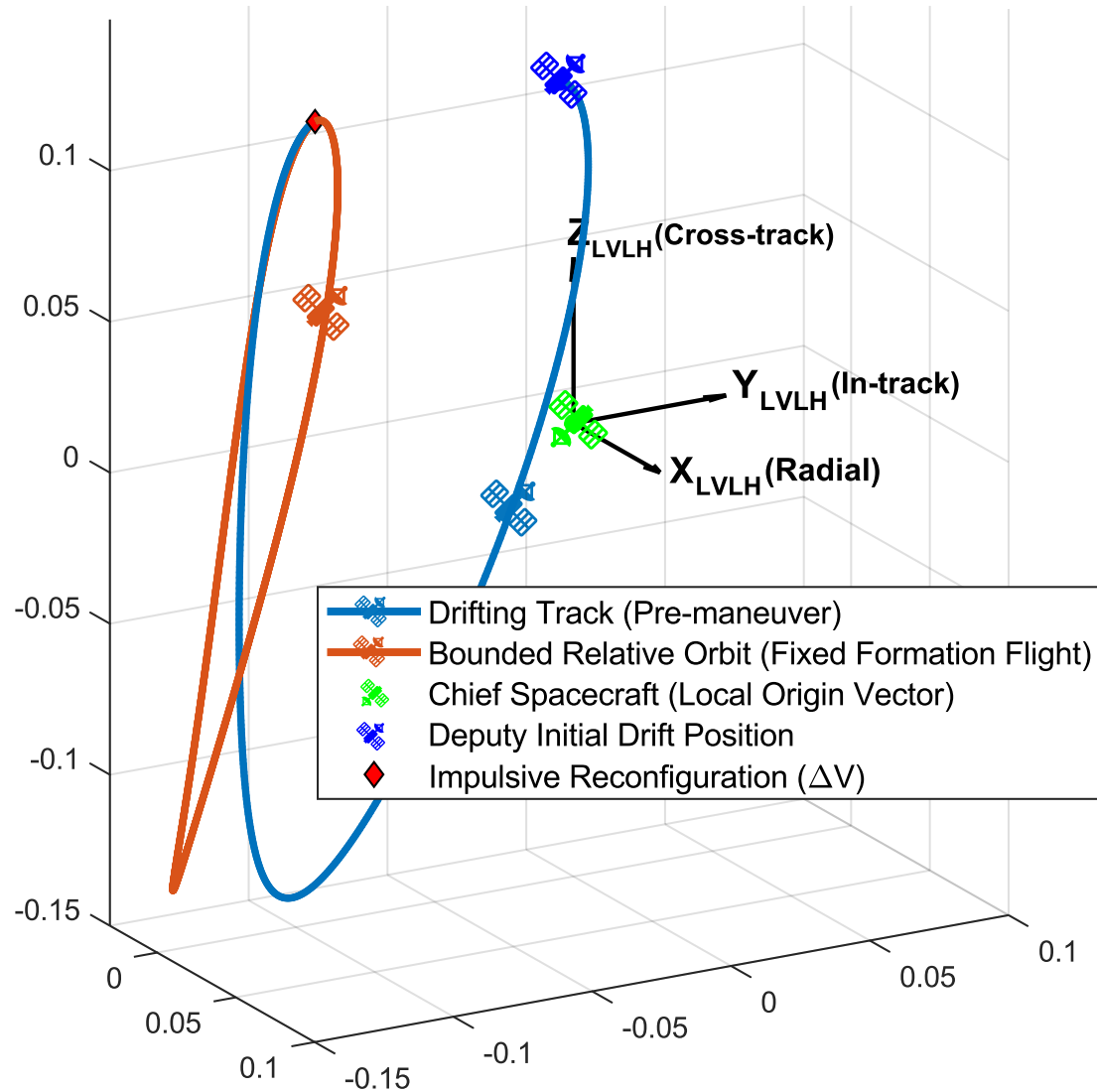


Maintenance & Reconfiguration



Problem Equivalence: Solved as a relative trajectory optimization, proximity rendezvous, and docking problem within the local frame.

- Active G&C:** Optimal **multi-impulse or continuous maneuvers** for station-keeping and reconfiguration. Formation flying demands guarantees of safety and/or convergence to the desired equilibrium.
- Distributed motion planning:** Decentralized optimization algorithms guaranteeing **constraint satisfaction** and formation requirements independently of full network state knowledge.
- Passive safety:** Designing **passively stable relative orbits** that prevent collisions (e.g., via eccentric/inclination vector separation or deploying differential drag surfaces).
- Optimization Constraints:** Enforcing geometric collision avoidance (keep-out zones), hardware thruster saturation, payload pointing restrictions, and proximity plume impingement.



State of the Art

Dynamics models [1]

- Cartesian state vector:** Linearization of exact non-linear equations into the Hill-Clohessy-Wiltshire eqs. for near-circular orbits or the Tschauner-Hempel eqs. for eccentric orbits. State Transition Matrice (STM) solutions for perturbed or eccentric orbits. **Simple, fast convergence for short-term control.** **Accuracy degrades with time and large relative distances.**
- Relative Orbital Elements:** Non-linear differences in orbital elements for advanced analytical propagation. Closed-form STM with perturbations for long-term modeling. **Valid for large separations and eccentric orbits; excellent geometric insight for formation design.** **Complex Cartesian mapping for sensors, equatorial singularities and linearization errors.**
- Mean Orbital Elements:** Filters out short-period periodic oscillations to isolate long-term secular drift and evolution, modeled through Gauss's Variational Equations. STMs solutions for perturbed and/or eccentric orbits. **Large step sizes and high stability for long-duration convergence.** **Unsuitable for precise, instantaneous collision avoidance or high-frequency control.**

G&C Strategies

- Relative Orbital Elements [2]:** Framework for guidance strategies and impulsive maneuvering, as well as passive safety and collision avoidance. **Less effective to manage position constraints through the optimization process.**
- Indirect methods: Calculus of variations** to convert the Optimal Control Problem into a Two-Point Boundary Value Problem (TPVBP). Necessary conditions for optimal guidance and control. **Initial guess sensitivity, convergence not guaranteed.**
- Model Predictive Control:** Relies on dynamics models of the process, then repeatedly optimizes over a **receding forward-looking time horizon**. Real-time enforcement of constraints and perturbations. **High computation in nonlinear cases.**
- G&C Neural Networks [3]:** Offline learning, enabling real-time onboard execution. Backward Generation of Optimal Examples to generate a huge **optimal state-G&C commands dataset**. **"Black box" lacking safety and stability guarantees.**
- Convex Programming:** Reformulates non-linear trajectory optimization into convex subproblems. Fast convergence for real-time onboard execution. **Linearization gaps and artificial infeasibility.**
- Dynamic Programming:** Bellman Principle of Optimality. Multi-stage subproblems to systematically compute globally optimal policies. **Complex implementation, relaxation needed for high-dimensional state spaces.**
- Control Lyapunov Functions / Control Barrier Functions:** CLF provide certifiably stable optimal guidance, while CBF enforce forward invariance of safe sets with minimal control (safety filter). **Difficult to build proper functions for complex systems.**

Preliminary Research

The LISA mission consists of three spacecraft flying in a massive, **Earth-leading/trailing orbit** on a **equilateral triangle formation**. The objective is to achieve a naturally stable compliant geometry for 10 years (science phase), relying solely on a drag-free attitude control system.

Formation Optimization
Optimizes the cartwheel orbital elements to achieve a stable formation that requires zero orbit control throughout the entire mission duration [4].

Transfer Optimization
Optimizes the interplanetary trajectory from Earth direct escape launch to the target cartwheel formation minimizing propellant consumption.

Our approach for Solar Electric Propulsion (SEP) transfer optimization:

- Indirect method with real ephemeris and mission parameters.
- TPBVP Single Shooting method with MATLAB's *fsolve*.
- Dual Homotopy to ensure solver convergence [5]:
 - Achieve very low-thrust: $T_{max}(\kappa) = \kappa T_{max}, \kappa \rightarrow 1$.
 - Smoothing control: energy- to fuel-optimal (bang-bang) $\epsilon \rightarrow 10^{-5}$.

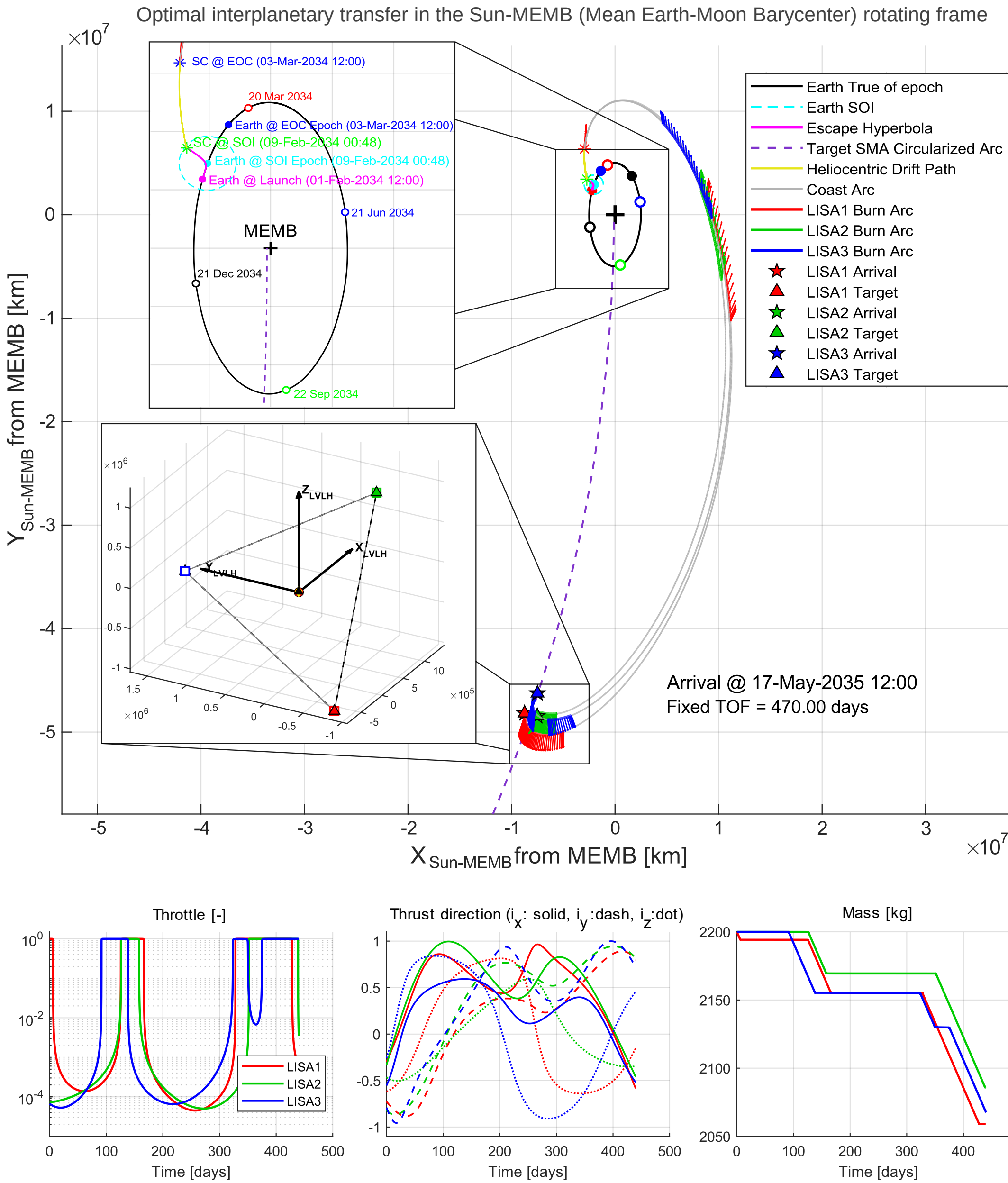
$$\min_{u,i} J = \lambda_0 \frac{T_{max}(\kappa)}{I_{sp} g_0} \int_{t_0}^{t_f} [u + \epsilon \ln(u(1-u))] dt$$

s.t.

$$\begin{aligned} \dot{\mathbf{r}} &= \mathbf{v}, \\ \dot{\mathbf{v}} &= -\frac{\mu}{r^3} \mathbf{r} + \frac{T_{max}}{m} u \mathbf{i}, \\ \dot{m} &= -\frac{T_{max}}{I_{sp} g_0} u, \end{aligned}$$

$$\begin{aligned} \mathbf{r}(0) &= \mathbf{r}_0, \quad \mathbf{r}(t_f) = \mathbf{r}_f \\ \mathbf{v}(0) &= \mathbf{v}_0, \quad \mathbf{v}(t_f) = \mathbf{v}_f \\ m(0) &= m_0, \quad u \in [0,1], \quad \|\mathbf{i}\| = 1 \end{aligned}$$

Assumptions for SEP transfer	
Wet mass	2200 kg
Maximum thrust	81 mN
Specific impulse	1660 s
Transfer duration	470 days
Commissioning time	30 days
v_∞ (Ariane 64)	300 m/s



[1] Sullivan, Joshua, Sebastian Grimberg, and Simone D'Amico. "Comprehensive Survey and Assessment of Spacecraft Relative Motion Dynamics Models." *JGCD* 40, no. 8 (2017): 1837–59.
[2] D'Amico, Simone. *Autonomous Formation Flying in Low Earth Orbit*. Ridderprint, 2010.
[3] Izzo, Dario, and Sebastien Origer. "Neural Representation of a Time Optimal, Constant Acceleration Rendezvous." *Acta Astronautica* 204 (March 2023): 510–17.
[4] Martens, Waldemar, and Eric Joffre. "Trajectory Design for the ESA LISA Mission." *The Journal of the Astronautical Sciences* 68, no. 2 (2021): 402–43.
[5] Taheri, Ehsan, and John L. Junkins. "Generic Smoothing for Optimal Bang-Off-Bang Spacecraft Maneuvers." *JGCD*, 41, no. 11 (2018): 2470–75.