Meshless CFD and Reduced Order Modeling of Cryogenic Sloshing in Conformal Tanks for Hydrogen Aviation

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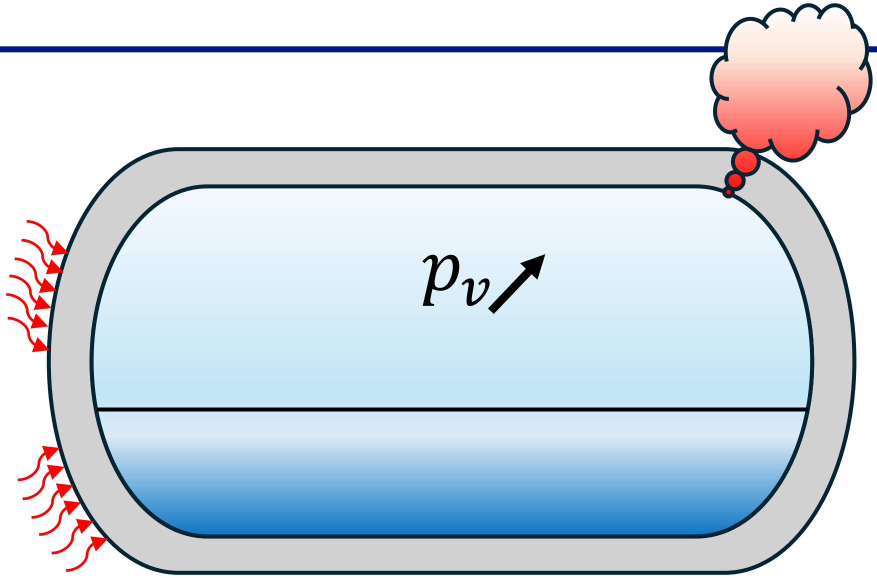
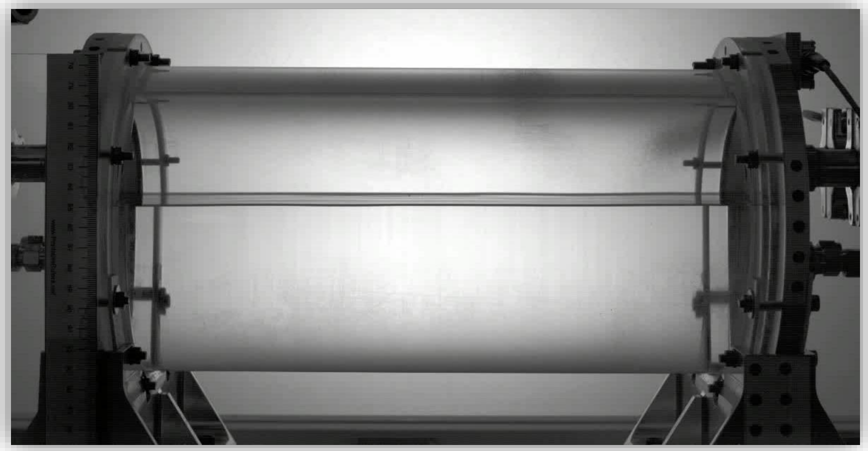
Andrea Ianiro
Miguel A. Mendez

PhD Aerospace Engineering UC3M
Doctoral Meetings 2026
2nd and 3rd June 2026

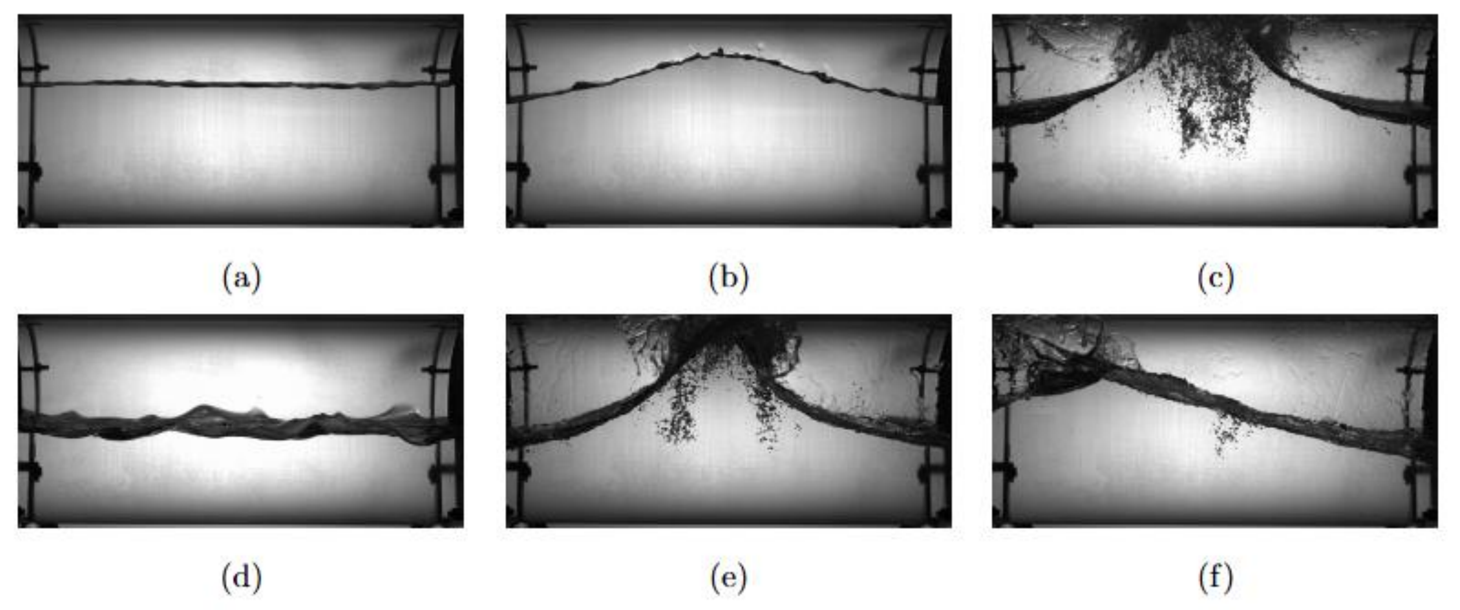
Cryogenic fuel for transport

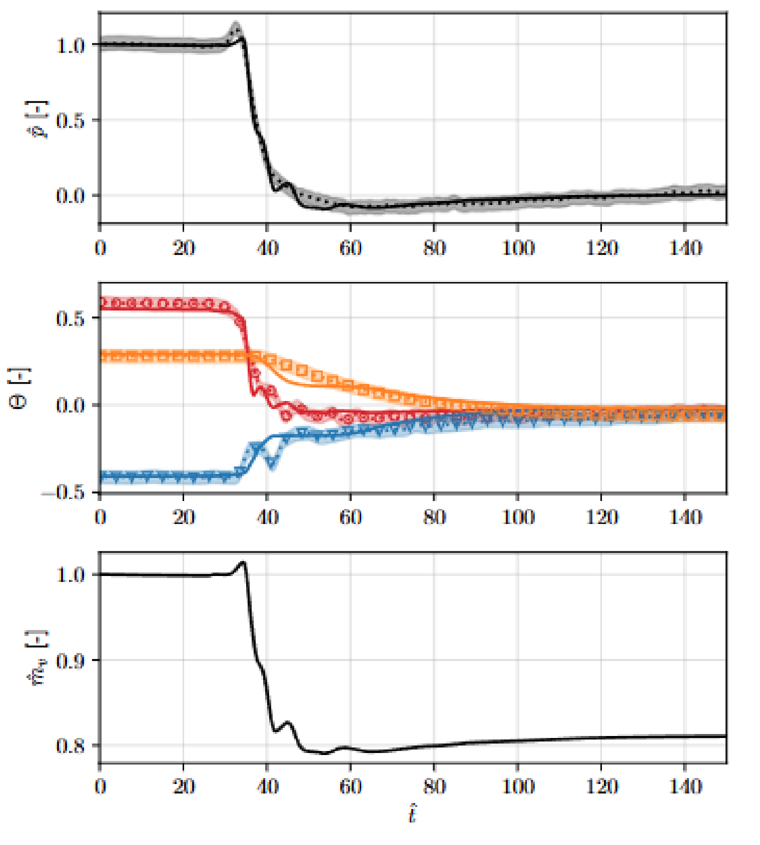




WP0

Experimental characterization of heat and mass transfer processes under aviation-relevant excitations

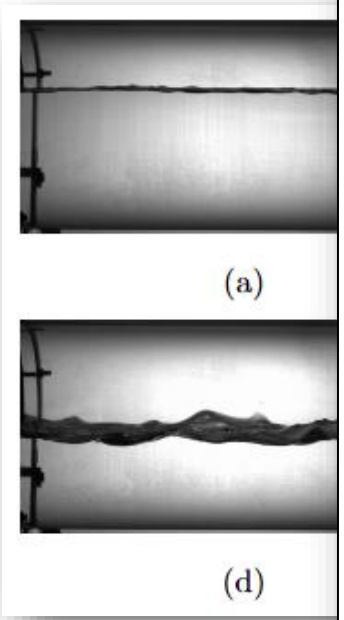


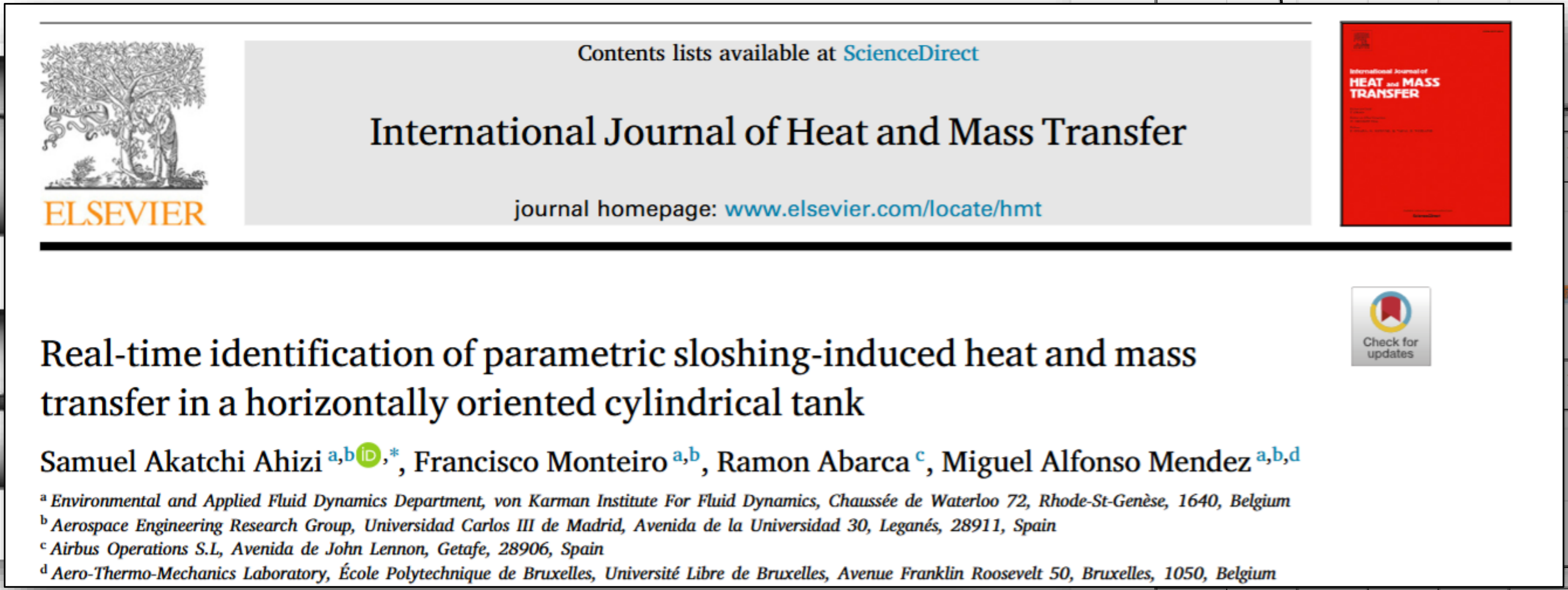


PhD Timeline

WP0

Experimental characterization of heat and mass transfer processes under aviation-relevant excitations





Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/hmt

Real-time identification of parametric sloshing-induced heat and mass transfer in a horizontally oriented cylindrical tank

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[a] *Environmental and Applied Fluid Dynamics Department, von Karman Institute For Fluid Dynamics, Chaussée de Waterloo 72, Rhode-St-Genèse, 1640, Belgium*
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[c] *Airbus Operations S.L, Avenida de John Lennon, Getafe, 28906, Spain*
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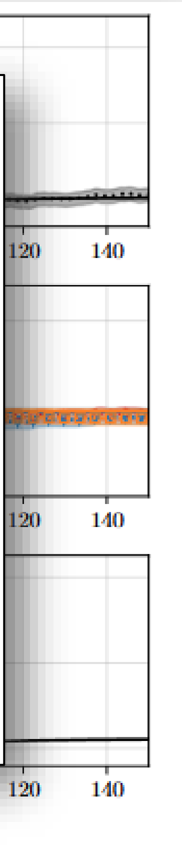

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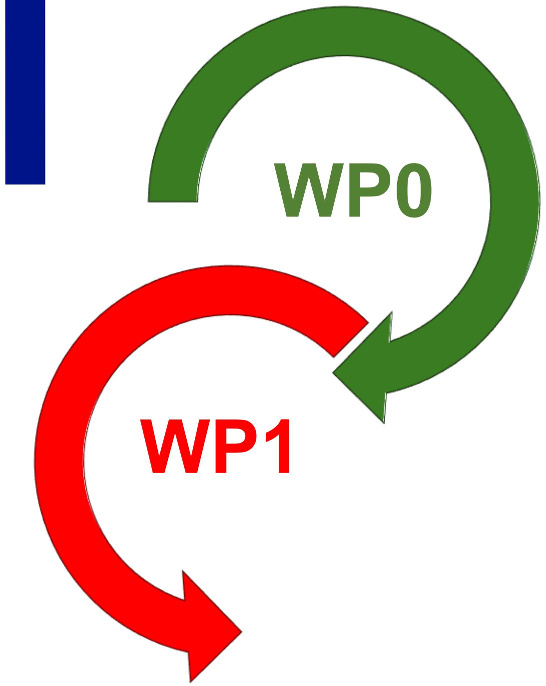

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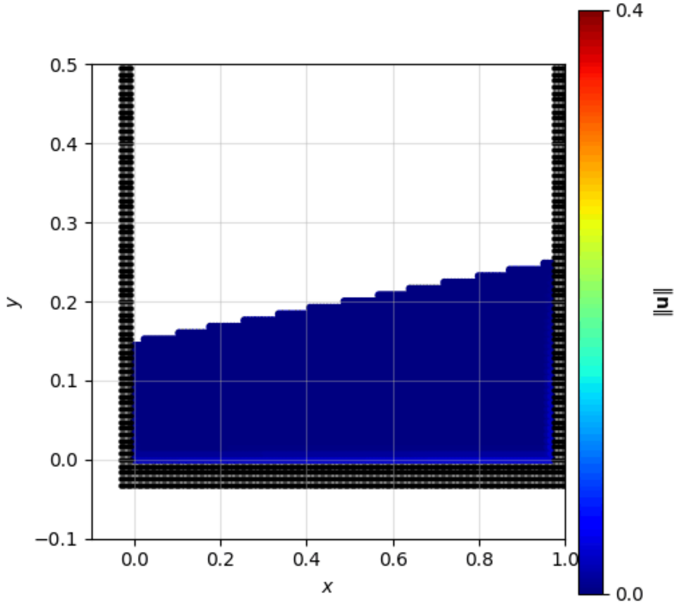
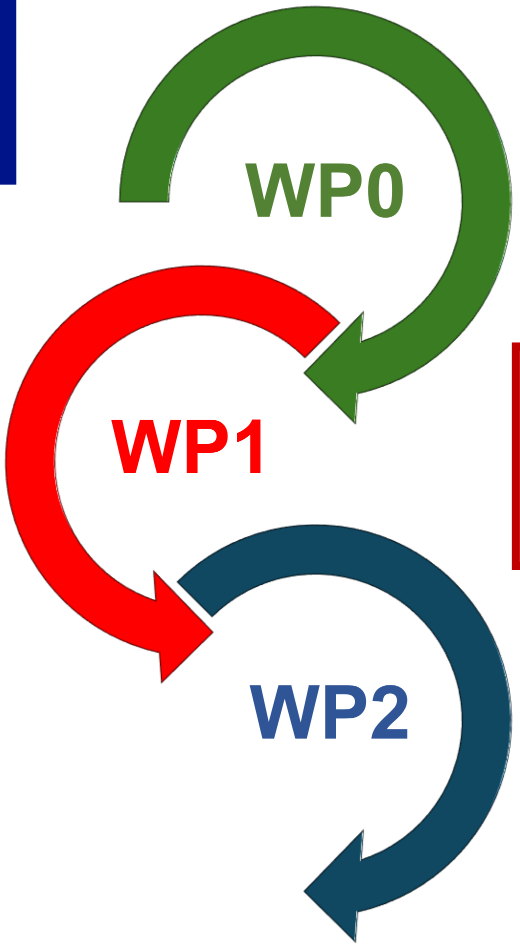
Exploration of **meshless methods** for non-isothermal sloshing modelling



PhD Timeline



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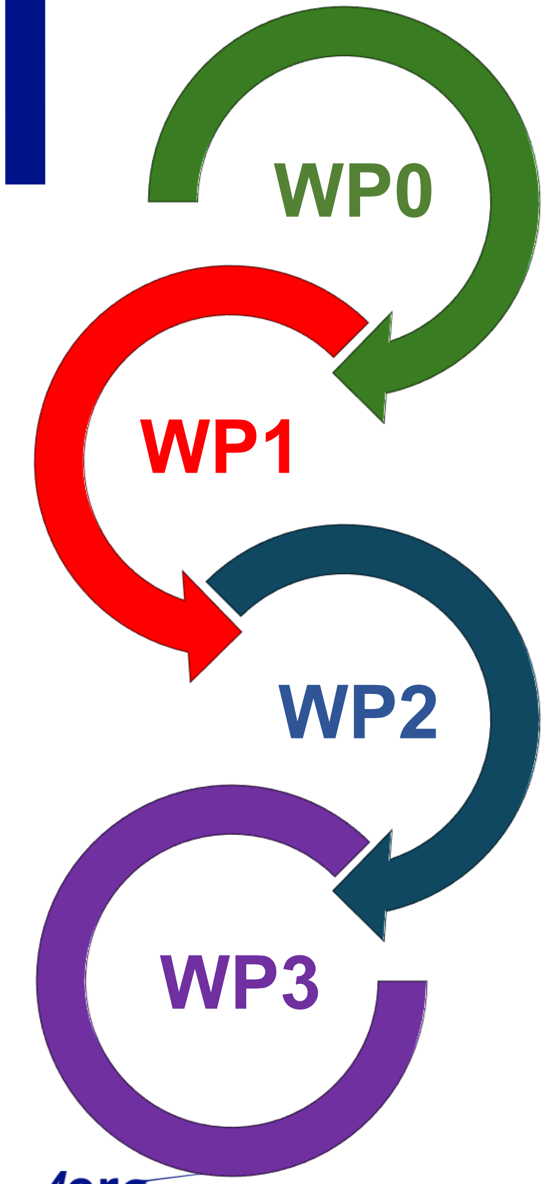
Development of data-driven Lagrangian **Reduced Order Models**

$$\mathbf{r}(t) = \mathbf{\Phi}_r \mathbf{q}_r(t)$$
$$\ddot{\mathbf{q}}_\mathbf{r} = \mathbf{\Phi}_r^T F(\mathbf{q}_r, \dot{\mathbf{q}}_r)$$

PhD Timeline



Experimental characterization of heat and mass transfer processes under aviation-relevant excitations

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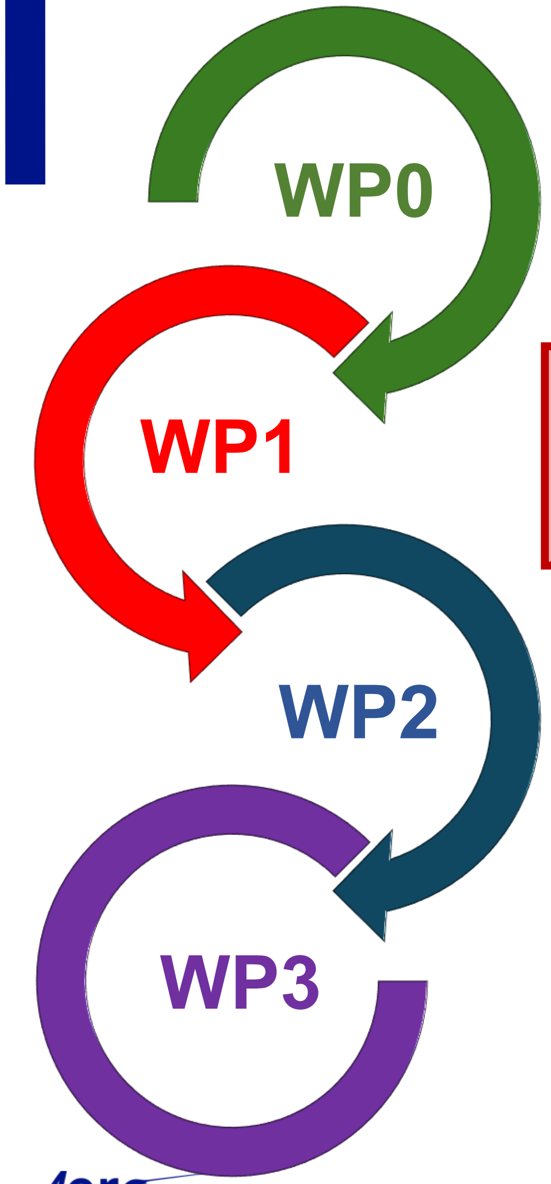

Experimental characterization of heat and mass transfer processes under aviation-relevant excitations

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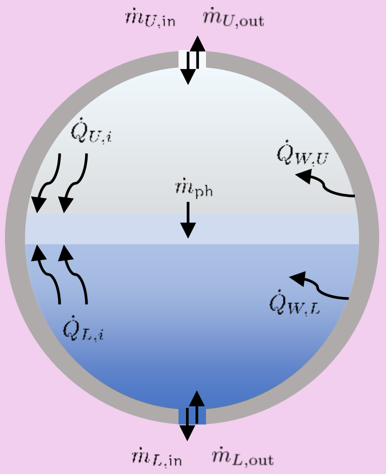
Non-isothermal sloshing modelling

Engineering models



Fast

Requires heavy tuning

Real-time data assimilation for the thermodynamic modeling of cryogenic storage tanks

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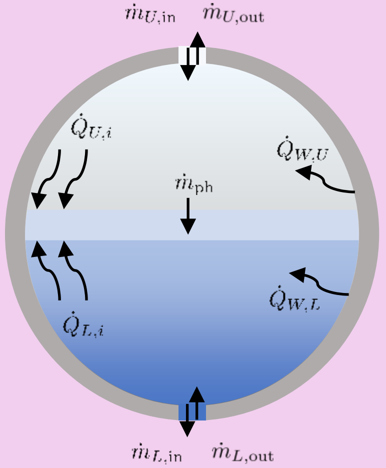
Non-isothermal sloshing modelling

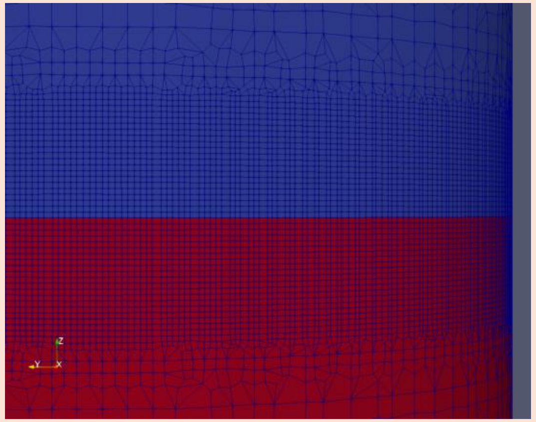
Engineering models



Fast

Requires heavy tuning

Eulerian methods



Established tool

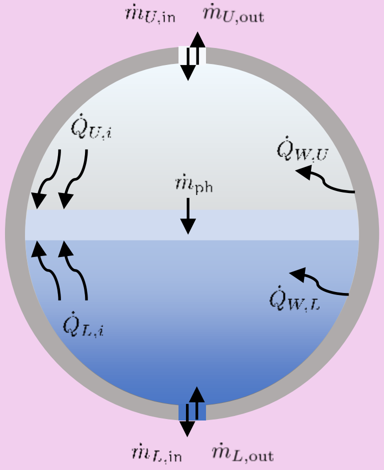
Difficult meshing

Diffuse interface

Parameters tuning

Non-isothermal sloshing modelling

Engineering models



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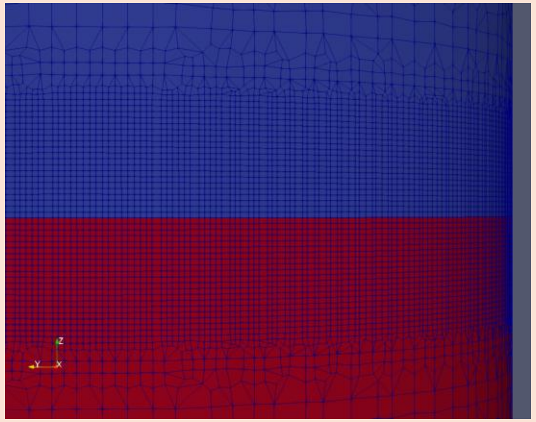

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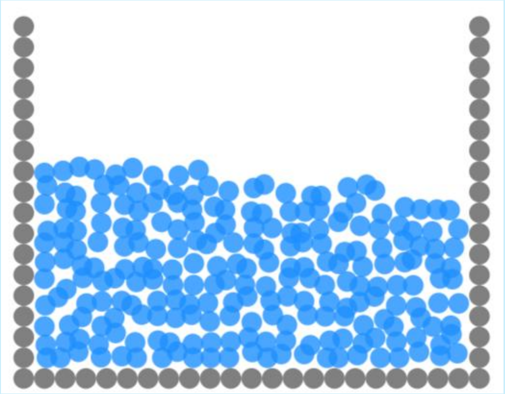


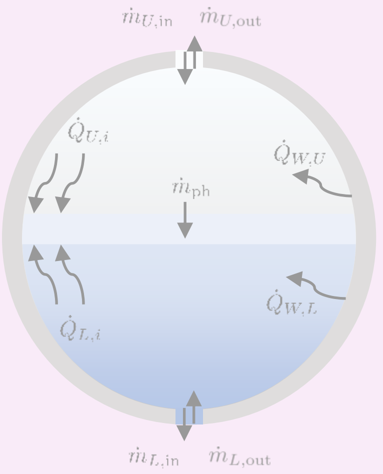
Established tool

No meshing

Limited phase change models

Non-isothermal sloshing modelling

Engineering models



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Requires heavy tuning

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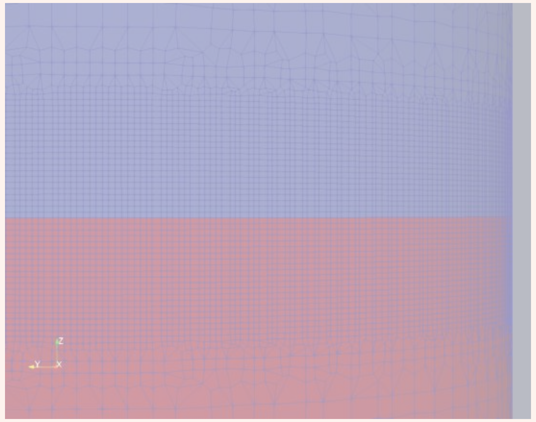

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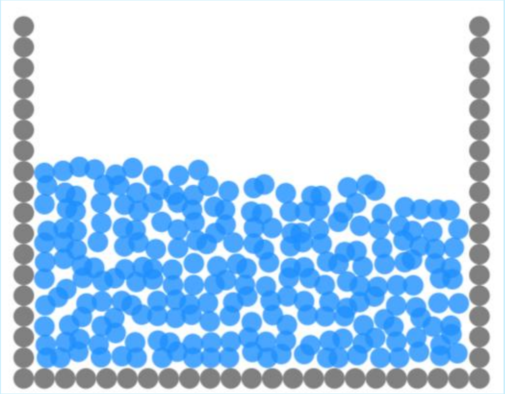
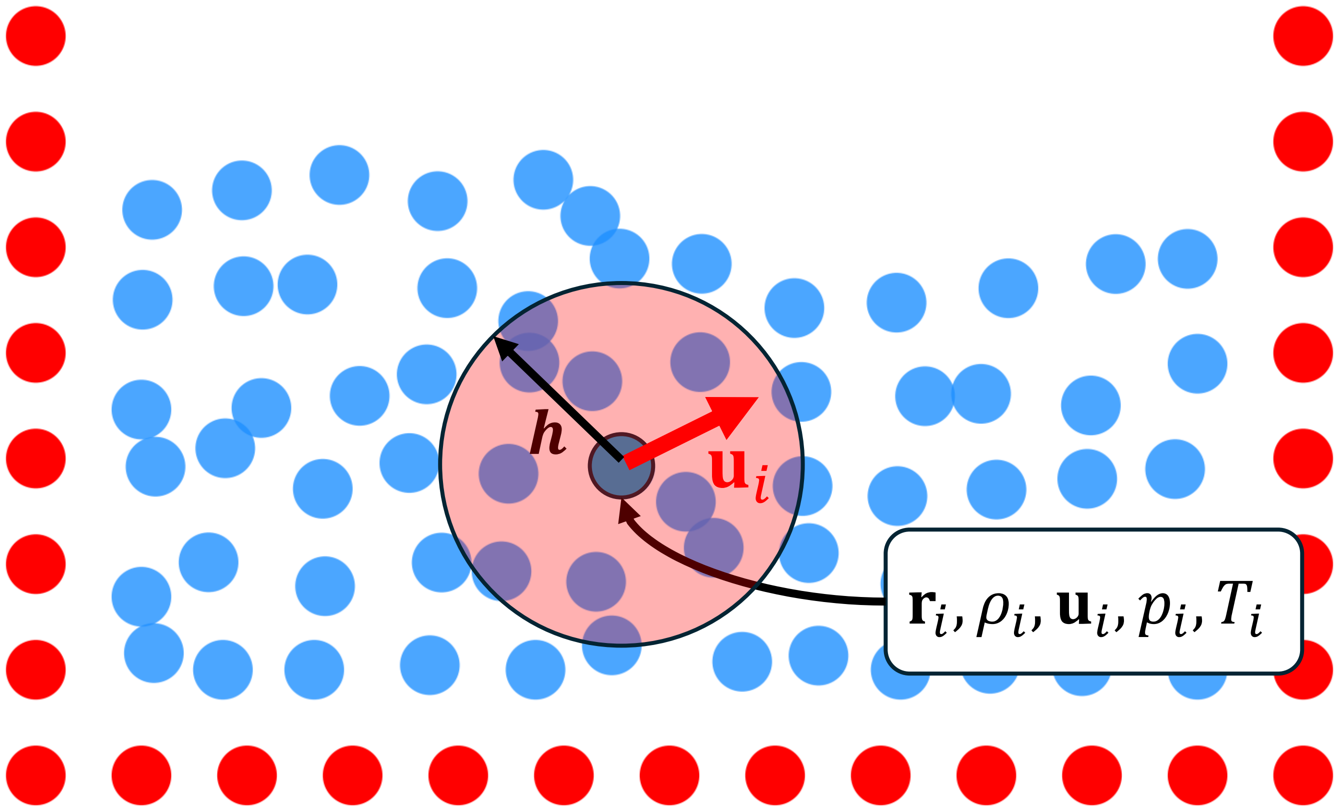
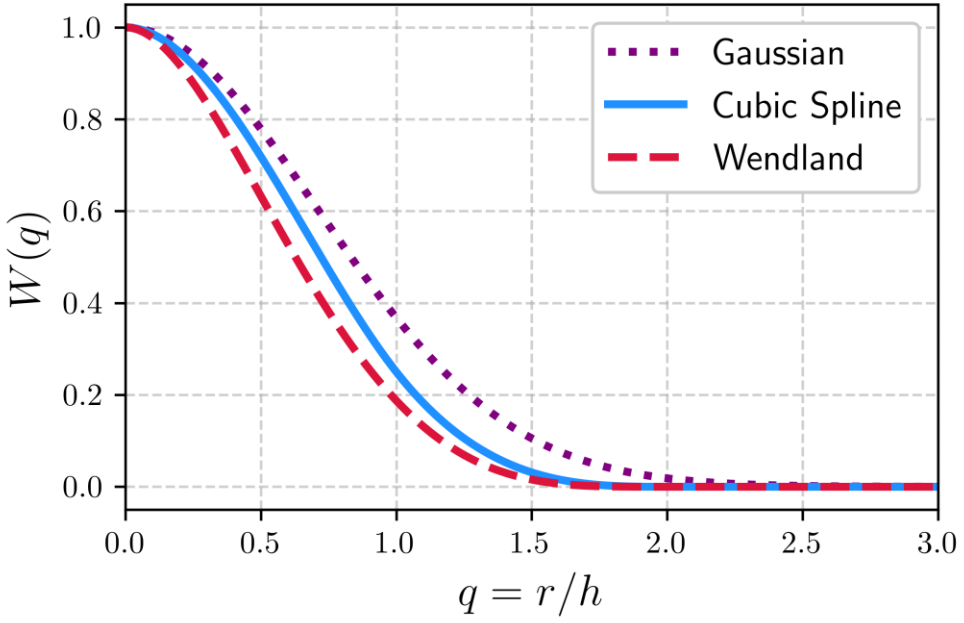


Established tool
No meshing
Limited phase change models

SPH concept

$$f(\mathbf{r}) \approx \sum_{j\in\Omega} f_j \frac{m_j}{\rho_j} W\left(\left\|\mathbf{r}-\mathbf{r}_j\right\|_2, h\right) \Rightarrow \nabla f(\mathbf{r}) \approx \sum_{j\in\Omega} f_j \frac{m_j}{\rho_j} \nabla W\left(\left\|\mathbf{r}-\mathbf{r}_j\right\|_2, h\right)$$





Governing equations

- Continuity equation:

$$\frac{D\rho}{Dt} = -\rho\nabla\cdot\mathbf{u}$$

$$\Rightarrow \frac{d\rho_i}{dt} = \sum_j m_j(\mathbf{u}_i - \mathbf{u}_j)\cdot\nabla_i W_{ij} + \mathcal{D}_i$$

- Momentum:

$$\frac{D\mathbf{u}}{Dt} = -\frac{\nabla p}{\rho} + \nu\nabla^2\mathbf{u} + \mathbf{a}_{buo} + \mathbf{a}_{ext}$$

$$\Rightarrow \frac{d\mathbf{u}_i}{dt} = \sum_j -m_j\frac{p_i + p_j}{\rho_j\rho_j}\nabla_i W_{ij} + \sum_j \mathbf{\Pi}_{ij} + \mathbf{a}_{buo,i} + \mathbf{a}_{ext}$$

- Energy:

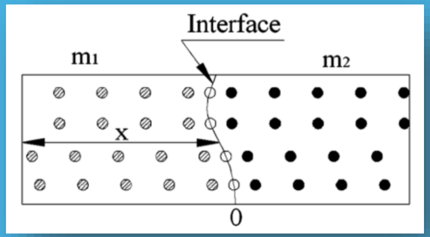
$$\rho\frac{De}{Dt} = \nabla\cdot(\kappa\nabla T) - p(\nabla\cdot u) + \phi$$

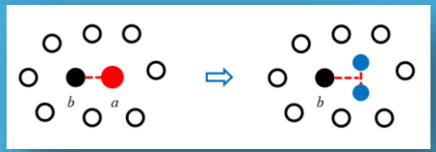
$$\Rightarrow c_p\frac{dT_i}{dt} = \sum_j \frac{2\kappa m_j(T_i - T_j)}{\rho_j\|\mathbf{r}_i - \mathbf{r}_j\|_2}(\mathbf{r}_i - \mathbf{r}_j)\cdot\nabla_i W_{ij}$$

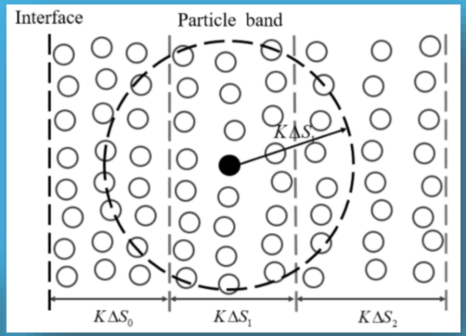
SPH evaporation modelling

- Large density jump at the interface: $\rho_l \gg \rho_g$

Topological manipulation [1,2,3]

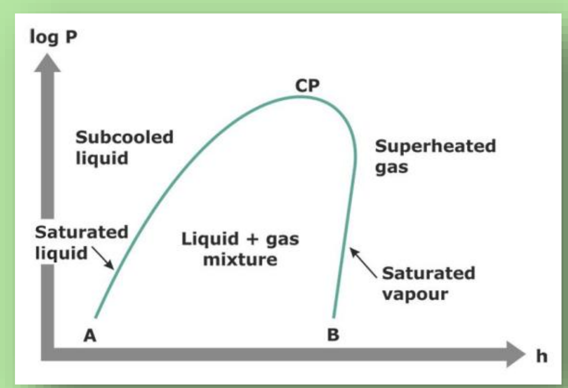






Unified EOS [4]

$$p = f(\rho, T)$$



[1] Li et al (2022)
[2] Wang et al (2022)
[3] Das and Das (2015)
[4] Ray et al (2017)

Proposed approach

Interfacial heat exchanges:

$$\dot{Q}_{v,s} = A_s h_c (T_v - T_{\text{sat}})$$

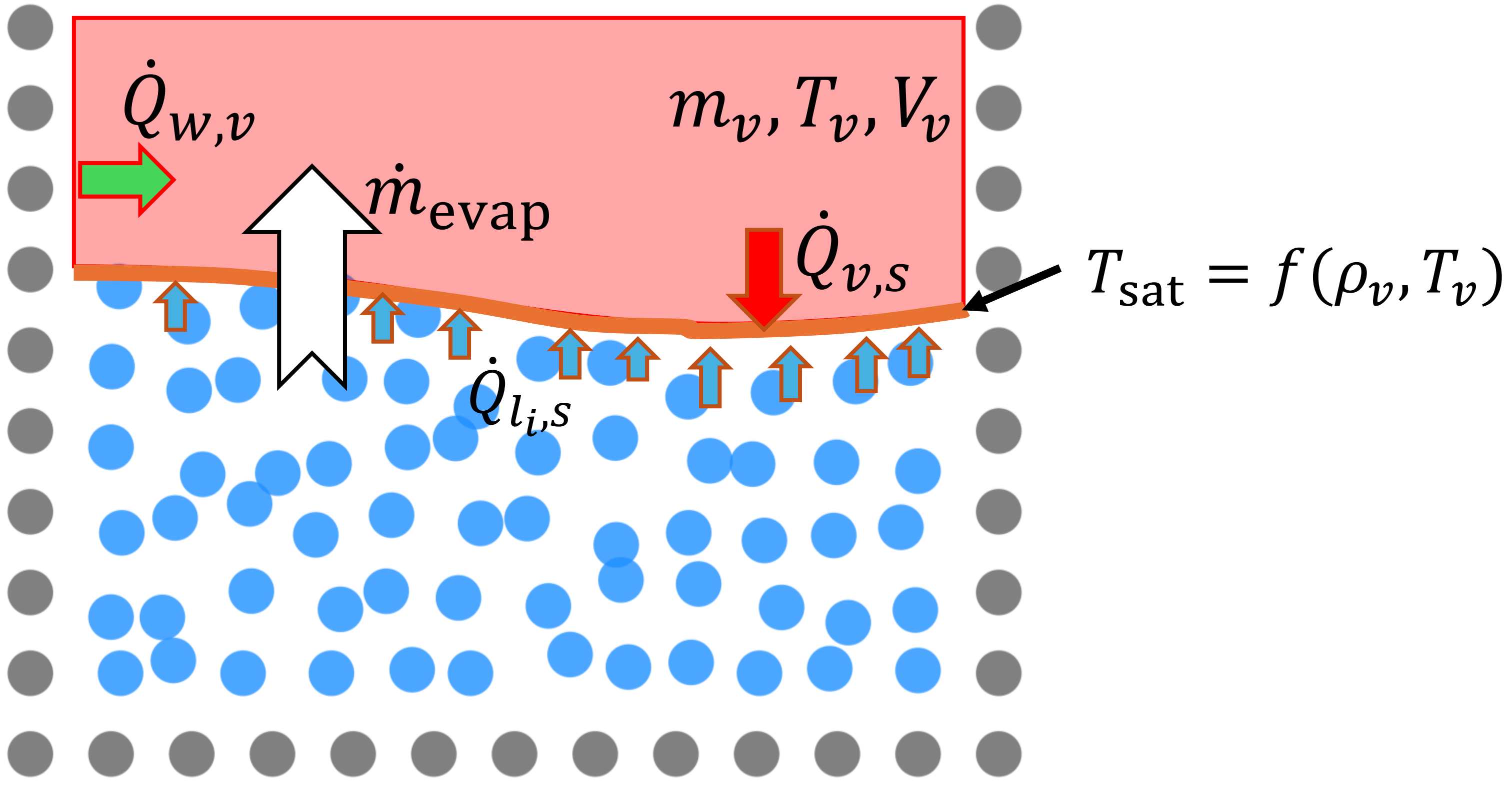
$$\dot{Q}_{l_i,s} = A_{\text{eff}} \frac{\kappa}{\Delta x} (T_i - T_{\text{sat}})$$

Robin BC at the interface:

$$\left.\frac{dT_i}{dt}\right|_{\in \delta\Omega} = \frac{\dot{Q}_{l_i,s}}{m_i c_p}$$

Isothermal walls:

$$\left.\frac{dT_i}{dt}\right|_{\text{walls}} = 0$$



Energy balance: $\dot{m}_{\text{evap}} = \dfrac{\dot{Q}_{v,s} + \sum_{i \in \delta\Omega} \dot{Q}_{l_i,s}}{\mathcal{L}_v}$

Global particle mass reduction: $\dfrac{dm_i}{dt} = -\dfrac{\dot{m}_{evap}}{n_p}$

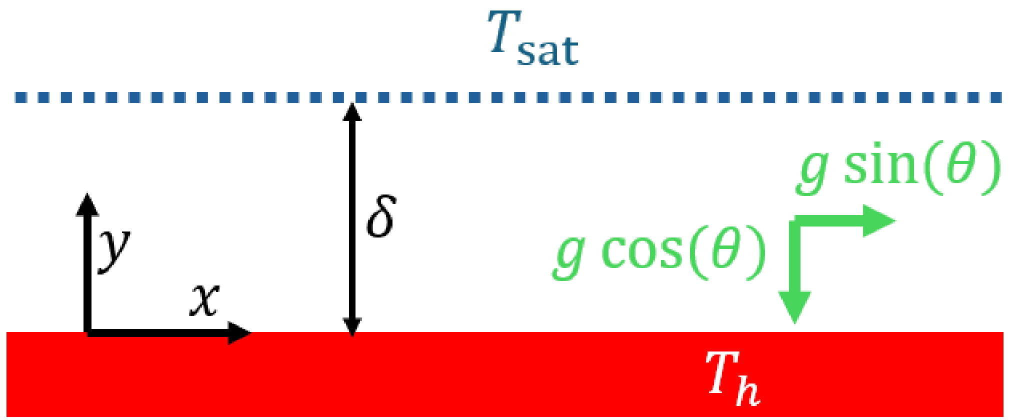
Implementation

- Cell-list neighbor search algorithm (2D/3D)
- Different wall treatment (DBC, GWBC, mDBC)
- Verlett Symplectic time integration scheme
- Implemented in Python -JAX:
  - GPU portability
  - Automatic Differentiation capabilities

Phase change validation

Nusselt film

- Initial thin film thickness δ_0
- Super-heated wall: $T_h > T_{\text{sat}}$
- Laminar regime: $\text{Re} = \frac{g sin(\theta) \delta_0^3}{3\nu^2} \ll 1$



Without evaporation: steady state

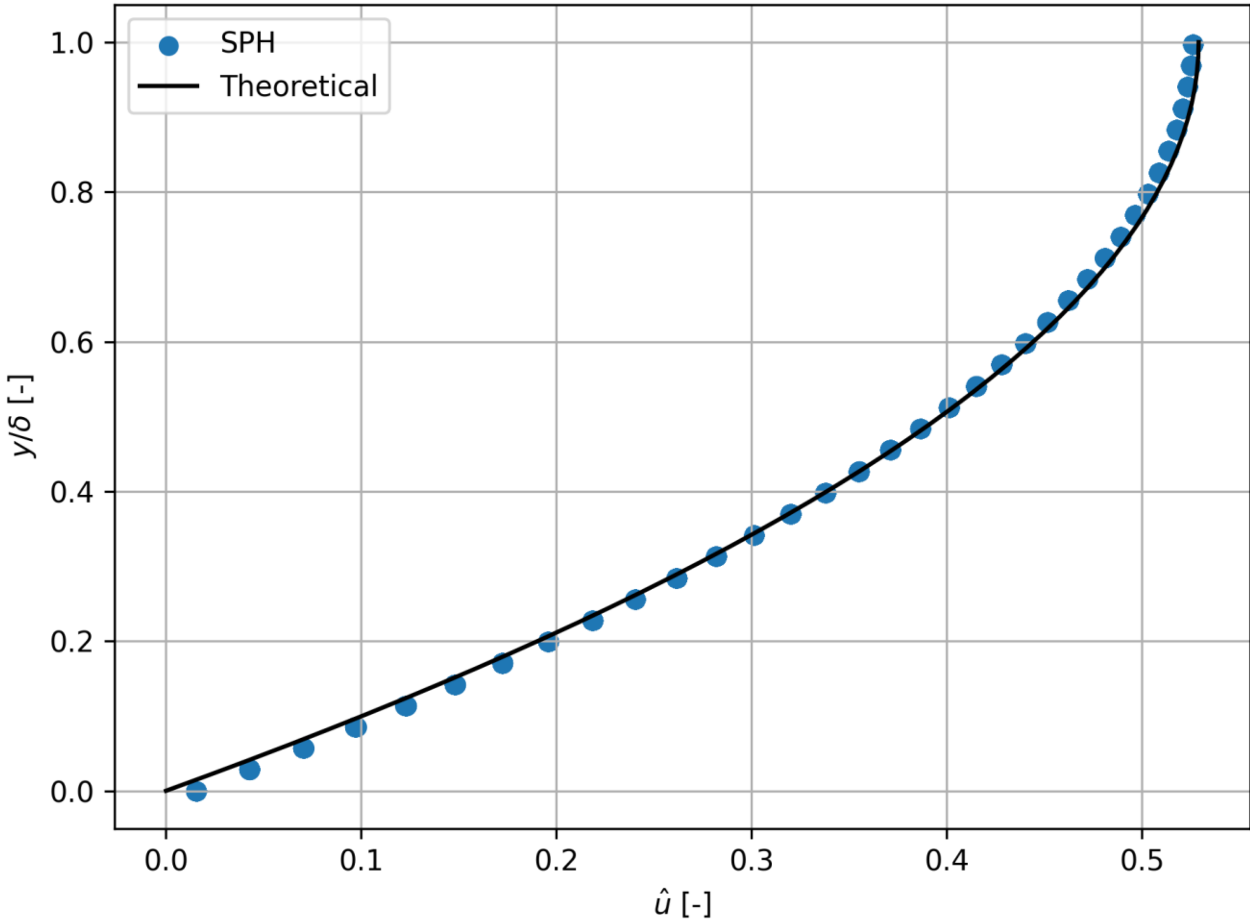
- $T(y) = T_h - \frac{T_h - T_{\text{sat}}}{\delta_0} y$
- $u(y) = \frac{g\sin(\theta)}{\nu}\left(\delta_0 y - \frac{y^2}{2}\right)$

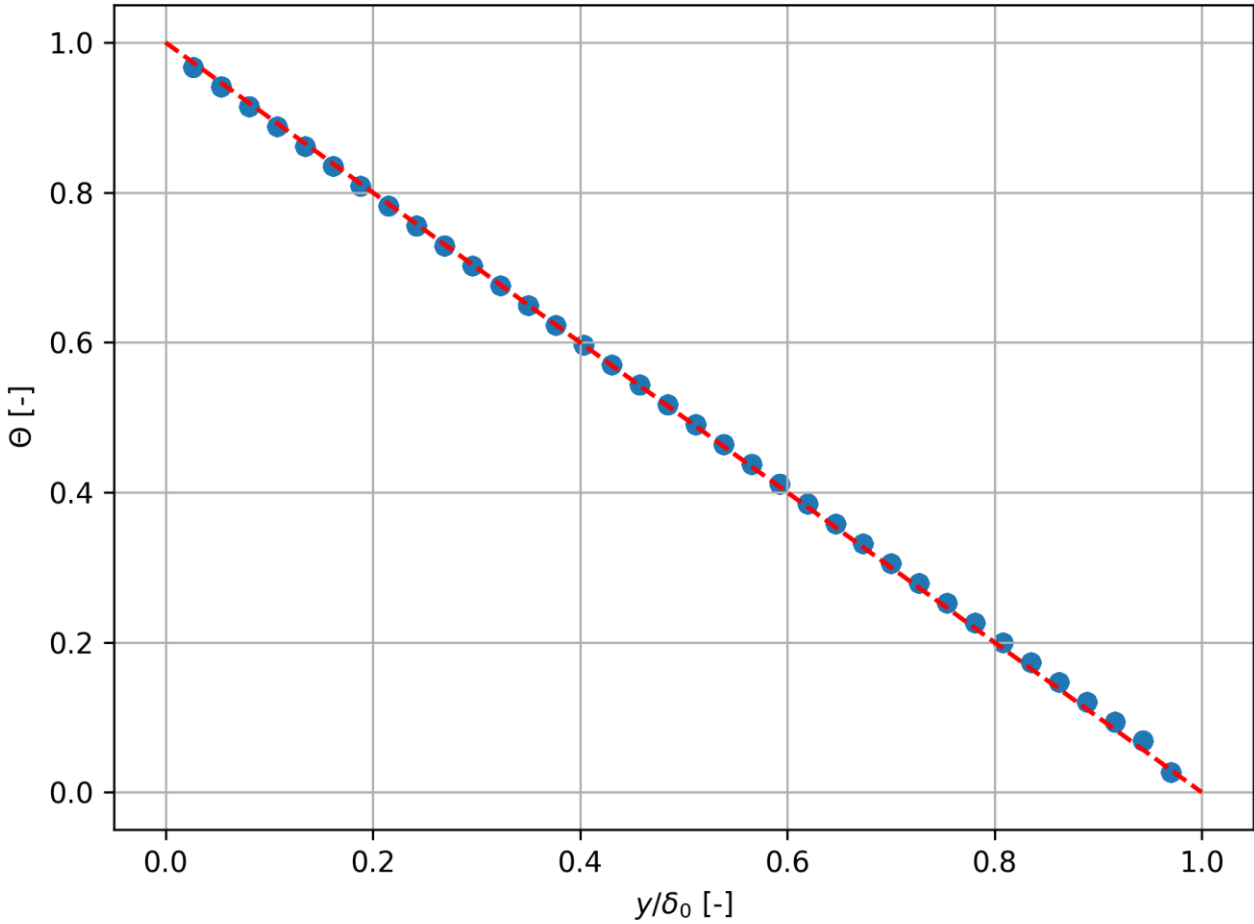
With evaporation: modified Stephan problem:

- $\delta(t) = \delta_0\sqrt{1 - 2\,\text{Ste Fo}}$

Phase change validation

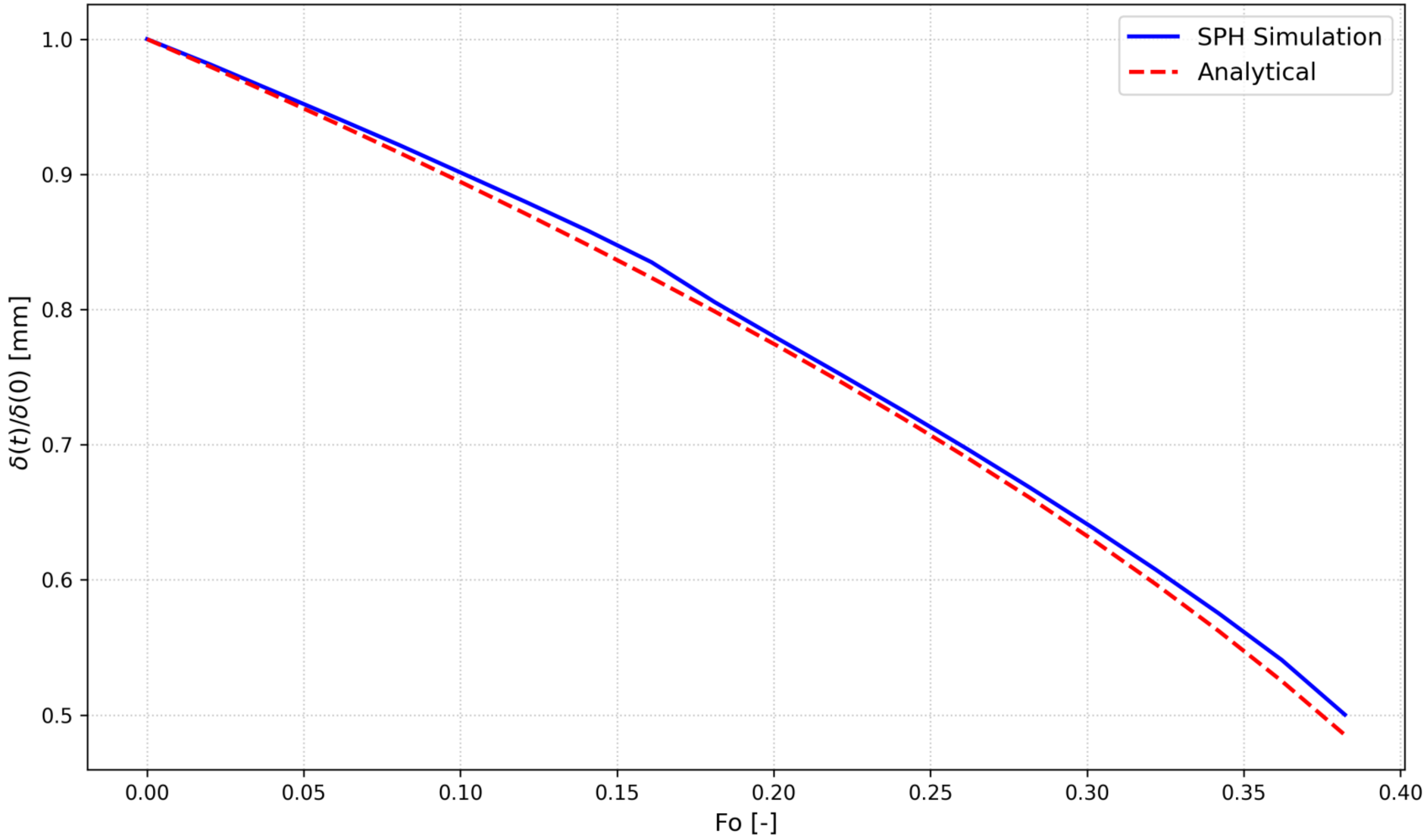
- Without evaporation the steady state is recovered





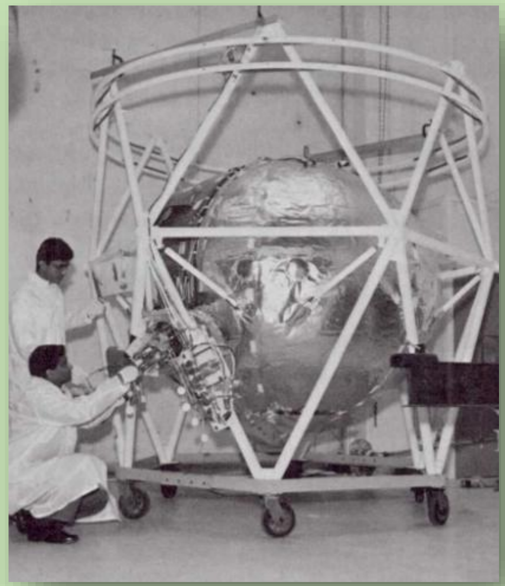
Phase change validation

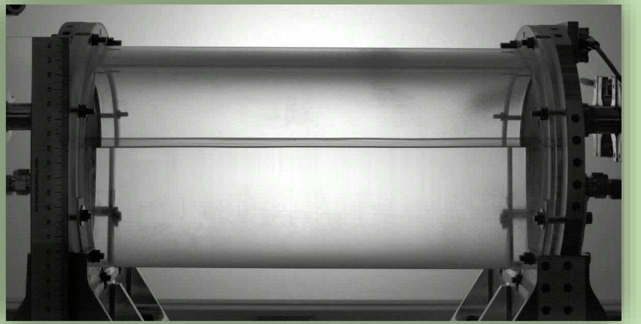
- With evaporation, the evolution of film thickness is captured



Next steps

Experimental validation of the solver





Exploration of Lagrangian ROM

$$\mathbf{r}(t) = \mathbf{\Phi}_r \mathbf{q}_r(t)$$
$$\ddot{\mathbf{q}}_\mathbf{r} = \mathbf{\Phi}_r^T F(\mathbf{q}_r, \dot{\mathbf{q}}_r)$$

Achievements

- Publication in *International Journal of Heat and Mass Transfer*
- Presentation at *French interfacial processes group meeting*
- Lecture on *Times series forecasting* at VK*I Hand's on ML series*
- Supervision of 2 master thesis

Thank you!
Questions?

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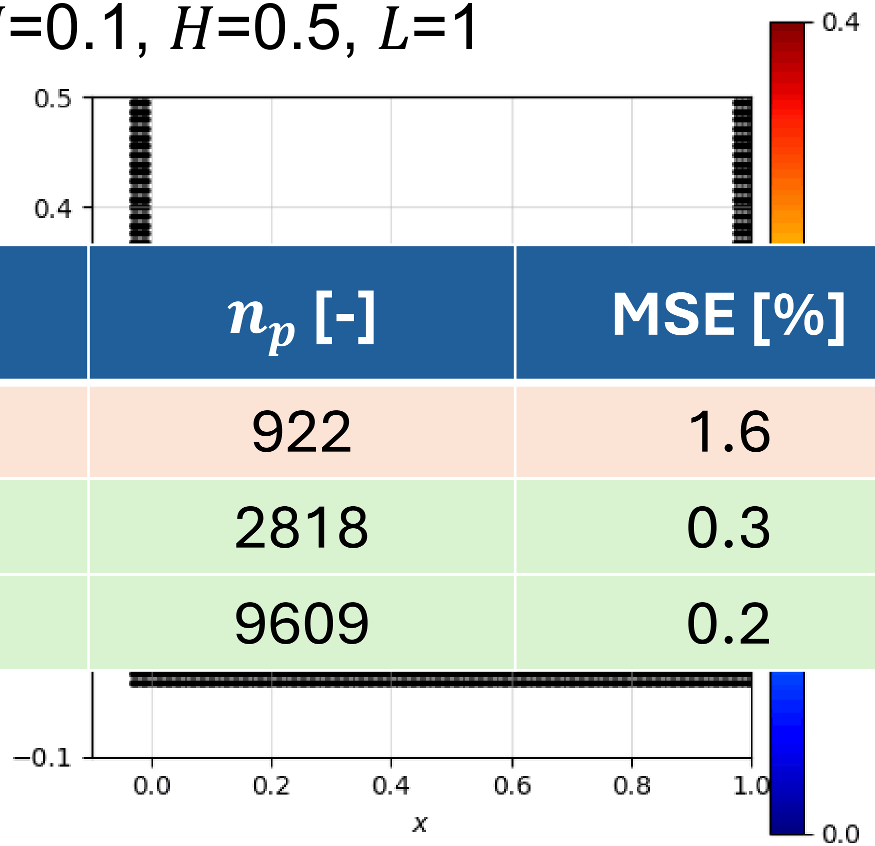
VLAIO Grant – LH2 for clean aviation
ERC - RETWIST

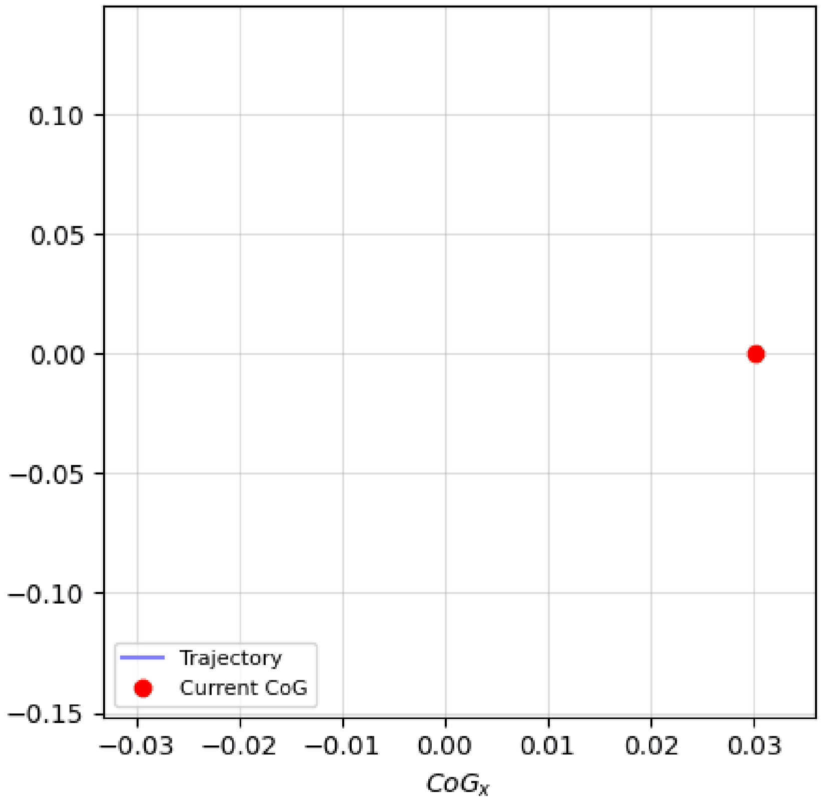
Isothermal sloshing validation

Free-damping of tilted interface

- CoG oscillating at 1st natural frequency: $\omega_1 = \sqrt{\frac{g\pi}{L}\tanh(\frac{\pi H}{L})}$
- ΔH=0.1, H=0.5, L=1



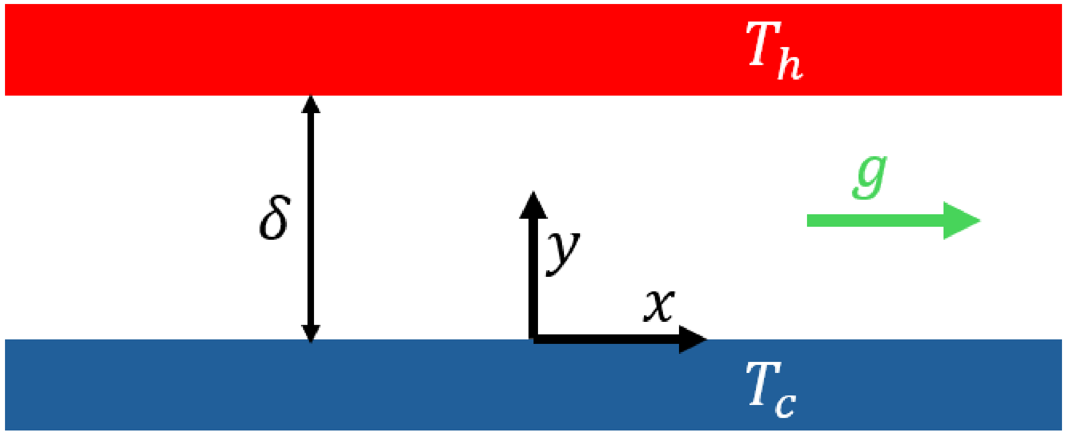
$\frac{\Delta x}{L}$ [-]	n_p [-]	MSE [%]
0.02	922	1.6
0.01	2818	0.3
0.005	9609	0.2



Thermal validation

Buoyancy-driven channel flow

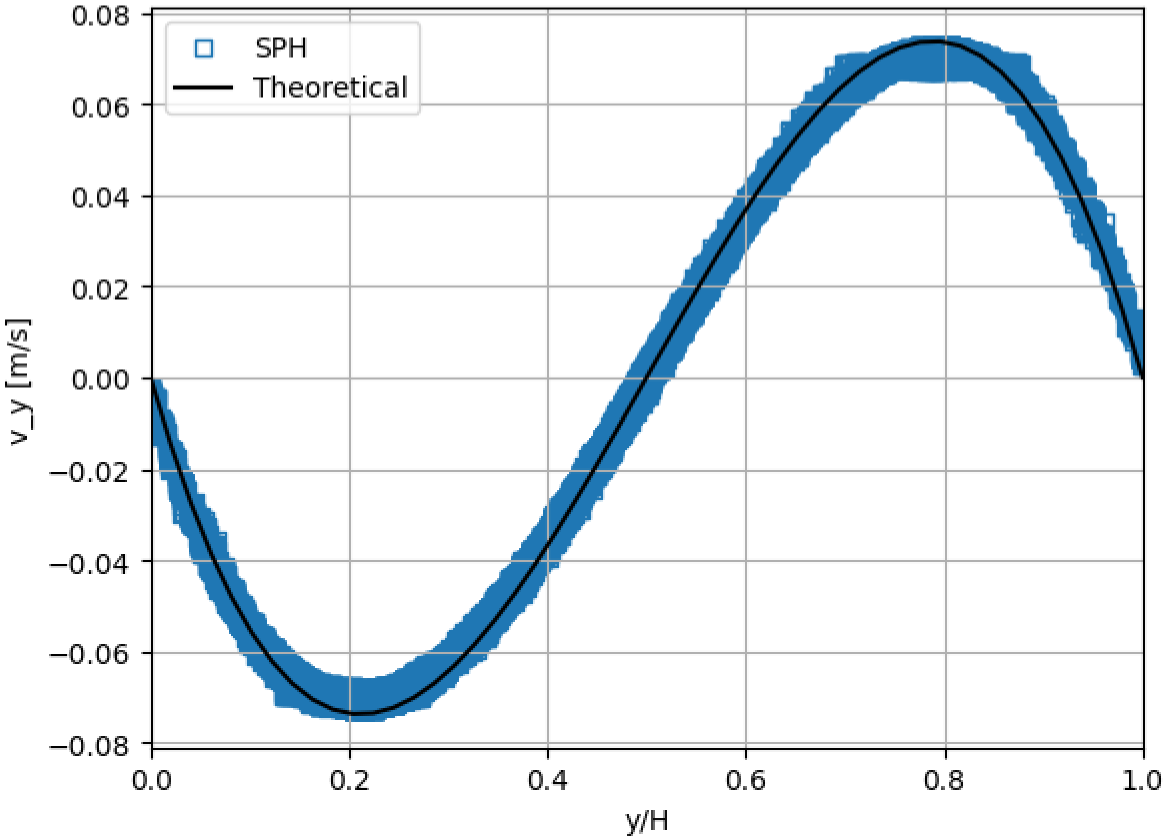
- Grashof: $\mathrm{Gr} = \frac{g\beta\Delta T\delta^3}{\nu^2} = 920$
- Prandtl: $\mathrm{Pr} = \frac{\nu}{\alpha} = 10$

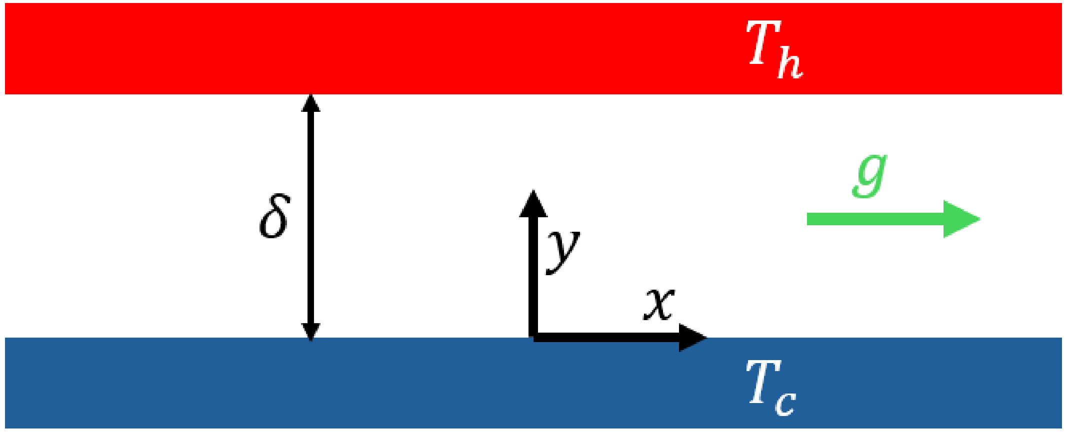


- Steady-state solution:
  - Linear temperature profile
  - Cubic velocity profile $u(y)\frac{\delta}{\nu} = -\frac{Gr}{12}\left[\frac{y}{\delta} - 3\left(\frac{y}{\delta}\right)^2 + 2\left(\frac{y}{\delta}\right)^3\right]$

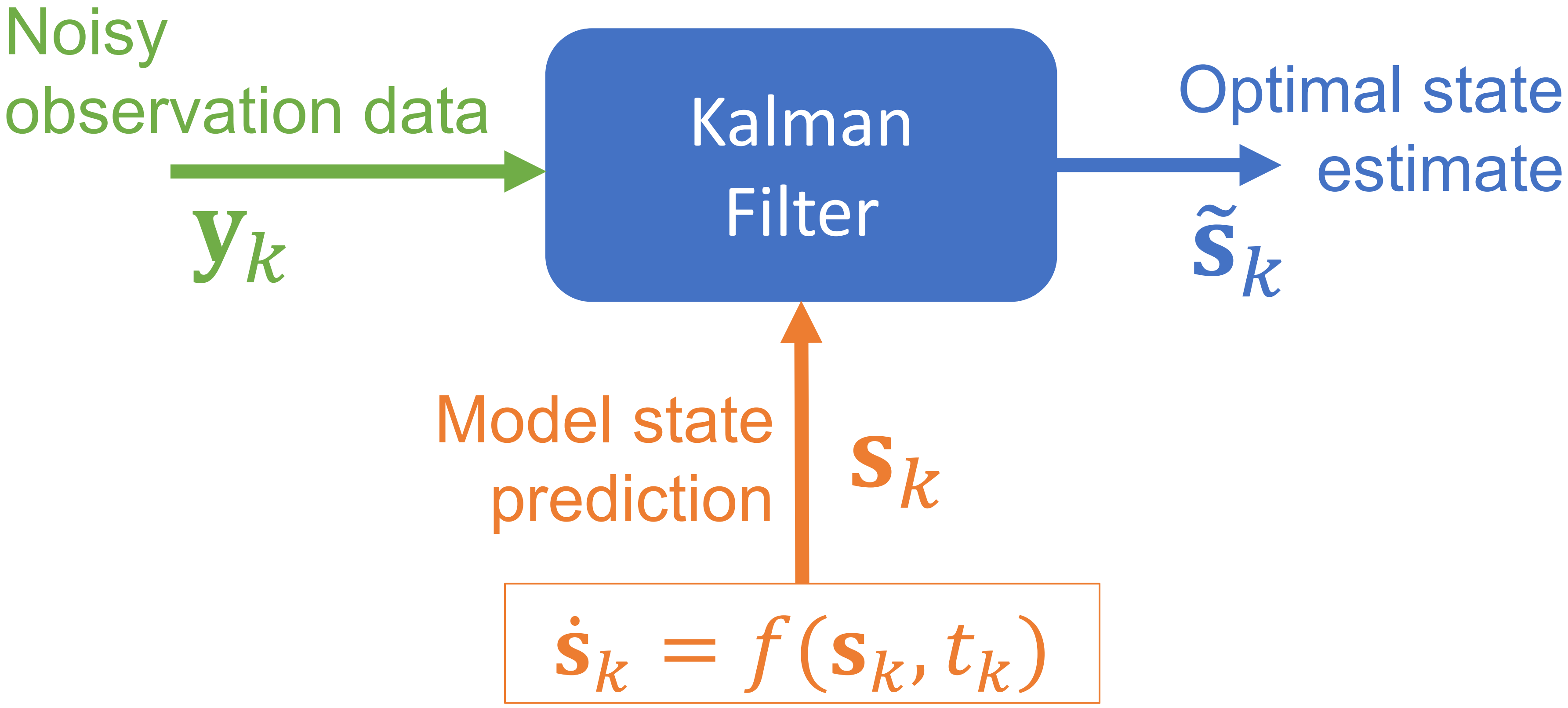
Thermal validation

- The steady-state velocity is retrieved





Kalman principle

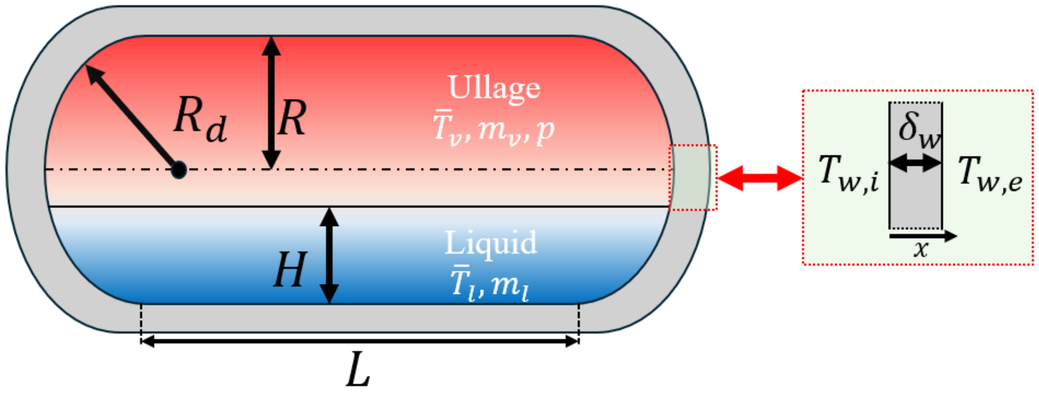


AEKF Nusselt inference

We have the observation vector $\mathbf{y}_k = [\bar{T}_{l,k}, \bar{T}_{v,k}, \bar{T}_{w,k}, p_k]^T$

We define

- an augmented state $\mathbf{s} = [\bar{T}_l, \bar{T}_v, T_w(x), p, m_l, h_{li}, h_{vi} h_{wl}, h_{wv}]^T$
- governed by a model such as $\dot{\mathbf{s}} = f(\mathbf{s})$



Ahizi, S. A., Monteiro, F., Abarca, R., and M. A. Méndez, Real-time identification of parametric sloshing-induced heat and mass transfer in a horizontally oriented cylindrical tank, IHMT, 2026.

Model governing equations

Conservation of fluid mass and energy:

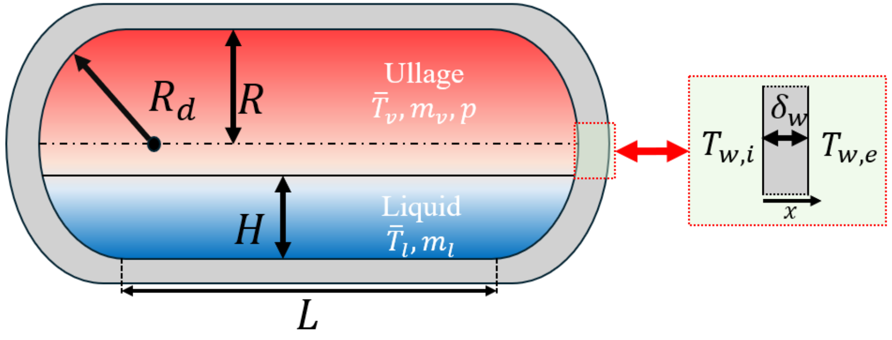
- $\frac{d}{dt}(m_l c_l T_l) = Q_{wl} - Q_{li} + \hbar_l^{\text{sat}} \dot{m}_c$
- $\frac{d}{dt}(m_v c_v T_v) = Q_{wv} - Q_{vi} + \hbar_v^{\text{sat}} \dot{m}_c$
- $\frac{dm_l}{dt} = -\frac{dm_v}{dt} = \dot{m}_c = -\frac{Q_{vi}+Q_{li}}{\mathcal{L}_v}$

Augmented closure

- $Q_{xy} = h_{xy} A_{xy} (T_x - T_y)$
- $\frac{dh_{xy}}{dt} = 0$

1D heat equation in the solid:

- Robin BC on the fluid side
- External heat flux on the outer face



AEKF Nusselt inference

The EKF provides a *state estimate* and *covariance*

$$\tilde{\mathbf{s}}_k = \boldsymbol{s}_k + \mathbf{K}_k(\mathbf{y}_k - \mathbf{G}\boldsymbol{s}_k)$$

and

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k\mathbf{G})\mathbf{P}_k^-(\mathbf{I} - \mathbf{K}_k\mathbf{G})^{\mathbf{T}} + \mathbf{K}_k\mathbf{R}\mathbf{K}_k^{\mathbf{T}}$$

with the *Kalman gain* and *covariance matrix update* such as:

$$\mathbf{K}_k = \mathbf{P}_k^-\mathbf{G}^{\mathrm{T}}\left(\mathbf{G}\mathbf{P}_k^-\mathbf{G}^{\mathrm{T}} + \mathbf{R}\right)^{-1} \text{ and } \mathbf{P}_k^- = \mathbf{A}_k\mathbf{K}_k\mathbf{A}_k^{\mathrm{T}} + \mathbf{Q}_k$$

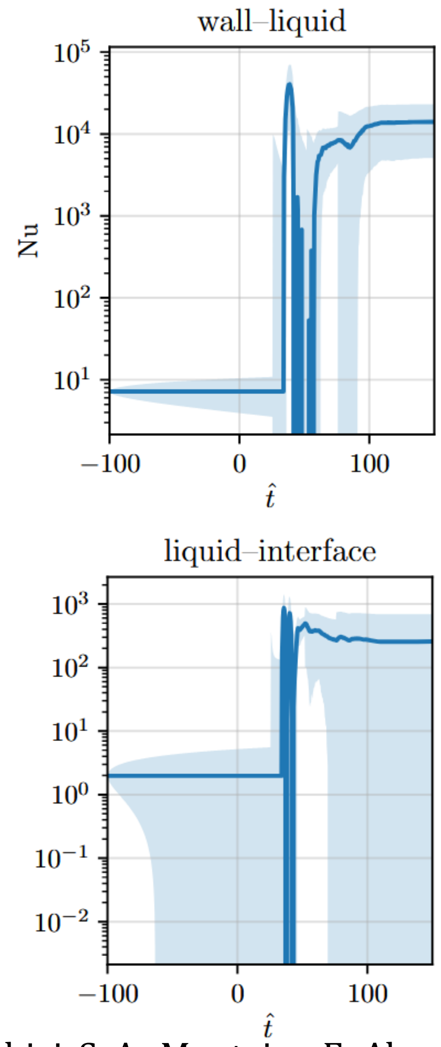
We use a *non-constant* process noise $\mathbf{Q}_k = diag(q_{phy}, q_{htc})$

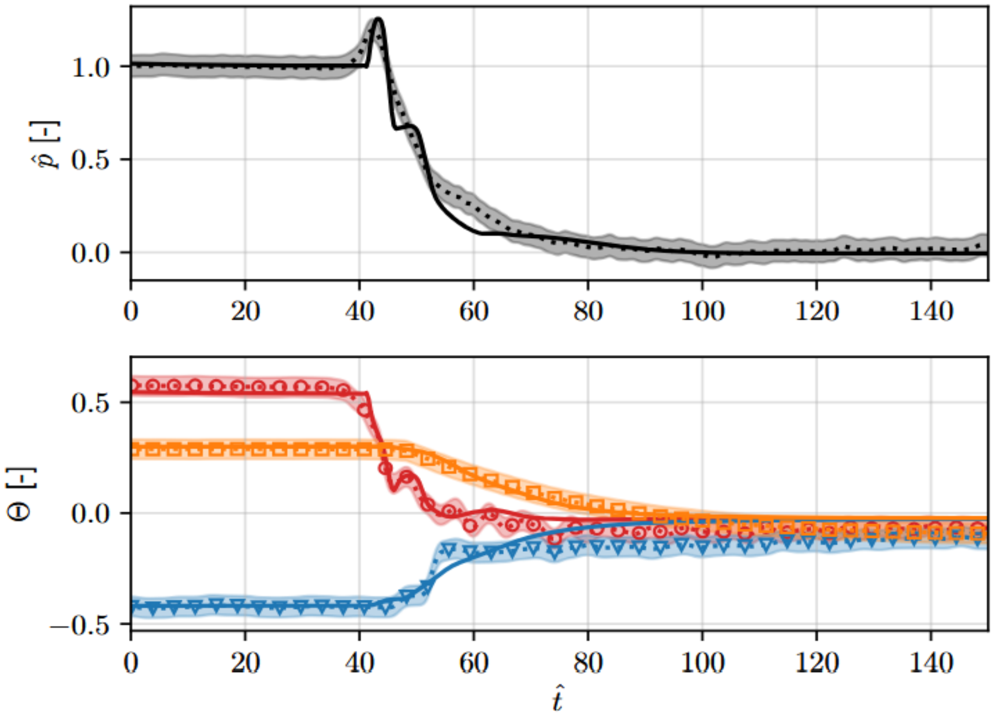
$$q_{htc} = \begin{cases} \mathcal{O}(1) & \text{if } |p_k - p_{k-1}| < 2.5\sigma_p \\ \mathcal{O}(10^6) & \text{if } |p_k - p_{k-1}| > 2.5\sigma_p \end{cases}$$

Implementation

- Crank-Nicholson time-stepping
- 100 initialization with random initial Nusselt
- Run in 5ms per step on AMD Ryzen 7
    - Real-time capabilities

AEKF inference results





Ahizi, S. A., Monteiro, F., Abarca, R., and M. A. Méndez, Real-time identification of parametric sloshing-induced heat and mass transfer in a horizontally oriented cylindrical tank, IHMT, 2026.

AEKF results

The modelling framework allows for further analysis

1. Contribution to pressure drop (≈80% phase change)

$$\frac{dp}{dt} = \underbrace{-\frac{1}{V_v}\left(\frac{\partial p}{\partial \rho}\right)_T \dot{m}_c}_{(dp/dt)_m} + \underbrace{\left(\frac{\partial p}{\partial T}\right)_\rho \frac{d\overline{T}_v}{dt}}_{(dp/dt)_T}$$

2. Sensitivity analysis

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