Turbulent characterization in Hall Thrusters

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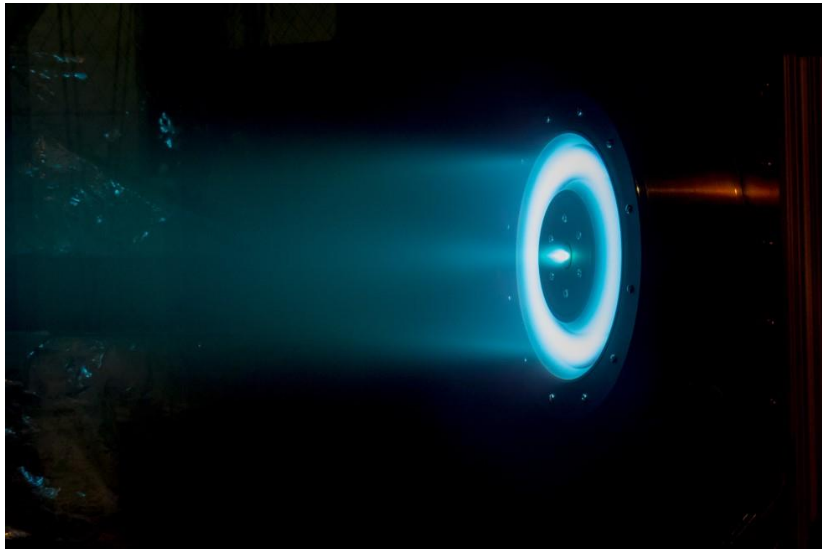
PhD Doctoral Meetings
PhD Program in Aerospace Engineering

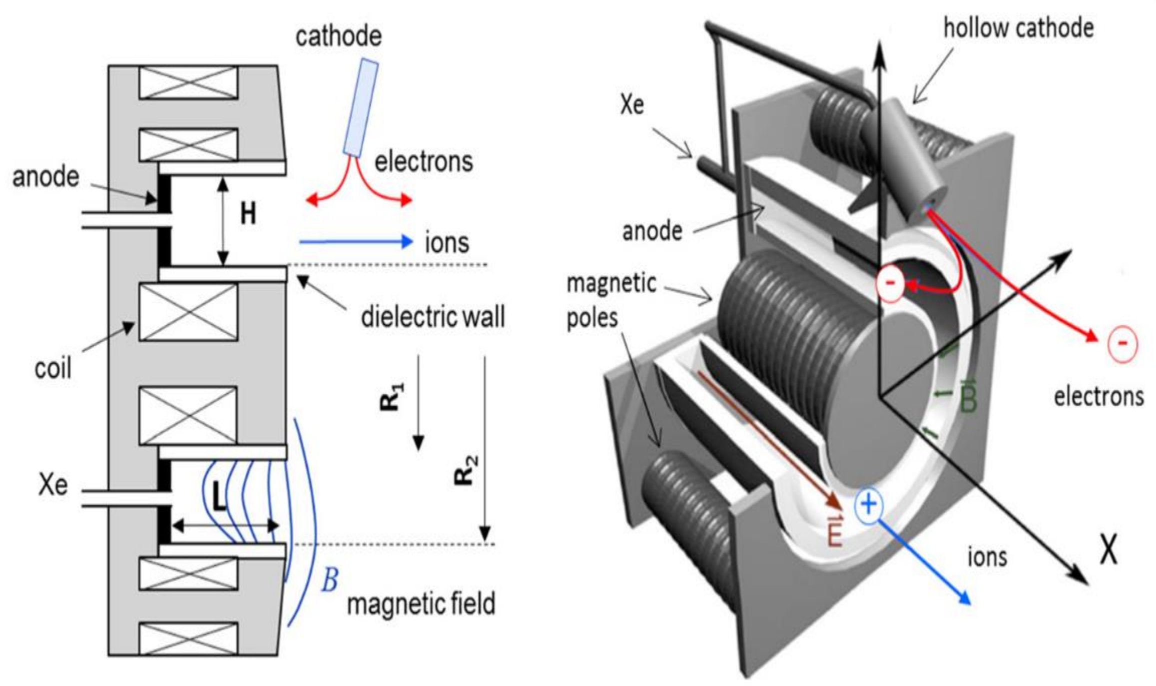
Contents

Motivation

Hall Effect Thruster (HET)



Used for in-space operations due to its high specific impulse compared to chemical propulsion.
Recognized as a **well-established** and **mature** technology



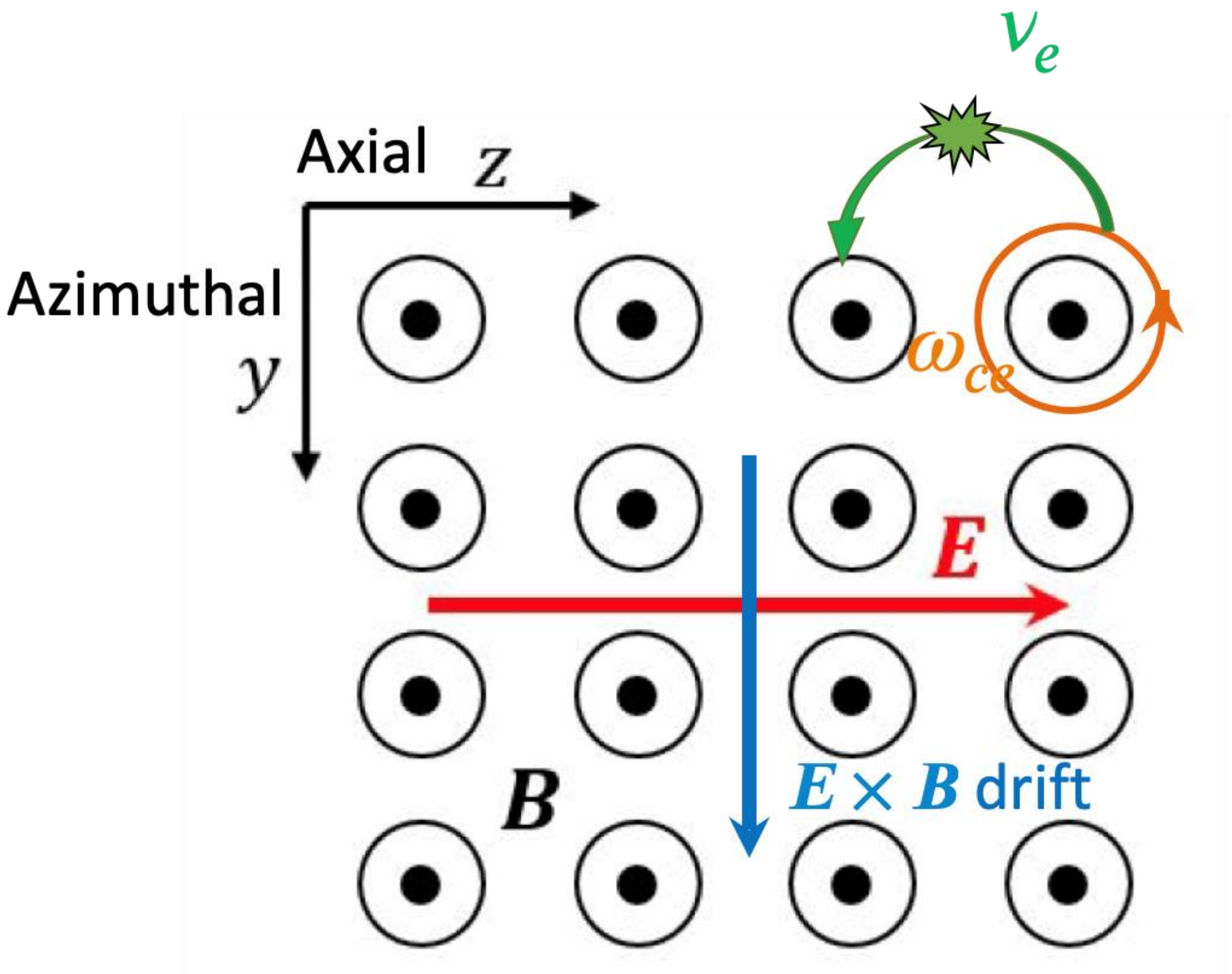
Role of each species and working principle:

- **Electrons** → Injected at cathode, (1) flow inside chamber to ionize neutrals (2) flow downstream to neutralize ion beam
- **Ions (Xe^+)** → Potential difference between cathode and anode accelerate them downstream
- **Neutrals (Xe)** → Injected at the anode, ionized in the chamber

Open problems

Anomalous electron transport and Plasma-wall interaction

Anomalous electron transport in ExB plasmas



1. Gyromotion around magnetic field lines
2. Radial motion along magnetic lines
3. Azimuthal drift in the ExB direction
4. Axial drift due to … collisions (?)

Electron cross-field drift observations show much larger drift. Electron transport possibly enhanced by:

- Plasma-wall interaction (?)
- **Azimuthal oscillations and turbulence** (?)

Main goal of the Thesis:
Theoretical study of main kinetic instabilities in Hall Thruster

What are the main instability modes triggering plasma turbulence in Hall Effect Thrusters?

Develop reliable and computationally feasible numerical models of the plasma discharge

How can we extrapolate those findings into simplified turbulence model for a Hall Thruster ?

This thesis expands on the work previously established by A. Marin-Cebrian and E. Bello-Benitez

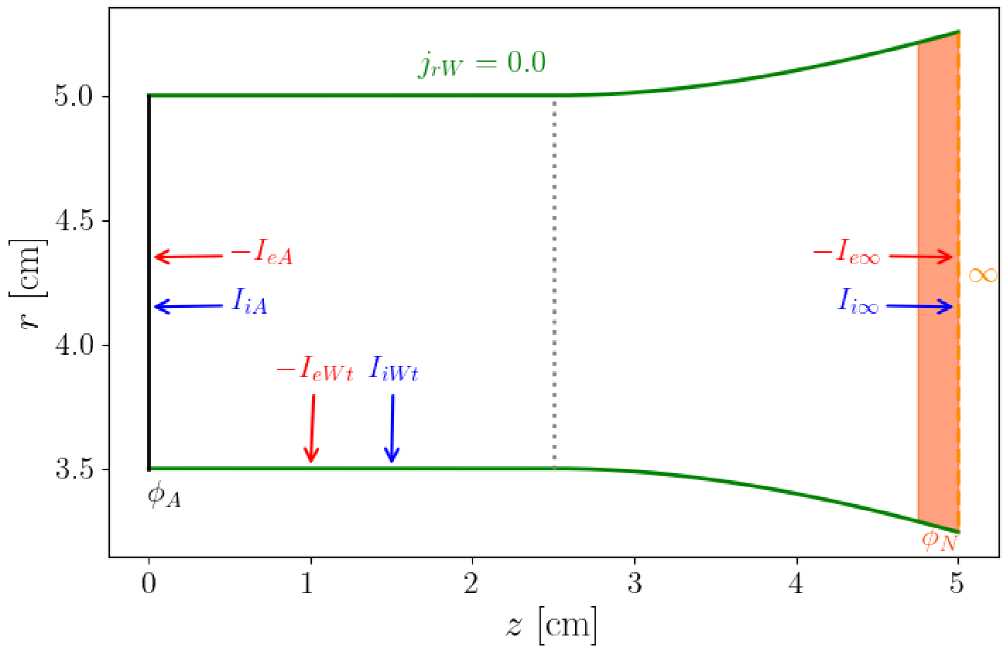
Improvement of PICASO axial-azimuthal code

Lacks some important physics

- Lack of wall interactions
- Simplified neutral treatment

Include plasma wall interactions, and neutrals on axial, and axial-azimuthal simulations of Hall thruster

A 2D PIC model: PICASO



➢ The PICASO code

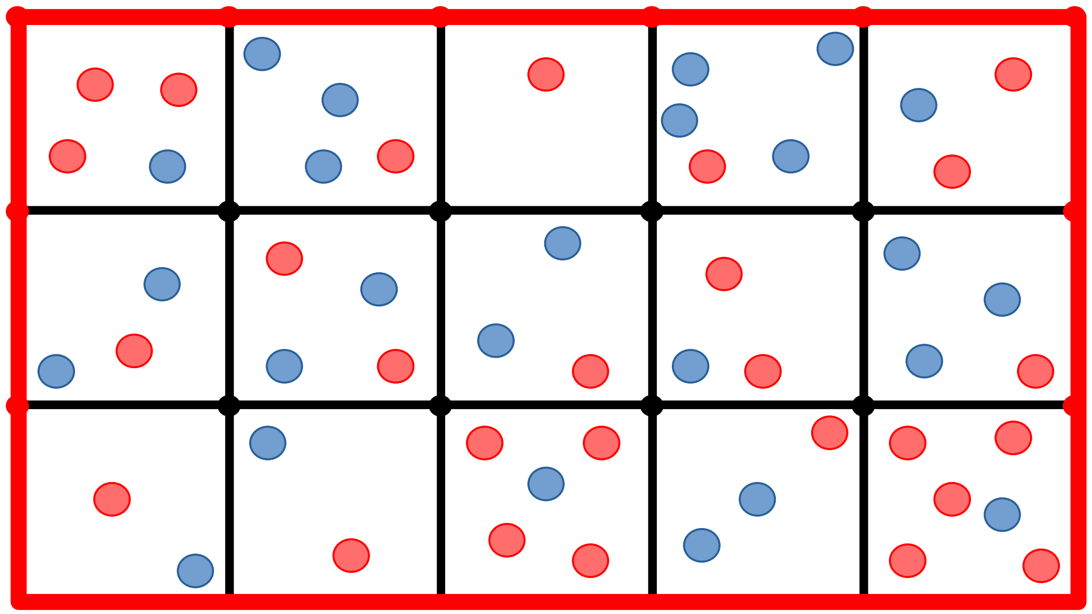
- Developed by E. Bello-Benitez (axial-azimuthal) and A. Marin-Cebrian (axial-radial)
- Solve the Boltzmann equation

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{e_s}{m_s}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \left(\frac{\delta f_s}{\delta t}\right)_{\text{coll}}$$

- Solve for the motion of macroparticles

$$\frac{\mathrm{d}\mathbf{v}_p}{\mathrm{d}t} = \frac{q_s}{m_s}(\mathbf{E}_p + \mathbf{v}_p \times \mathbf{B}_p)$$

$$\frac{\mathrm{d}\mathbf{x}_p}{\mathrm{d}t} = \mathbf{v}_p$$

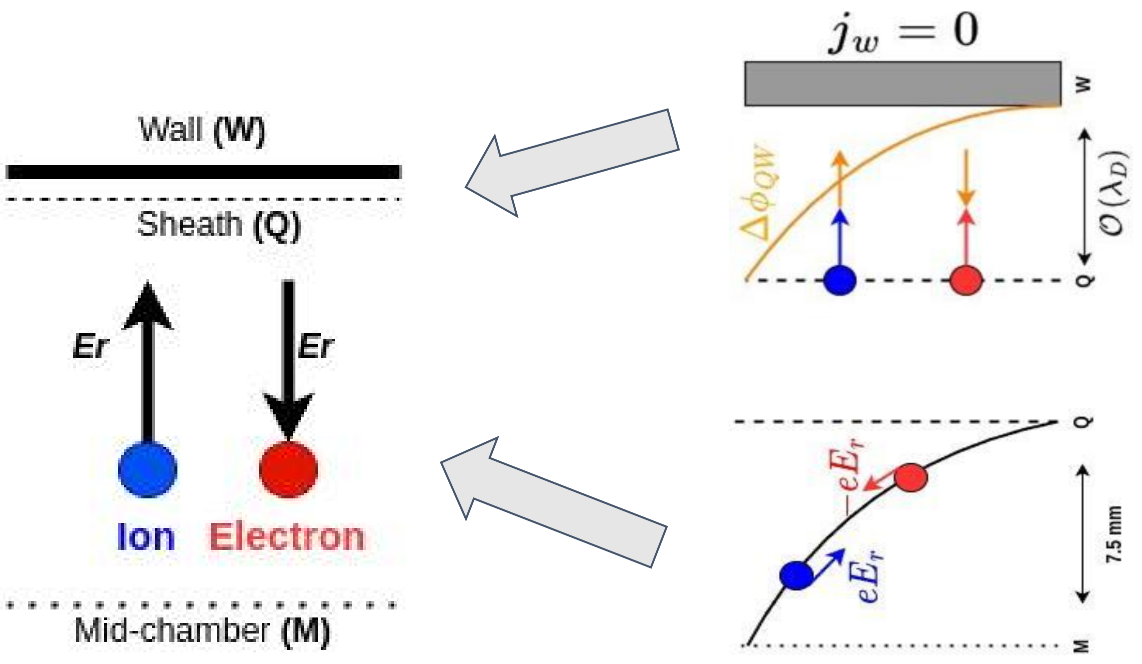


- External magnetic field and solution of electric field from Poisson

$$\nabla^2 \phi = -\frac{e}{\varepsilon_0}(n_i - n_e)$$

- Collisional parameter $\left(\frac{\delta f_s}{\delta t}\right)_{\text{coll}}$

Axial PIC simulation with wall effects



At the **wall** and in the **HET plume**, complex phenomena occur

- Ion recombination
- Collection and emission of electrons
- Plume divergence
- Dielectric walls

To capture these phenomena, we developed a model that:

- ❑ **Move particles in radial direction** with **analytical expressions** for **radial electric field (M–Q)** and **potential drop at the sheath (Q–W)**.
- ❑ **Remove and emits particles** that reach walls while ensuring **dielectric conditions** are met.
- ❑ **Includes plume divergence effects**

$$E_r = \frac{m_i \tilde{u_{ri}}}{e}\left(\frac{\partial \tilde{u_{ri}}}{\partial r} + S_{prod}\right) \qquad \phi_{sh} = \frac{T_e}{e}\ln\sqrt{\frac{m_i}{2\pi m_e}} - \delta\phi_{sh}$$

$$\tilde{u_{ri}}(z,r) = c_s(z)\left(\frac{r - r_M}{r_{W2} - r_M}\right)^3 \qquad \frac{\partial(\delta\phi_{sh})}{\partial t} = \frac{j_{rW}}{\tilde{c}}$$

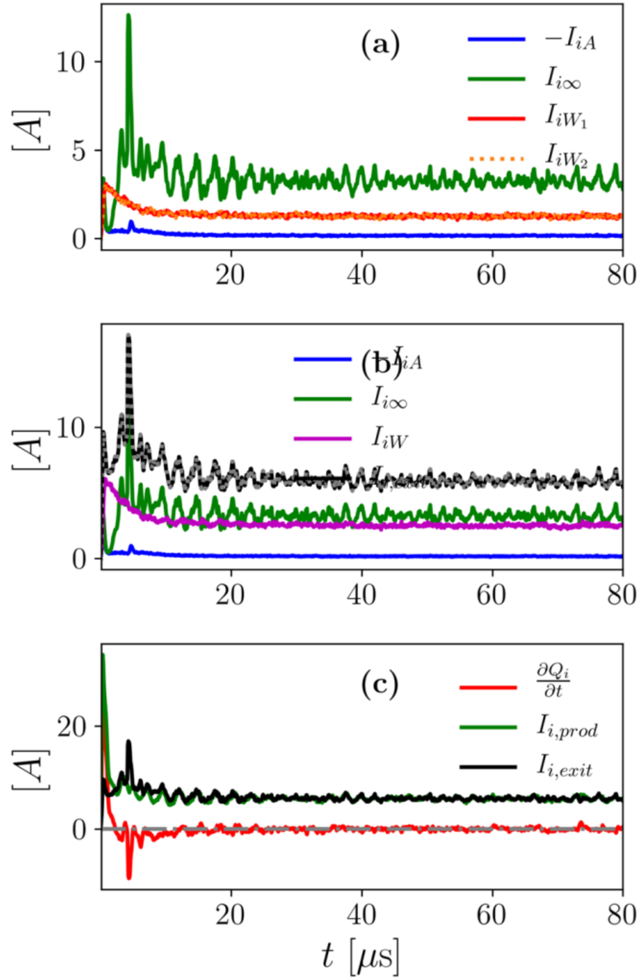
Overcoming previous limitations

- ➢ Particles move radially
- ➢ Self consistent solution ensuring a current free wall
- ➢ Plume divergence effects
- ➢ Secondary electron emission

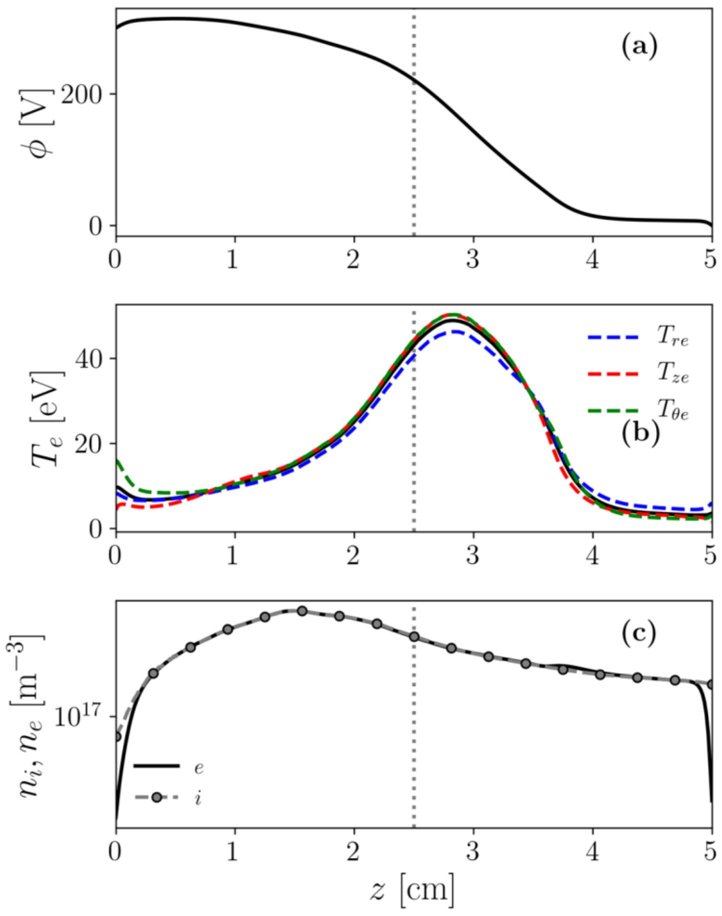
Cuerva et al. , "Towards self-consistent simulation of radial effects on an axial PIC model of the Hall discharge" IEPC, (2024)

Axial PIC simulation with wall effects

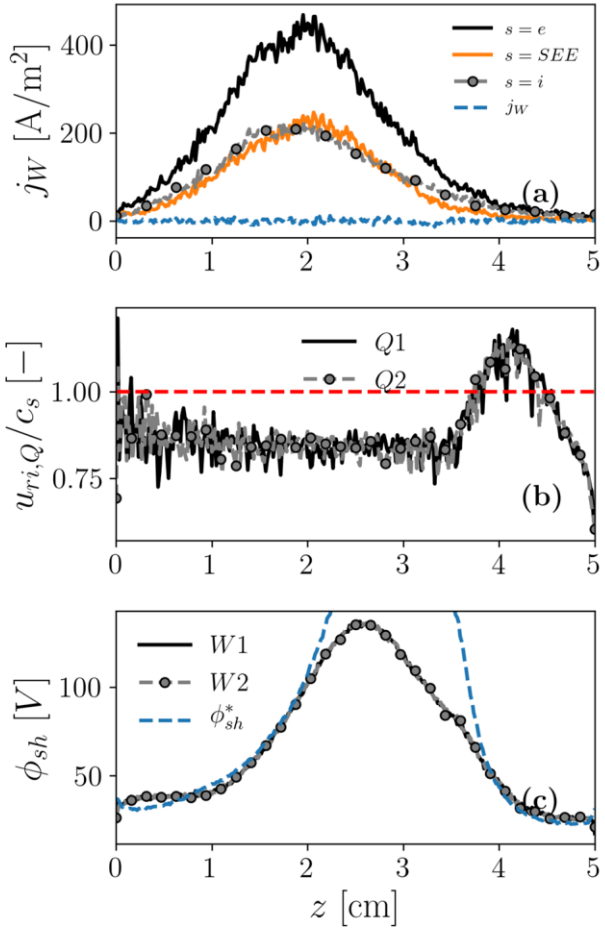
Steady-state solution



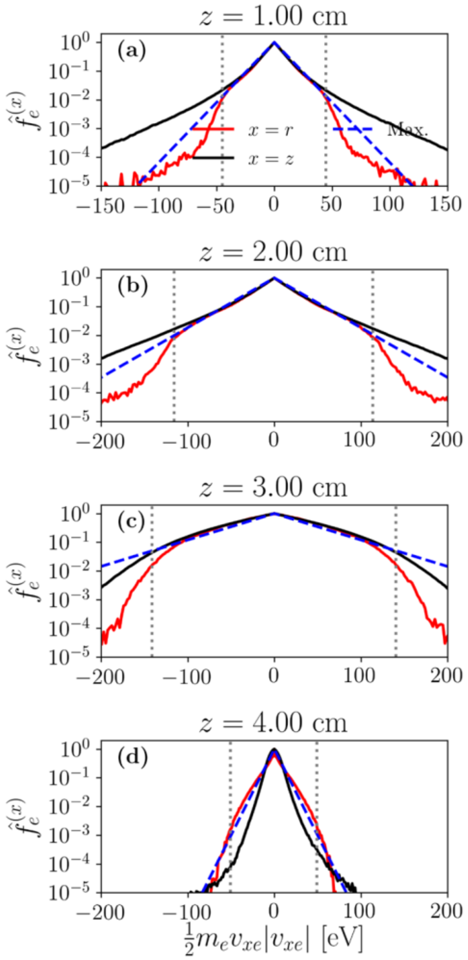
Axial profiles



Wall profiles



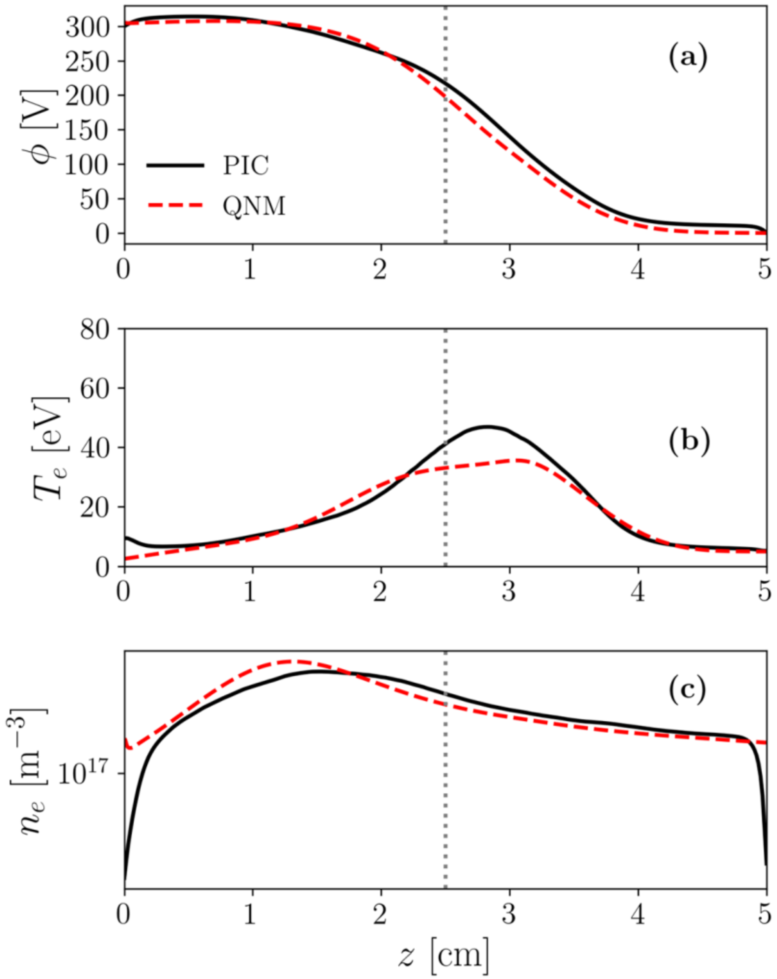
Electron VDFs

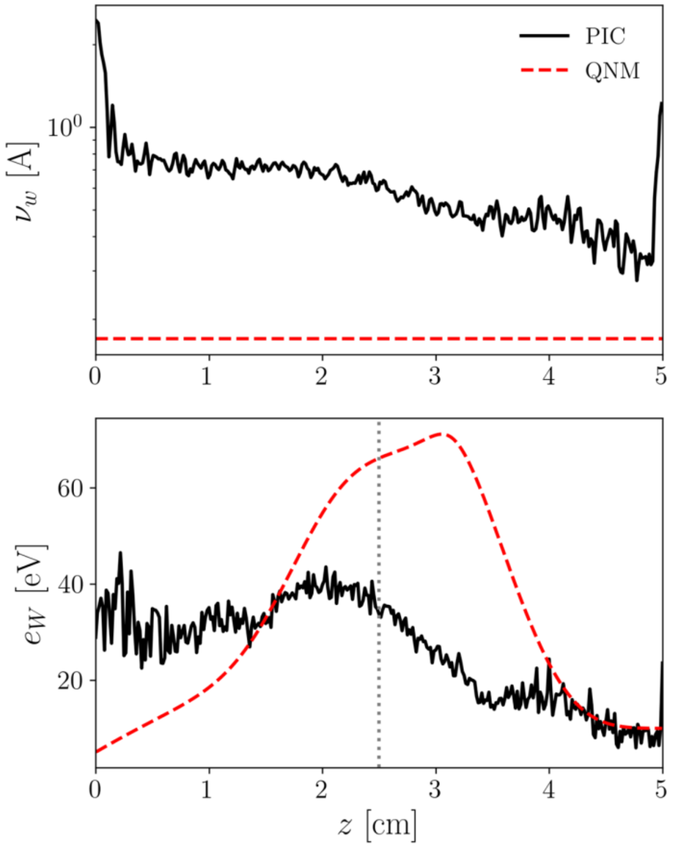


Poli et al. , "Time-dependent axial fluid model of the Hall thruster discharge and its plume," Journal of Physics D: Applied Physics, (2023)

Axial PIC simulation with wall effects

Quasineutral electron fluid model assumptions	Kinetic results
Maxwellian electron VDF.	**Non-Maxwellian** electron VDF.
Scalar pressure.	**Pressure** is a full **tensor**. Gyrotropic + Gyroviscous
Fourier law as heat flux closure.	**Complex energy flux**.





- **In fluid**, the wall collision parameter is constant along the axial direction, and the impact energy follows a 2*Te dependence.
- **PIC** results show a significantly higher effective wall collision rate compared to fluid models, with wall energy varying along the axial direction and showing no dependence on Te.

$$I_{d,PIC} \simeq 4.91A$$
$$I_{d,QNM} \simeq 4.86A$$

GPU-based kinetic simulations of Hall Thruster

CPU-based PIC codes on OpenMP have some numerical disadvantages

- Limited to 40 threads
- Augmented vacuum permittivity to ease up numerical constraints and obtain feasible computational times

Develop a new axial-azimuthal PIC code on GPU to study simplified Hall thruster scenarios with no artificial scaling

GPU kinetic simulations

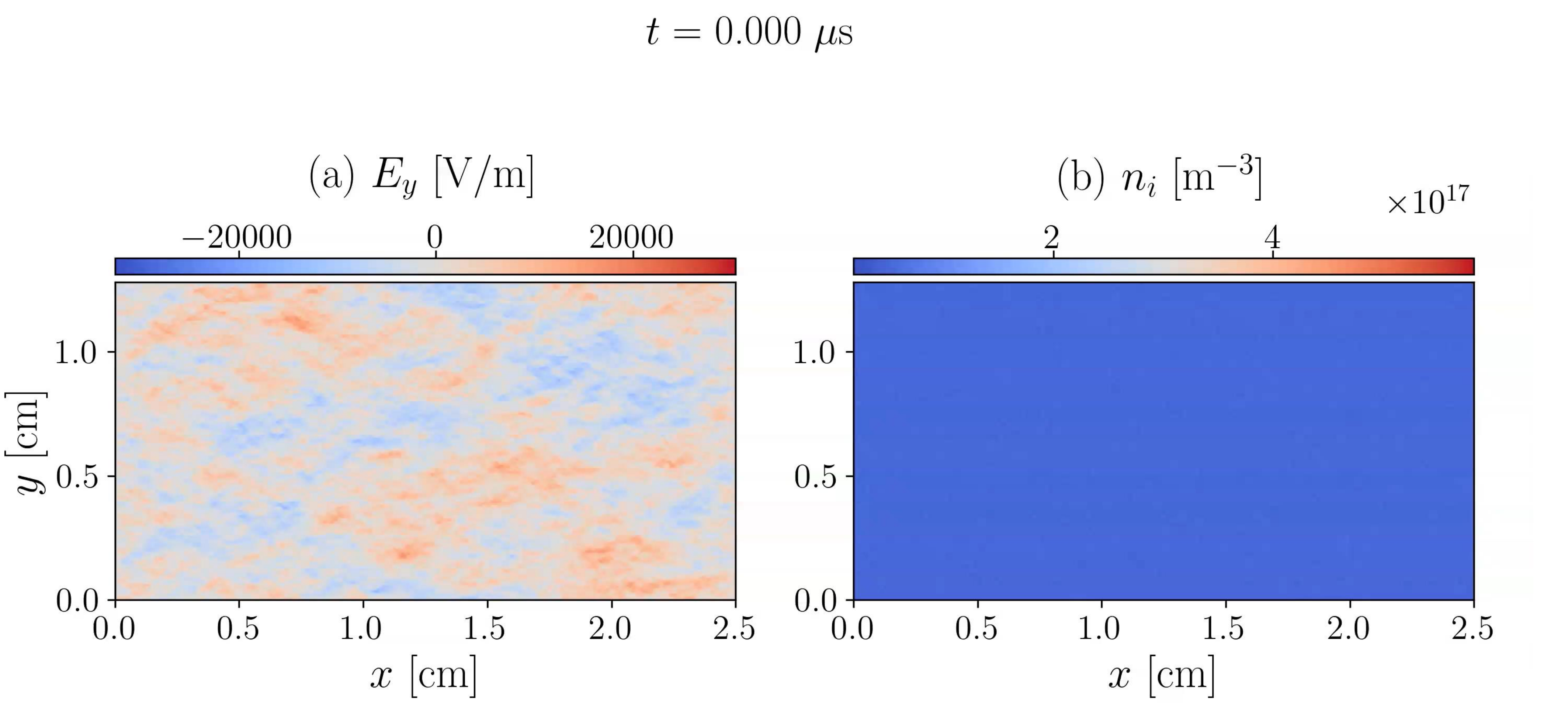
Limitations on CPU-based simulations of HET

- **Require fine resolution to resolve electron dynamics**
  - Debye length (~10^{-6} m)
  - Electron timescales (~10^{-12} s)
- **Must be run long enough to capture quasi-steady state**
  - Length scales ~10^{-1} m
  - Time scales ~10^{-3} s
- **Artificial scaling modify the onset of instabilities**

Why → GPUs ?

1. Allows for massively parallelization → up to 10x - 100x faster than CPU-based codes
2. No artificial scaling on the vacuum permittivity
3. Allows high resolution + long-time simulations
4. Improves statistical accuracy with more particles
5. Support fast parametric studies for design and optimization

GPU kinetic simulations



GPU kinetic simulations

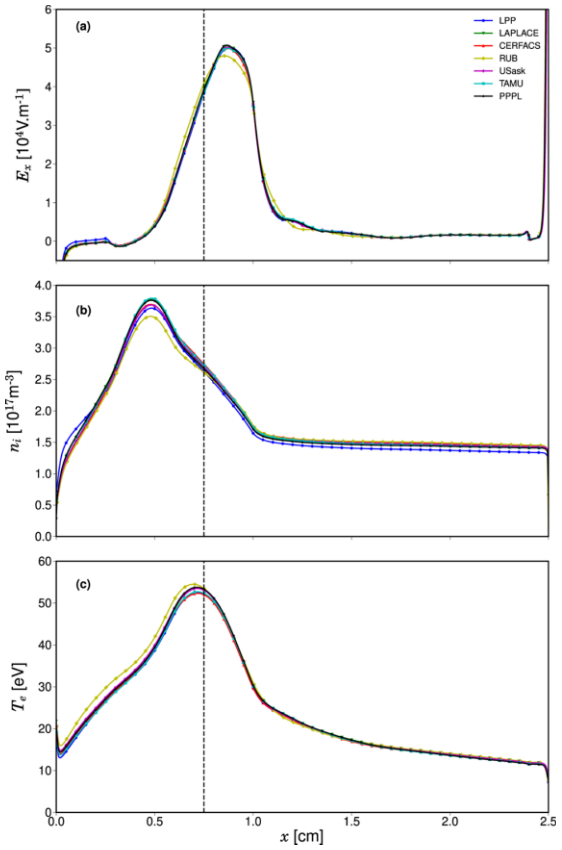
T Charoy *et al.* , “2D axial-azimuthal particle-in-cell benchmark for low-temperature partially magnetized plasmas”. 2019 *Plasma Sources Sci. Technol.*

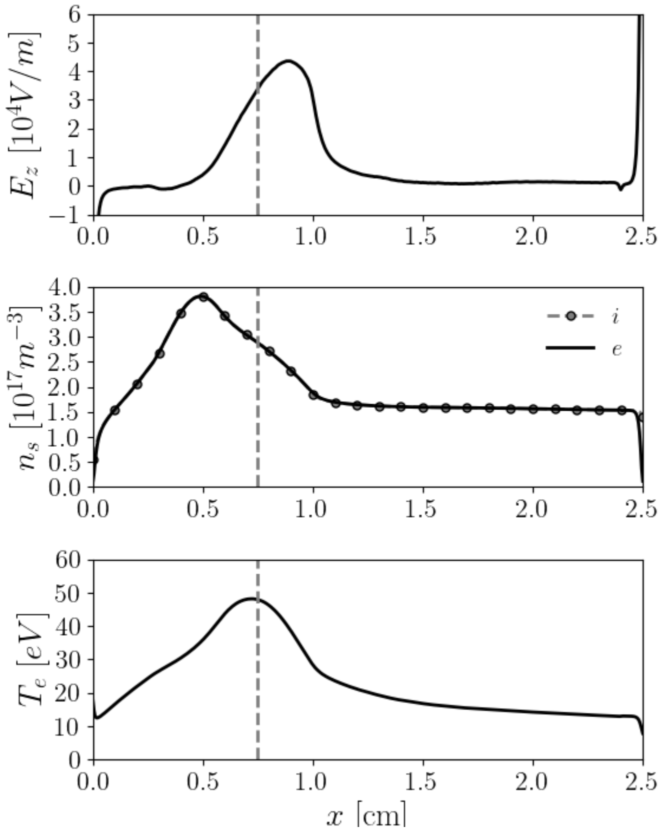
Marks, T.A., Gorodetsky, A.A. GPU-accelerated kinetic Hall thruster simulations in WarpX. J Electr Propuls 4, 34 (2025)

Simulate Case 2 of the **LANDMARK low-temperature plasma benchmark**.
Focuses on capturing **key kinetic effects in a Hall effect thruster discharges**:

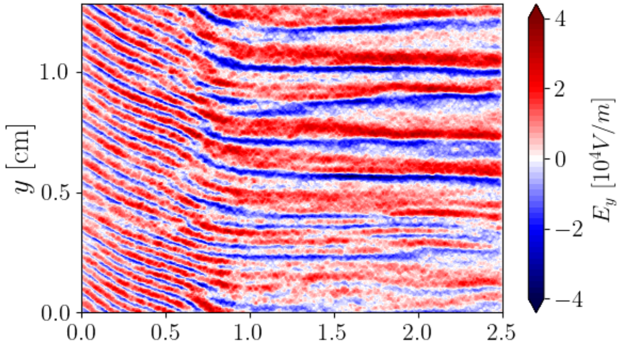
- **Onset and growth of azimuthal instabilities.**
- Plasma acceleration in the axial direction

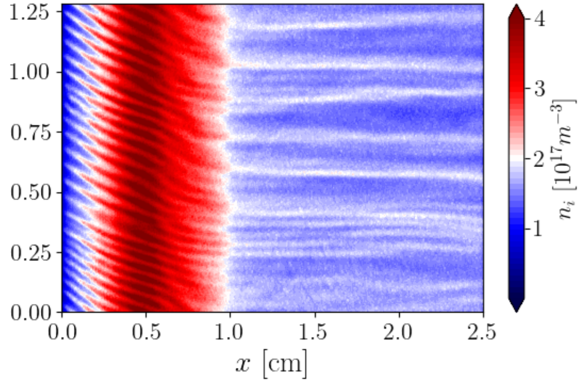
Benchmark of multiple PIC codes from the **plasma propulsion community**





Agreement with published results and validates the code for future axial-azimuthal simulations of the Hall thruster.





Performance of GPU compared to PIC codes

PICASO-G (A30)	1.88 days
PPPL (112 CPU)	2.5 days
WarpX (H100/ V100)	1.81 days/ 3.81 days

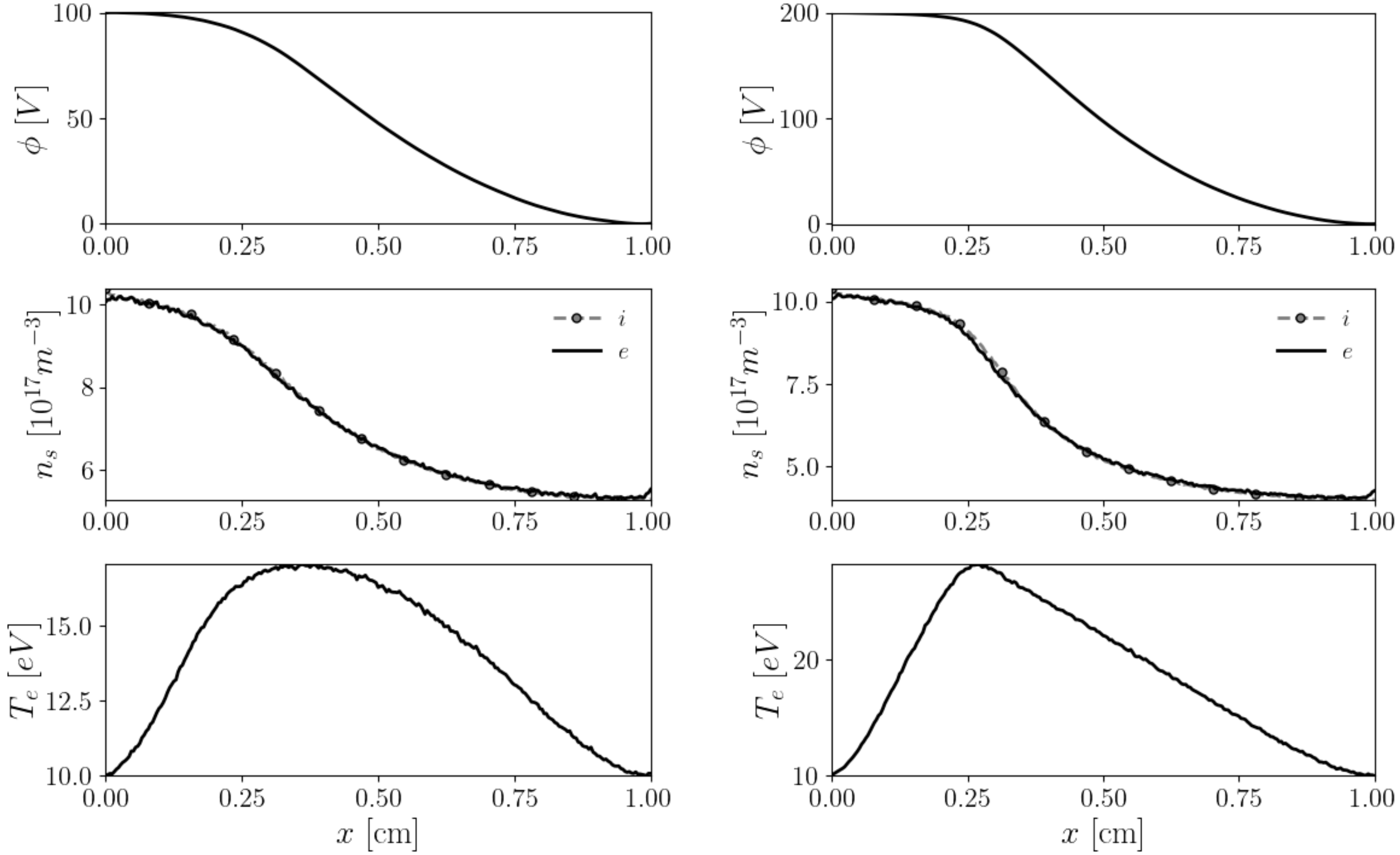
GPU kinetic simulations

Benchmark has some limitations:

- Highly **dependant** on **ionization source** profile
- **Low magnetic field** compared to real-HET
- **Growth and saturation of instabilities are difficult to analyze**

Growth and saturation of micro-instabilities on HET **are not fully understood**

Focus on **SPT100 acceleration region** with near-operational parameters → **Steady-state solutions without anomalous transport**
Representative axial profiles → **Strong baseline for realistic operation**



Summary and current/future work

Axial PIC simulation with wall effects

- **Implementation of a simplified model for plasma wall interactions**, capable of
  - Dielectric walls in steady state
  - Secondary electron emission
  - Plume divergence effects
- Include this model on axial-azimuthal Hall Thruster simulations
- Develop a self-consistent fluid model for the neutral dynamics

GPU-based kinetic simulations

- Benchmark the GPU code with the axial-azimuthal benchmark case and obtain significantly enhance in the performance compared to other CPU and GPUs codes
- New simulations which overcome the limitations of this benchmark and shade some light on the main instability mechanisms driving anomalous transport in Hall Effect Thruster.

Results

Conferences

- “Towards self-consistent simulation of radial effects on an axial PIC model of the Hall discharge” at 38th International Electric Propulsion Conference, Toulouse, France, June 2024.
- “Non-linear ECDI and Anomalous Transport in ExB plasmas via GPU-Accelerated PIC Simulations”, at 39th International Electric Propulsion Conference, London, England, September 2025.

Manuscript preparation

- “Towards self-consistent simulation of radial effects on an axial PIC model of the Hall discharge”, under preparation.

Thank you! Questions?

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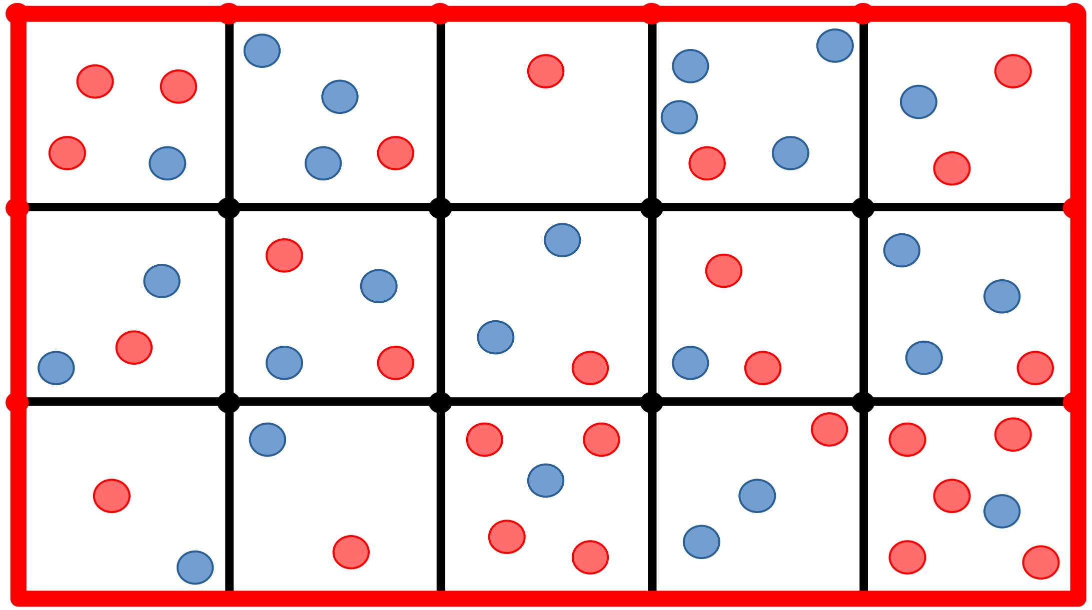
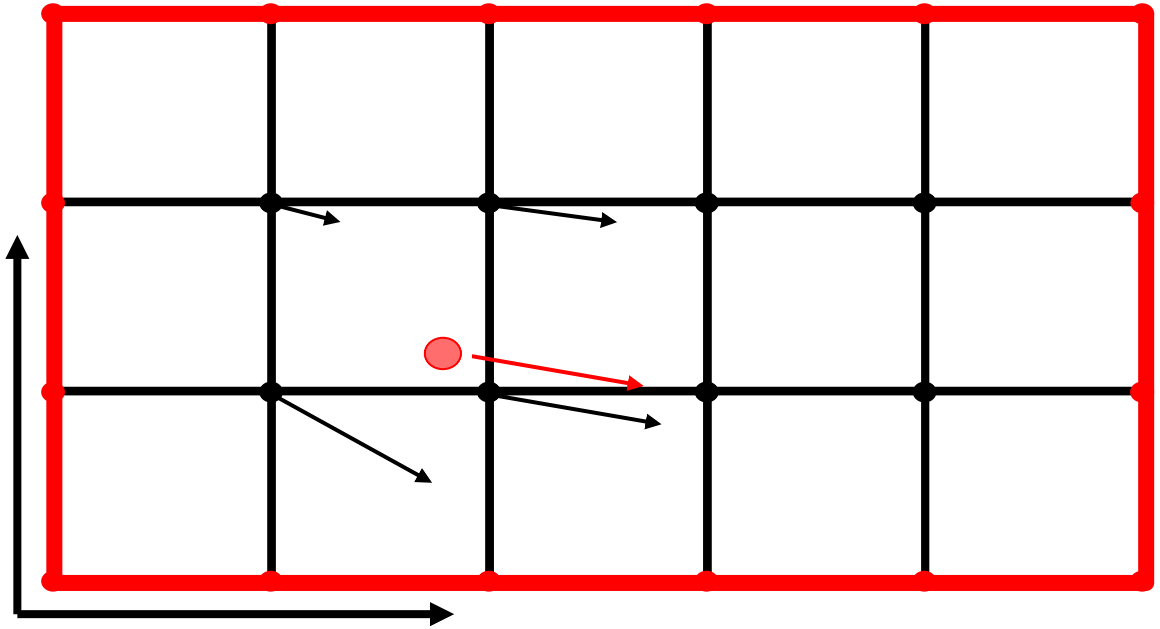
Backup slides

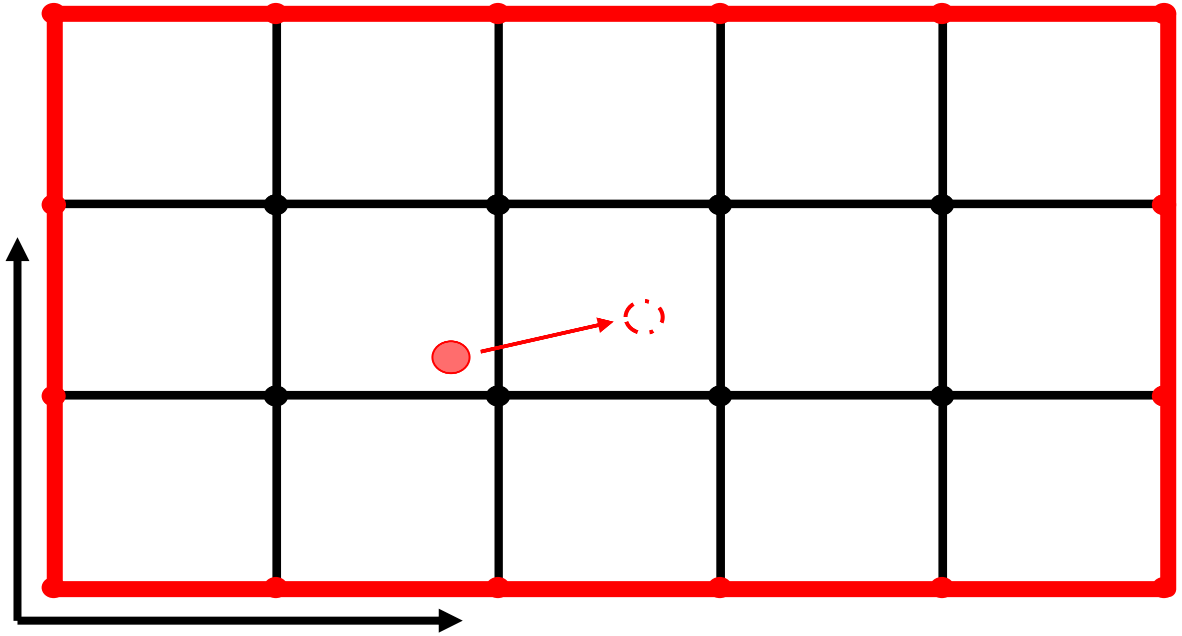
A 2D PIC model: PICASO

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\nabla_x f+\frac{q}{m}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\nabla_v f=\left[\frac{\partial f}{\partial t}\right]_{coll}$$

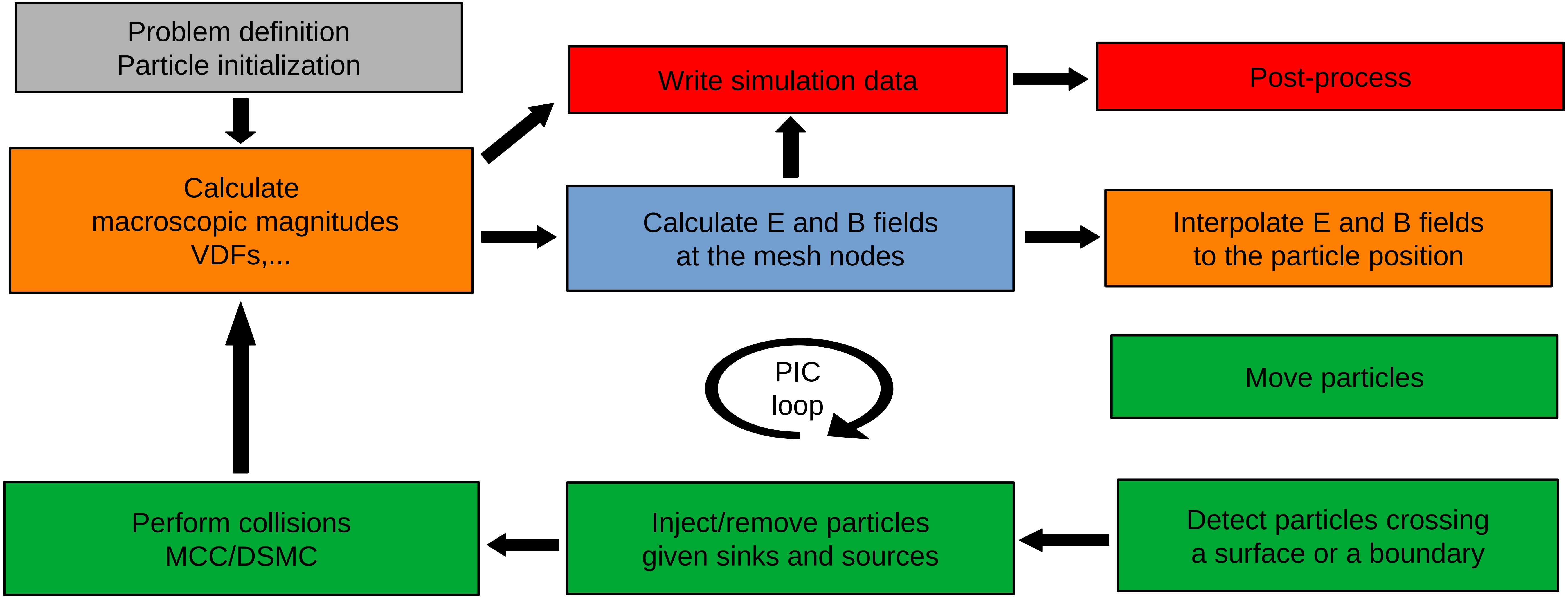
Simulated macroparticles = 10^5–10^8 real particles





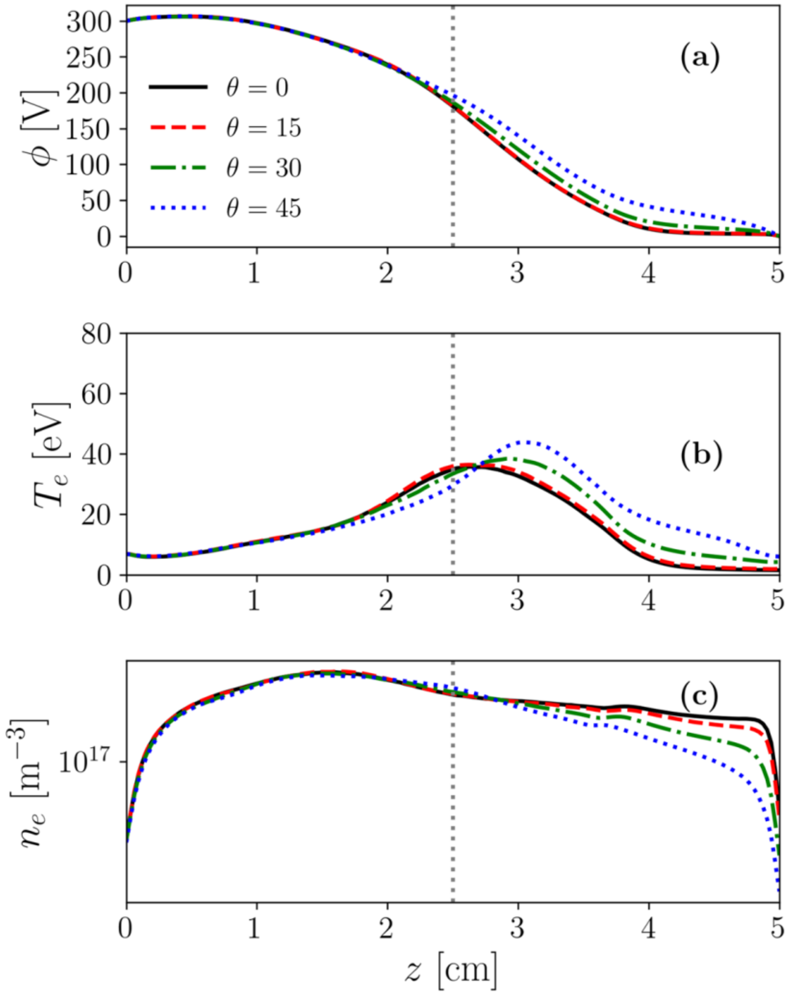


The particle-in-cell approach

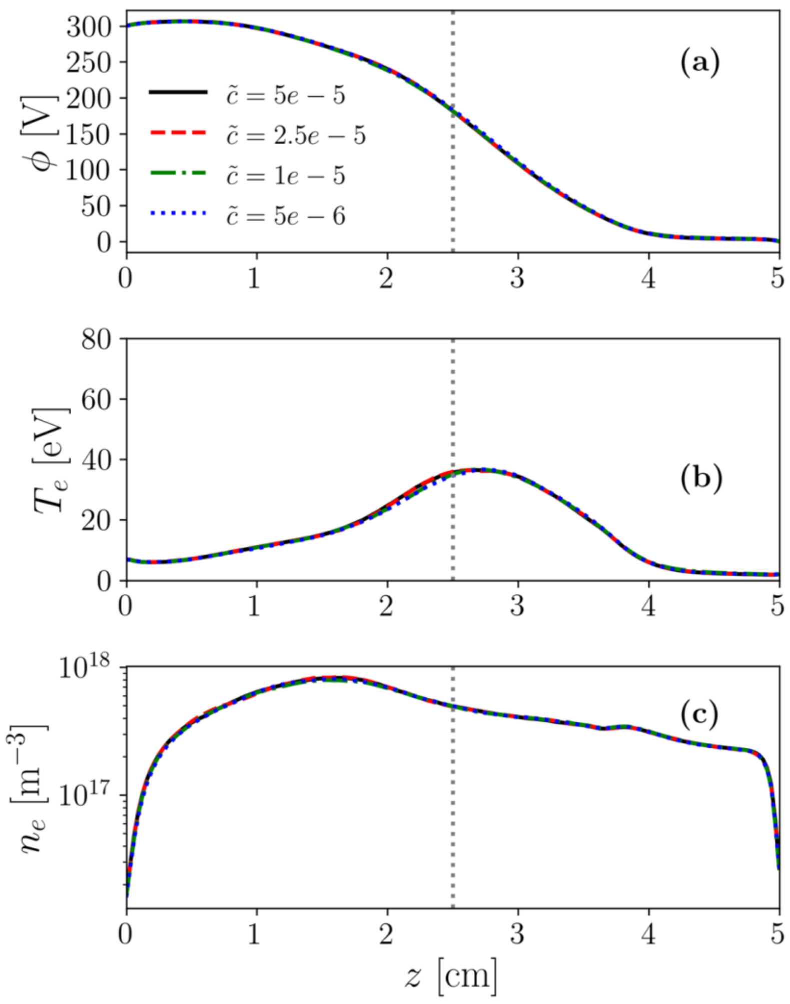


Axial PIC simulation with wall effects

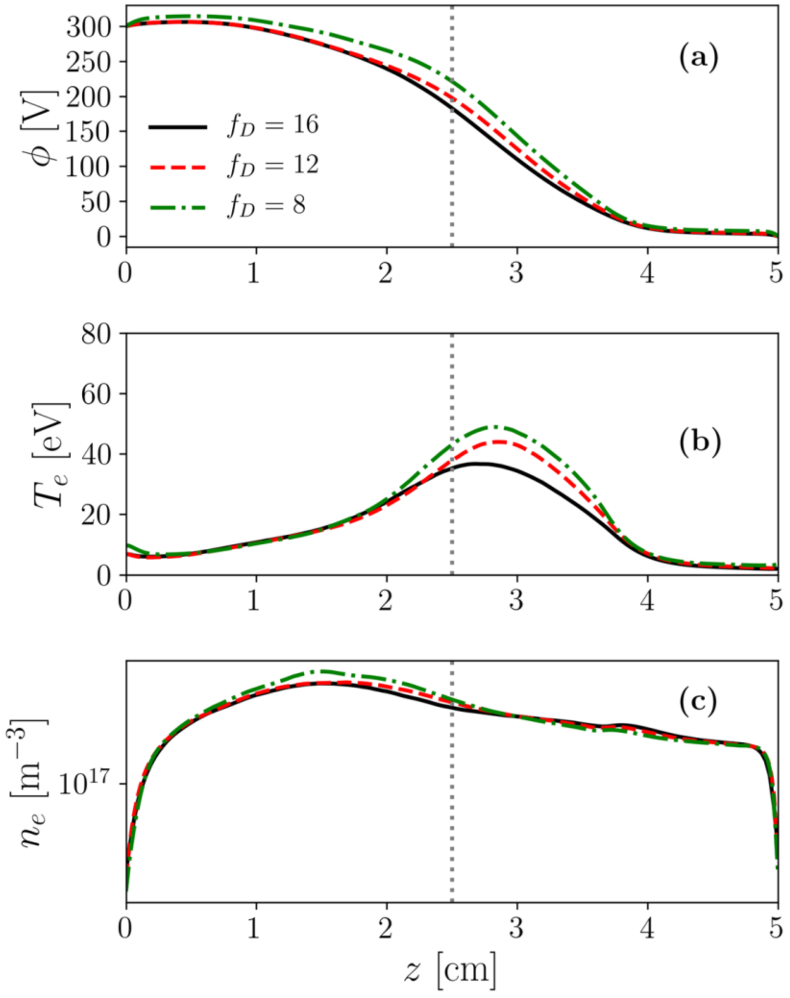
Plume divergence effects



Capacitance invariance



Augmented vacuum permittivity



ECDI: Anomalous transport

Type of instability where free energy is provided by the relative drift between ions and electrons → Driven by an ion beam streaming across a magnetic field (HET conditions)

Linear theory:

- ➢ Homogeneous equilibrium
- ➢ Propagation perpendicular to magnetic field
- ➢ Maxwellian magnetized electrons with ExB drift
- ➢ Cold unmagnetized ions

$$g(\omega_e, b_e; \omega_{ce}) = \exp(-b_e)\left[I_0(b_e) + 2\sum_{m=1}^{\infty}\frac{\omega_e^2 I_m(b_e)}{\omega_e^2 - m^2\omega_{ce}^2}\right].$$

$$\frac{n_{e1}}{n_0} = [1 - g(\omega_e, b_e; \omega_{ce})]\frac{e\phi_1}{T_{e0}}$$

$$\frac{n_{i1}}{n_0} = \frac{k^2 c_s^2}{\omega_i^2}\frac{e\phi_1}{T_{e0}}$$

$$1 + k^2\lambda_{D0}^2 = \frac{k^2 c_{s0}^2}{\omega_i^2} + g(\omega_e, b_e; \omega_{ce}).$$

Solutions of the dispersion relation

1. Ion acoustic wave $g \ll k^2 c_{s0}^2/\omega_i$ $\quad \omega_{\mathrm{IA}} = \frac{k c_{s0}}{\sqrt{1 + k^2\lambda_{D0}^2}},$
1. Berstein waves $g \gg k^2 c_{s0}^2/\omega_i$ $\quad \omega_i = \pm\frac{k c_{s0}}{\sqrt{1 + k^2\lambda_{D0}^2 - g(\omega_e)}}.$
1. Unstable acoustic mode → ECDI $\quad g > 1 + k^2\lambda_{D0}^2,$