Turbulent transport characterization in Hall Effect Thrusters

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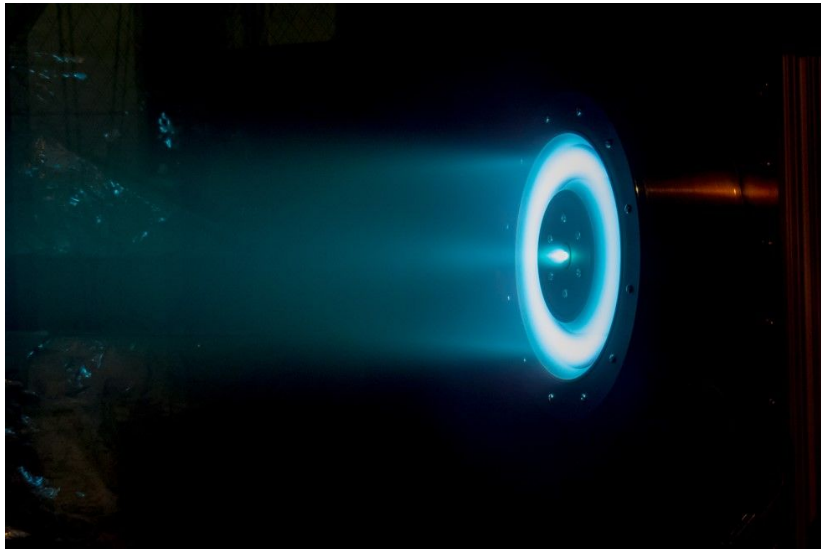
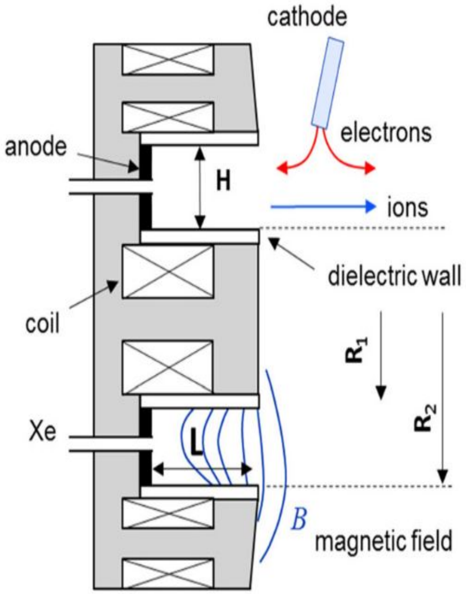
2nd and 3rd June 2026

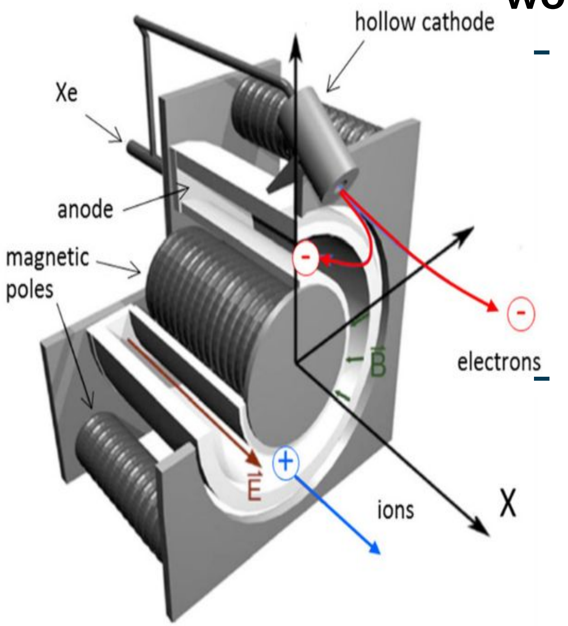
Motivation

Hall Effect Thruster (HET)

Used for in-space operations due to its high specific impulse compared to chemical propulsion.
Recognized as a **well-established** and **mature** technology







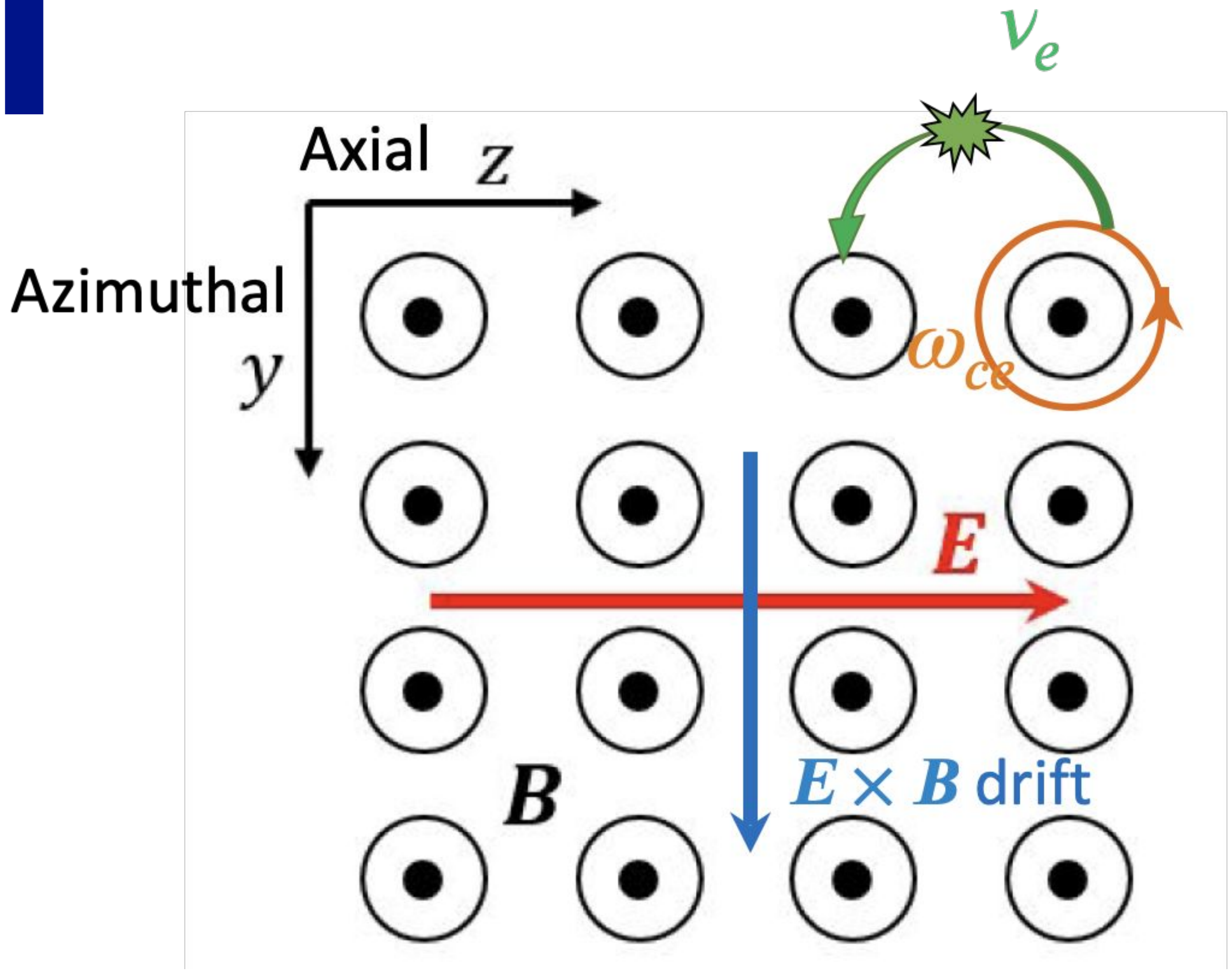
Role of each species and working principle:

- **Electrons** → Injected at cathode, (1) flow inside chamber to ionize neutrals (2) flow downstream to neutralize ion beam
- **Ions (Xe^+)** → Potential difference between cathode and anode accelerate them downstream
- **Neutrals (Xe)** → Injected at the anode, ionized in the chamber

Open problems

Anomalous electron transport and Plasma-wall interaction

Anomalous electron transport in ExB plasmas



1. Gyromotion around magnetic field lines
2. Radial motion along magnetic lines
3. Azimuthal drift in the ExB direction
4. Axial drift due to … collisions (?)

Electron cross-field drift observations show much larger drift. Electron transport possibly enhanced by:

- Plasma-wall interaction (?)
- **Azimuthal oscillations and turbulence** (?)

Main goal of the thesis

Numerical study of main kinetic instabilities in Hall Thruster

What are the main instability modes triggering plasma turbulence in Hall Effect Thrusters?

Develop reliable and computationally feasible numerical models of the plasma discharge

How can we extrapolate those findings into simplified turbulence model for a Hall Thruster ?

This thesis expands on the work previously established by **A. Marin-Cebrian** and **E. Bello-Benitez**

Particle in cell approach

Basically, I like to move particles!

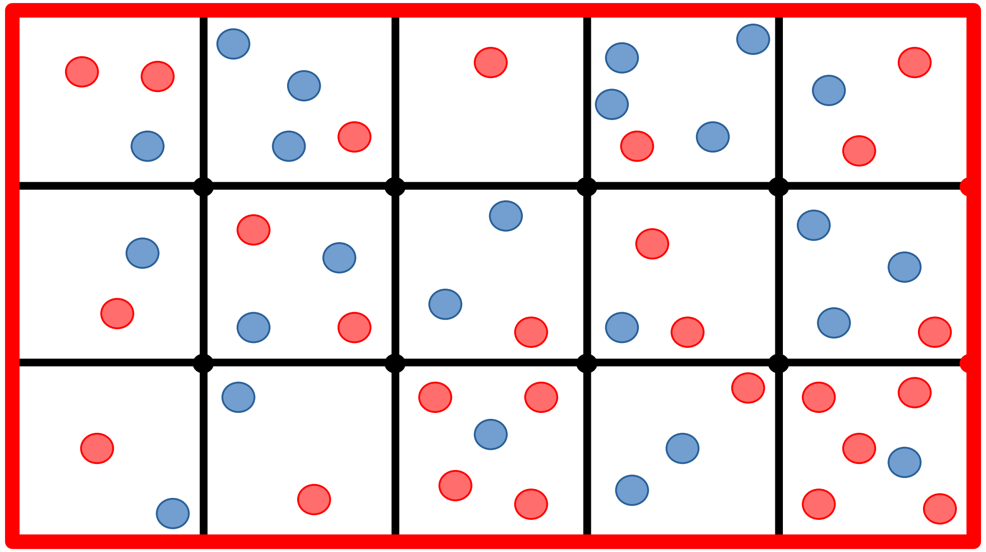
$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{e_s}{m_s}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

$$\nabla^2 \phi = -\frac{e}{\varepsilon_0}(n_i - n_e)$$

$$\frac{\mathrm{d}\mathbf{v}_p}{\mathrm{d}t} = \frac{q_s}{m_s}(\mathbf{E}_p + \mathbf{v}_p \times \mathbf{B}_p)$$

$$\frac{\mathrm{d}\mathbf{x}_p}{\mathrm{d}t} = \mathbf{v}_p$$

Simulated macroparticles = 10^5-10^8 real particles



Improvement of PICASO axial-azimuthal code

Lacks
imp
ph

all

- Si
neutra
treatme
- Comparis
with fluid
odels

Include
neutrals
on axial, and a
ations of Hall
thruster

GPU-based kinetic simulations of Hall Thruster

CPU-based PIC codes on OpenMP have some numerical disadvantages

- Limited to 40 threads
- Augmented vacuum permittivity to ease up numerical constraints and obtain feasible computational times

Develop a new axial-azimuthal PIC code on GPU to study simplified Hall thruster scenarios with no artificial scaling

GPU kinetic simulations

uc3m PHD

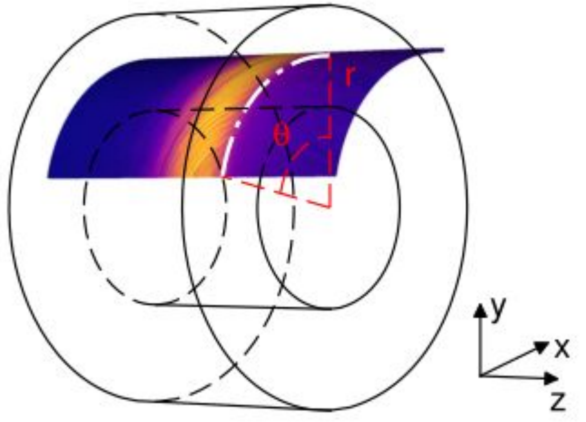
Limitations on CPU-based simulations of HET

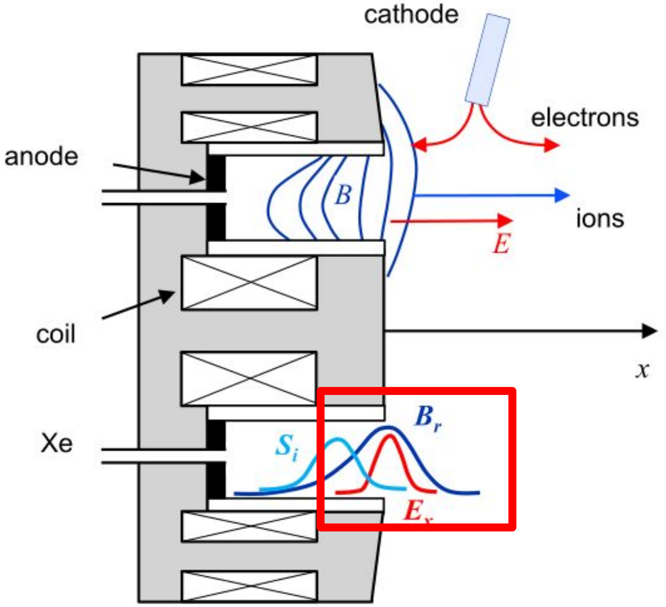
- **Require fine resolution to resolve electron dynamics**
  - Debye length (~10^{-6} m)
  - Electron timescales (~10^{-12} s)
- **Must be run long enough to capture quasi-steady state**
  - Length scales ~10^{-1} m
  - Time scales ~10^{-3} s
- **Artificial scaling modify the onset of instabilities**

Why GPUs ?

1. **Allows for massively parallelization → up to 10x - 100x faster than CPU-based codes**
2. **No artificial scaling on the vacuum permittivity**
3. **Allows high resolution + long-time simulations**
4. **Improves statistical accuracy with more particles**
5. **Support fast parametric studies for design and optimization**

Simplified scenario



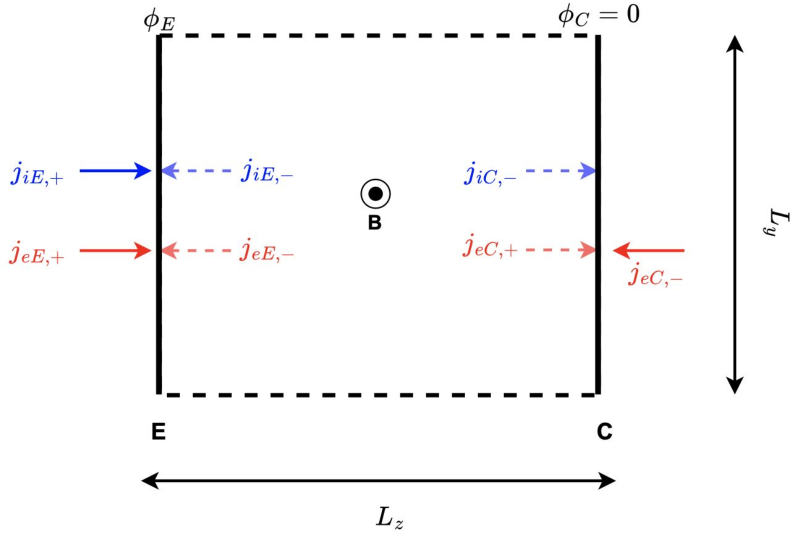


Anomalous cross field electron transport

Acceleration region of Hall Thrusters → driven by azimuthal instabilities drive

$$\Delta\phi = 150V \quad B_0 = 50G$$

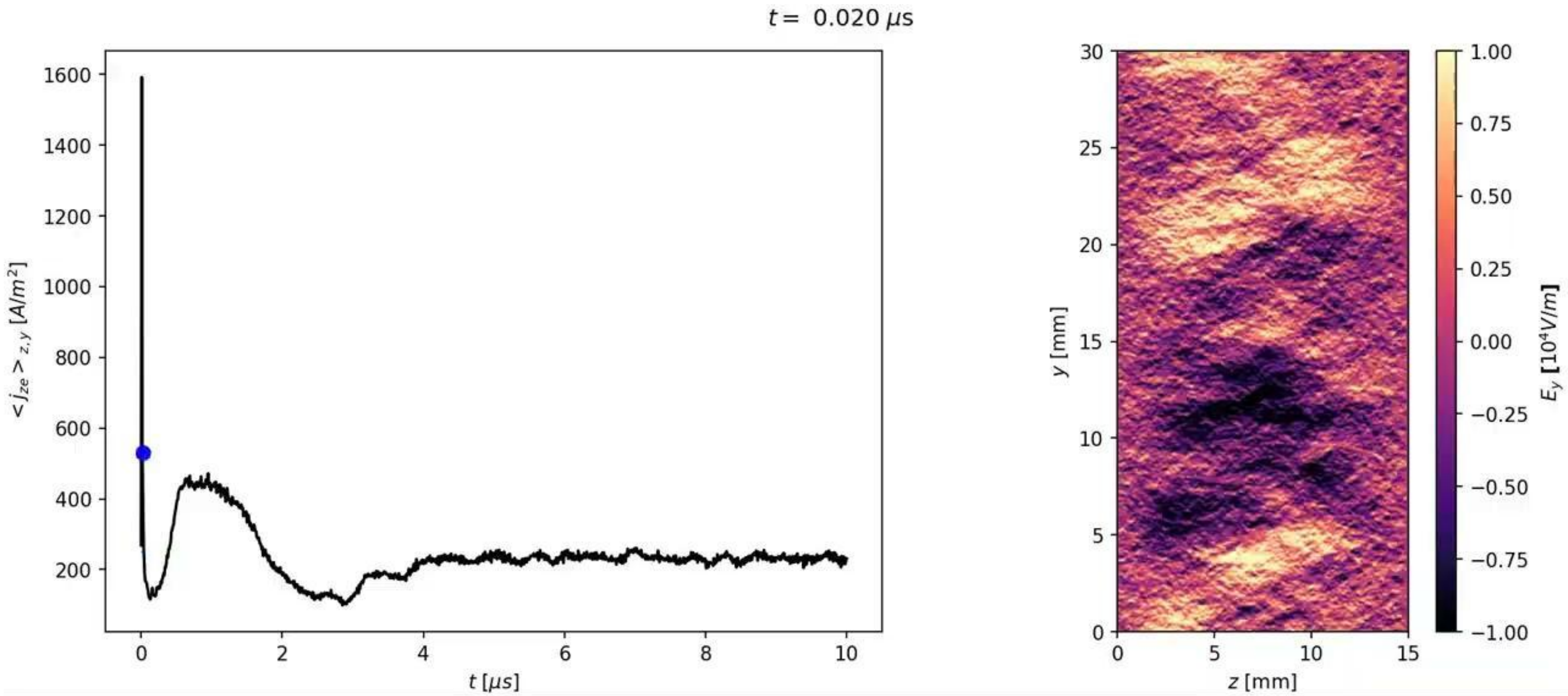
$$n_0 = 2 \times 10^{17} m^{-3}$$



2D(z,θ) kinetic simulation of the acceleration region

- **Simplified configuration** with the **minimum number of free parameters**
- **Analysis of the impact** of instabilities **without added complexity** of magnetic field gradients, collisions, etc ..
- **Parametric studies** → scaling laws and dimensional analysis

Transient response



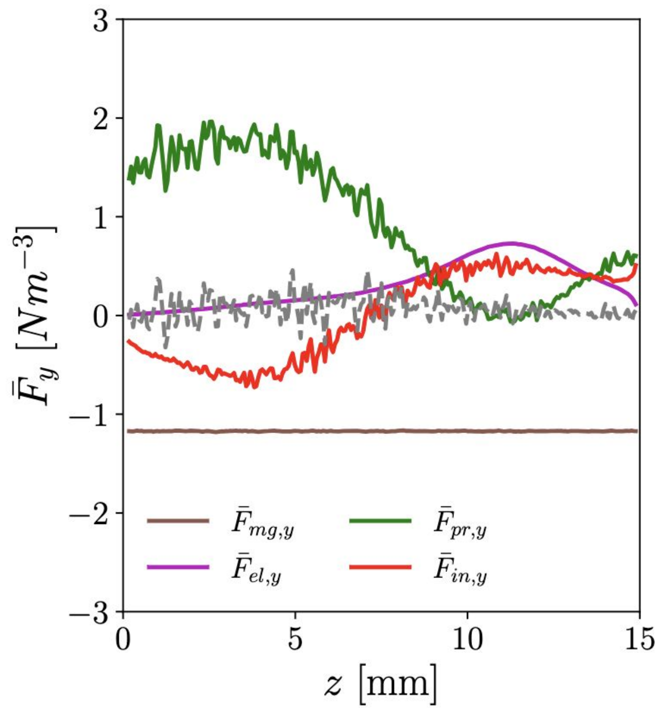
Time evolution of anomalous current → generated by the onset and saturation of azimuthal instabilities.

Time-average response

Electron azimuthal momentum balance

Generated by instabilities — **Finite Larmor radius effects**

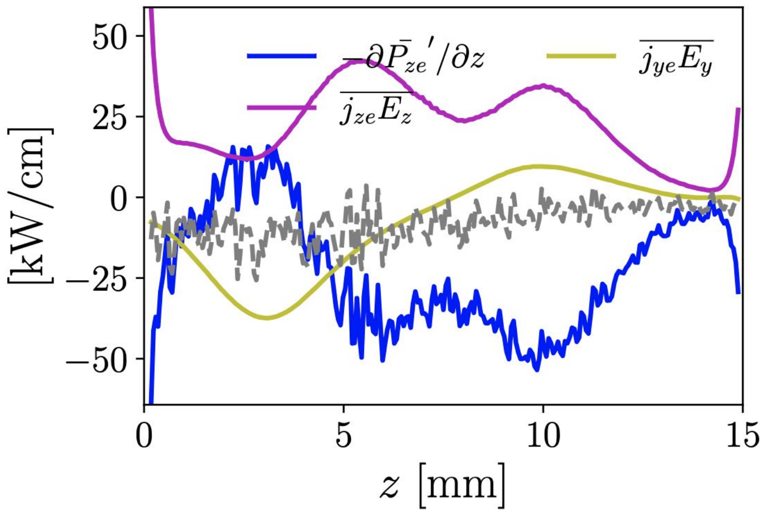
$$-j_{ze}B = \bar{F}_{ano} \equiv \bar{F}_{el,y} + \bar{F}_{pr,y} + \bar{F}_{in,y} \qquad \frac{\partial \bar{M}_{zy}}{\partial z} = \bar{F}_{pr,y} + \bar{F}_{in,y}$$



- **Electric**, **pressure** and **inertia** forces balance magnetic force.
- **Pressure** is of gyro viscous character
- **Electric force** coincides with generation of large coherent structures in 2D maps

Electron energy balance

$$\nabla \cdot \mathbf{P}''_\mathbf{s} = \mathbf{j_s} \cdot \mathbf{E} = j_{sz}E_z + j_{sy}E_y$$



- **Turbulent contribution** from the **correlation between azimuthal current and electric field**
- **Shapes of jzs*Ez are not homothetic due to correlations between jze and Ez**

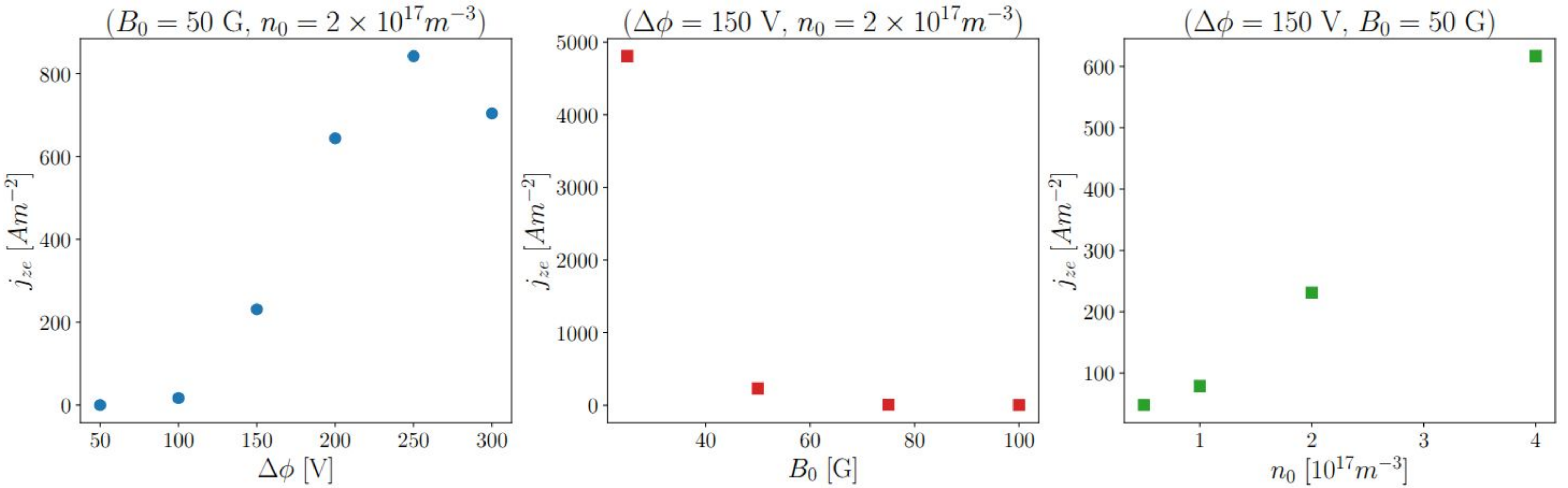
Parametric study

Selecting **three key input parameters** and vary them to see their effect on the anomalous electron current

$\Delta\phi$ → **Non-monotonic behavior** → increasing ExB drift does not imply higher anomalous current

B_0 → **Classical results with magnetic field** → increasing magnetic field drastically reduces anomalous current

n_0 → Anomalous current proportional to increasing electron density



Numerical thermalization

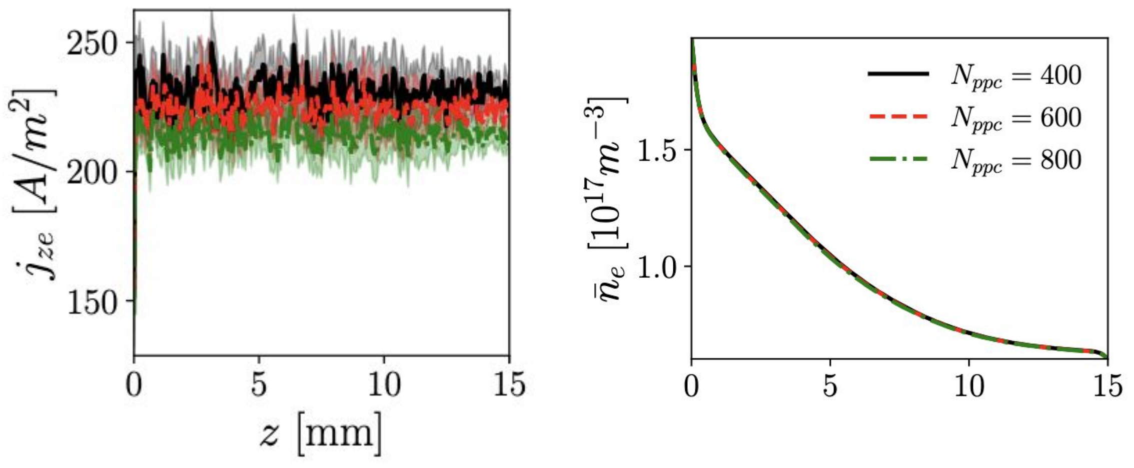
Jubin et al. , "Numerical thermalization in 2D PIC simulations: Practical estimates for low-temperature plasma simulations," Phys. Plasmas 31, 023902 (2024)

A non-physical relaxation process where discrete "macroparticle" noise mimics physical collisions, forcing the plasma toward a **Maxwellian equilibrium**.

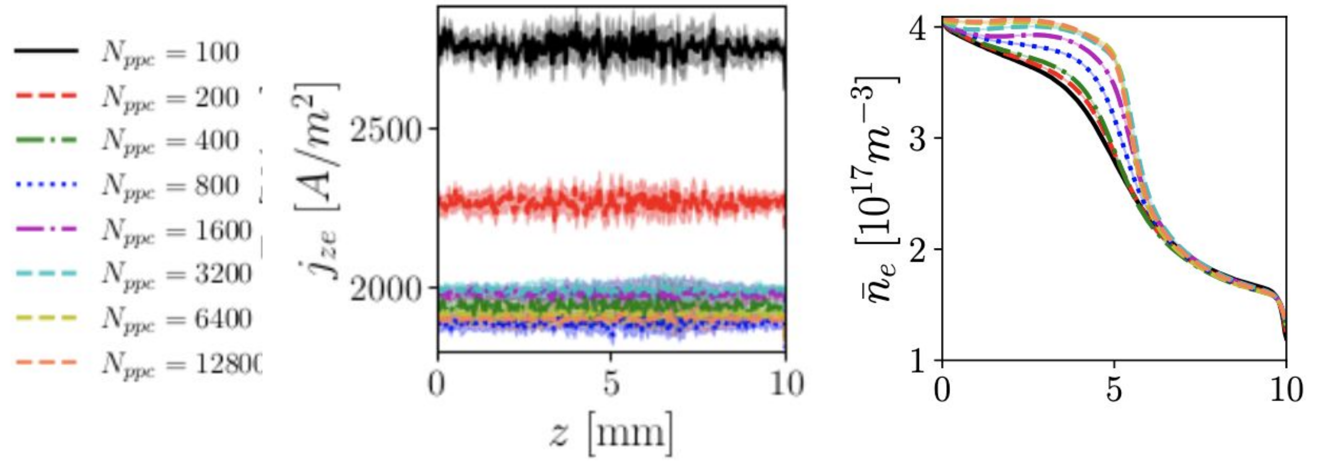
Artificial Momentum Exchange: Numerical noise provides a fake momentum sink, creating a **"numerical anomalous current"**

Over-prediction of Cross-Field Transport: Numerical collisions allow electrons to "jump" across magnetic field lines → **overestimation of the electron current**

New simulations



Previous simulations



Summary and current/future work

GPU-based kinetic simulations

- ❑ **2D PIC simulations** capture saturated azimuthal instabilities in the Hall thruster acceleration region.
- ❑ Resulting **Ey fluctuations** drive an ExB axial drift, generating fully anomalous axial electron current.
- ❑ Important phenomena observed in **momentum and energy balances**
- ❑ **Parametric study:** behavior of instabilities and electron current dependent on magnetic field, potential drop and electron density → obtain scaling laws ??
- ❑ **Numerical thermalization**: Mandatory to analyze the effect of the number of particles in this simple simulations as numerical thermalization could overpredict the anomalous electron current.

Axial PIC simulation with wall effects (NOT SHOWN)

- ❑ *Implementation of a simplified model for plasma wall interactions, capable of*
  - ❑ *Dielectric walls in steady state*
  - ❑ *Secondary electron emission*
  - ❑ *Plume divergence effects*
- ❑ *Include this model on axial-azimuthal Hall Thruster simulations*
- ❑ *Explore the difference between fluid and PIC → ongoing preparing for publication*

Achievements

Conferences

- "Towards self-consistent simulation of radial effects on an axial PIC model of the Hall discharge" at 38th International Electric Propulsion Conference, Toulouse, France, June 2024
- "Non-linear ECDI and Anomalous Transport in ExB plasmas via GPU-accelerated PIC simulations", at 39th International Electric Propulsion Conference, London, England, September 2025.
- "Azimuthal instabilities driving electron transport in Hall Thrusters: Insights from GPU PIC simulations", at 10th Space Propulsion Conference, Bari, Italy, May 2026

Manuscript preparation

- "The role of Radial Dynamics in Axial PIC Models of Hall Thrusters", under preparation
- "Coherent Structures and Anomalous Electron Transport in Cross-Field Plasmas", under preparation

Thank you!
Questions?

Turbulent transport characterization in Hall Effect Thrusters

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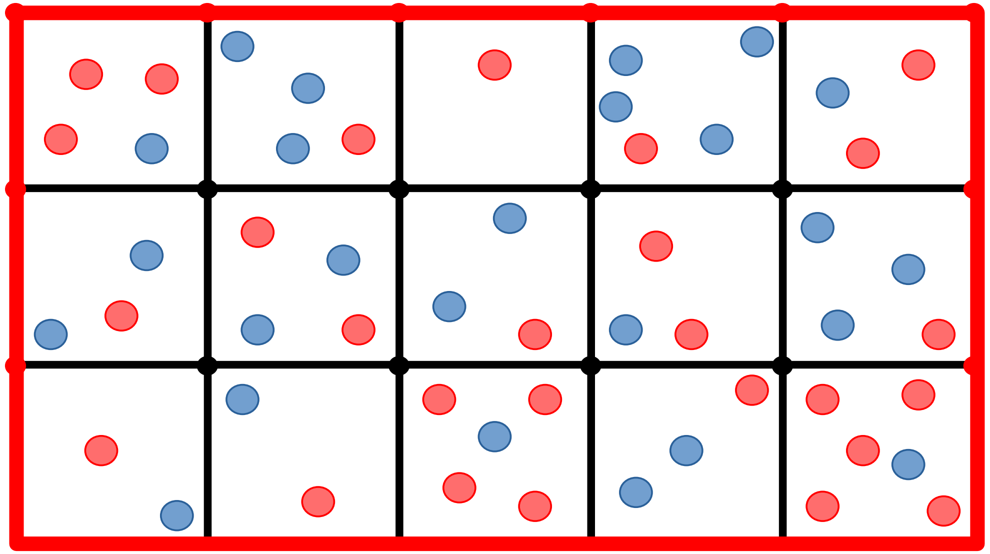
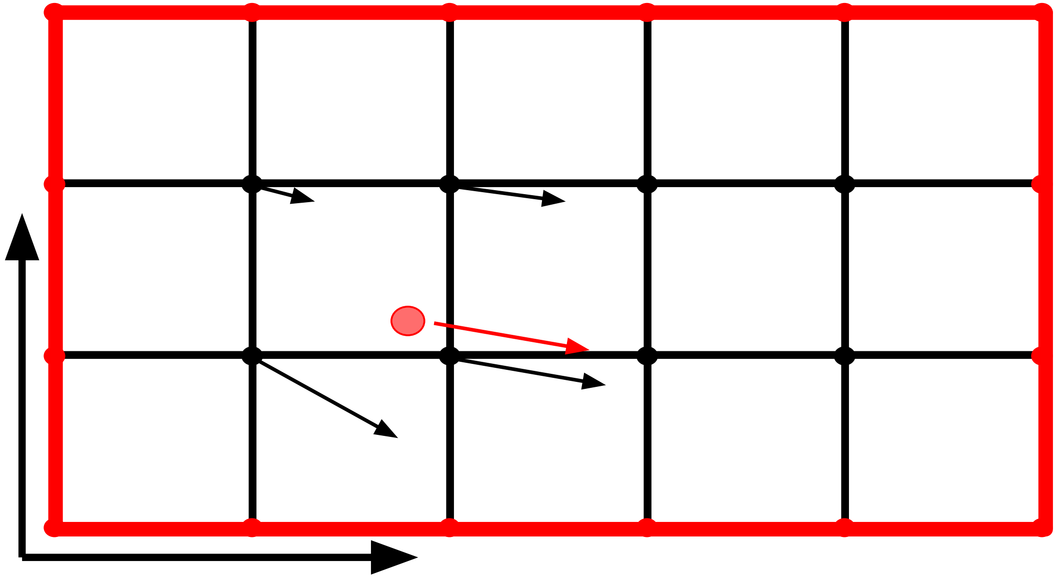
Backup slides

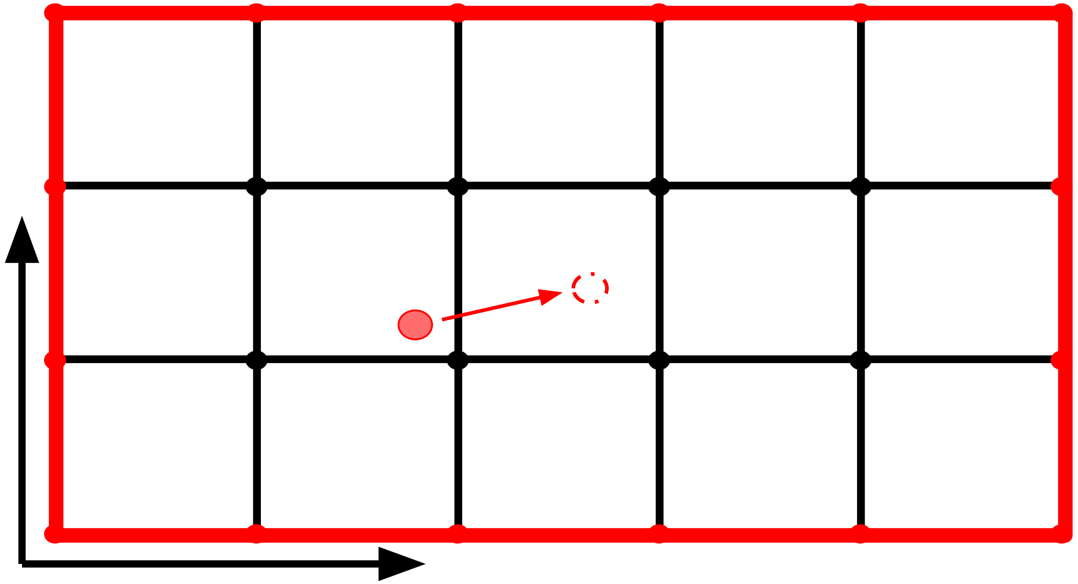
A 2D PIC model: PICASO

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_x f + \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \cdot \nabla_v f = \left[\frac{\partial f}{\partial t}\right]_{coll}$$

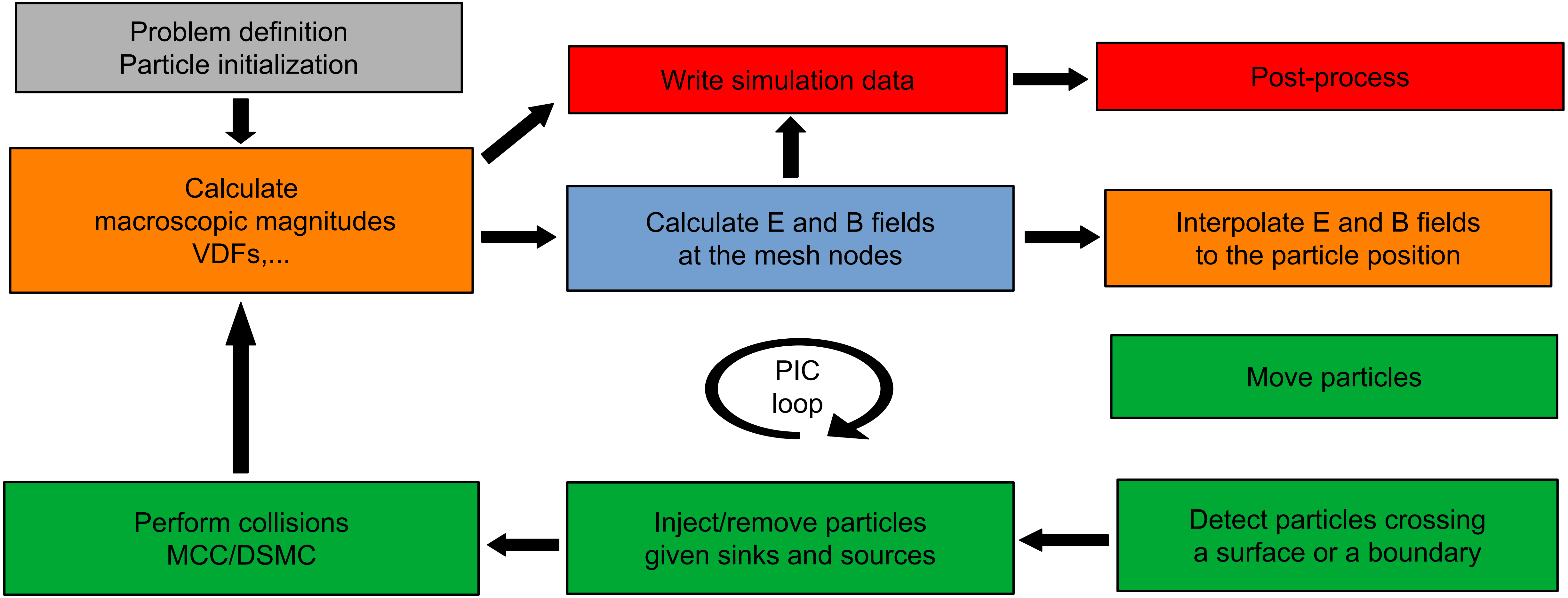
Simulated macroparticles = 10^5–10^8 real particles







The particle-in-cell approach



A 2D PIC model: PICASO

- **The PICASO code**
  - Developed by E. Bello-Benitez (axial-azimuthal) and A. Marin-Cebrian (axial-radial)
  - Solve the Boltzmann equation

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{e_s}{m_s}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \left(\frac{\delta f_s}{\delta t}\right)_{\text{coll}}$$

  - Solve for the motion of macroparticles

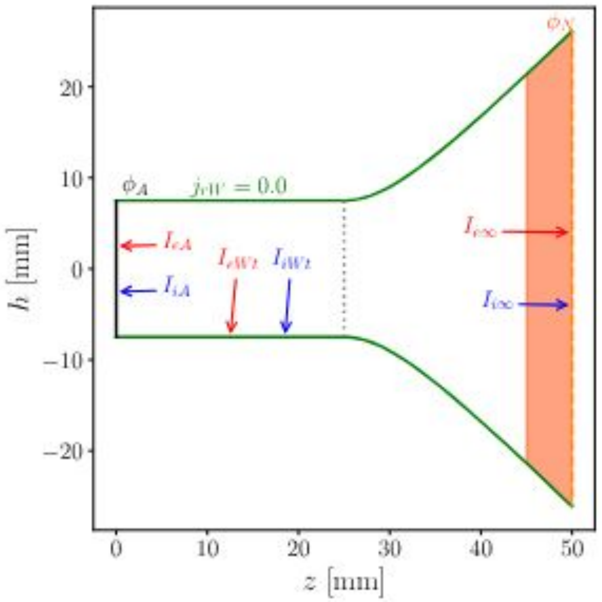
$$\frac{\mathrm{d}\mathbf{v}_p}{\mathrm{d}t} = \frac{q_s}{m_s}(\mathbf{E}_p + \mathbf{v}_p \times \mathbf{B}_p)$$

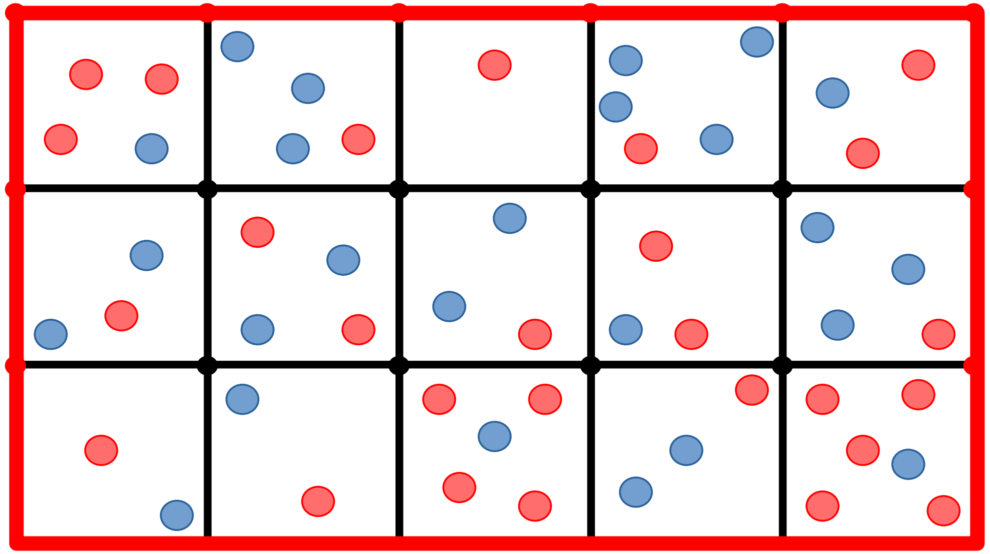
$$\frac{\mathrm{d}\mathbf{x}_p}{\mathrm{d}t} = \mathbf{v}_p$$

  - External magnetic field and solution of electric field from Poisson

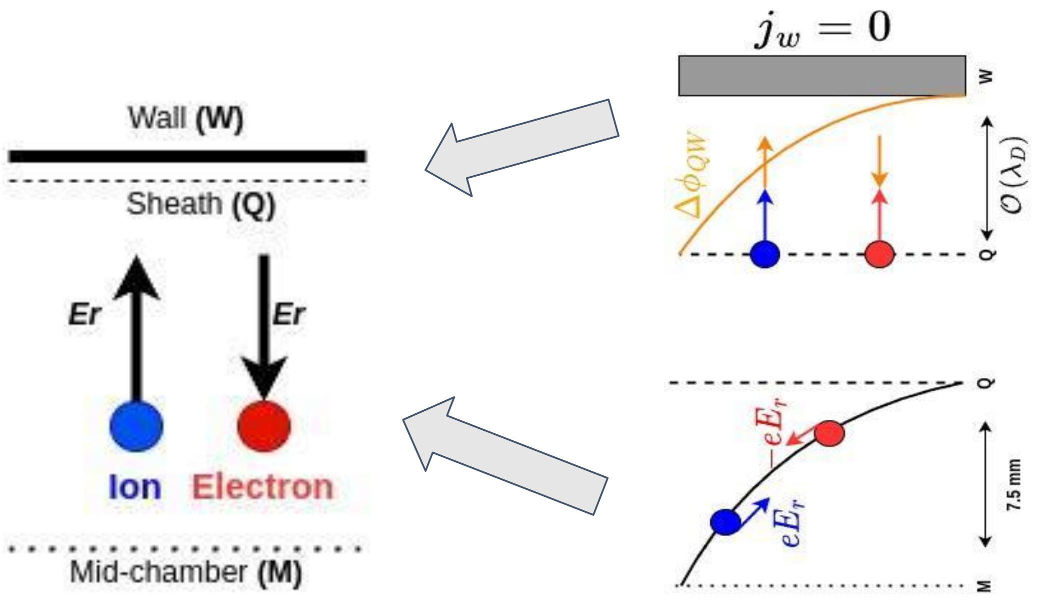
$$\nabla^2 \phi = -\frac{e}{\varepsilon_0}(n_i - n_e)$$

  - Collisional parameter $\left(\frac{\delta f_s}{\delta t}\right)_{\text{coll}}$





Axial PIC simulation with wall effects



At the **wall** and in the **HET plume**, complex phenomena occur

- Ion recombination
- Collection and emission of electrons
- Plume divergence
- Dielectric walls

To capture these phenomena, we developed a model that:

- **Move particles in radial direction** with **analytical expressions** for **radial electric field (M-Q)** and **potential drop at the sheath (Q-W)**.
- **Remove and emits particles** that reach walls while ensuring **dielectric conditions** are met.
- **Includes plume divergence effects**

$$E_r = \frac{m_i \tilde{u_{ri}}}{e}\left(\frac{\partial \tilde{u_{ri}}}{\partial r} + S_{prod}\right) \qquad \phi_{sh} = \frac{T_e}{e}\ln\sqrt{\frac{m_i}{2\pi m_e}} - \delta\phi_{sh}$$

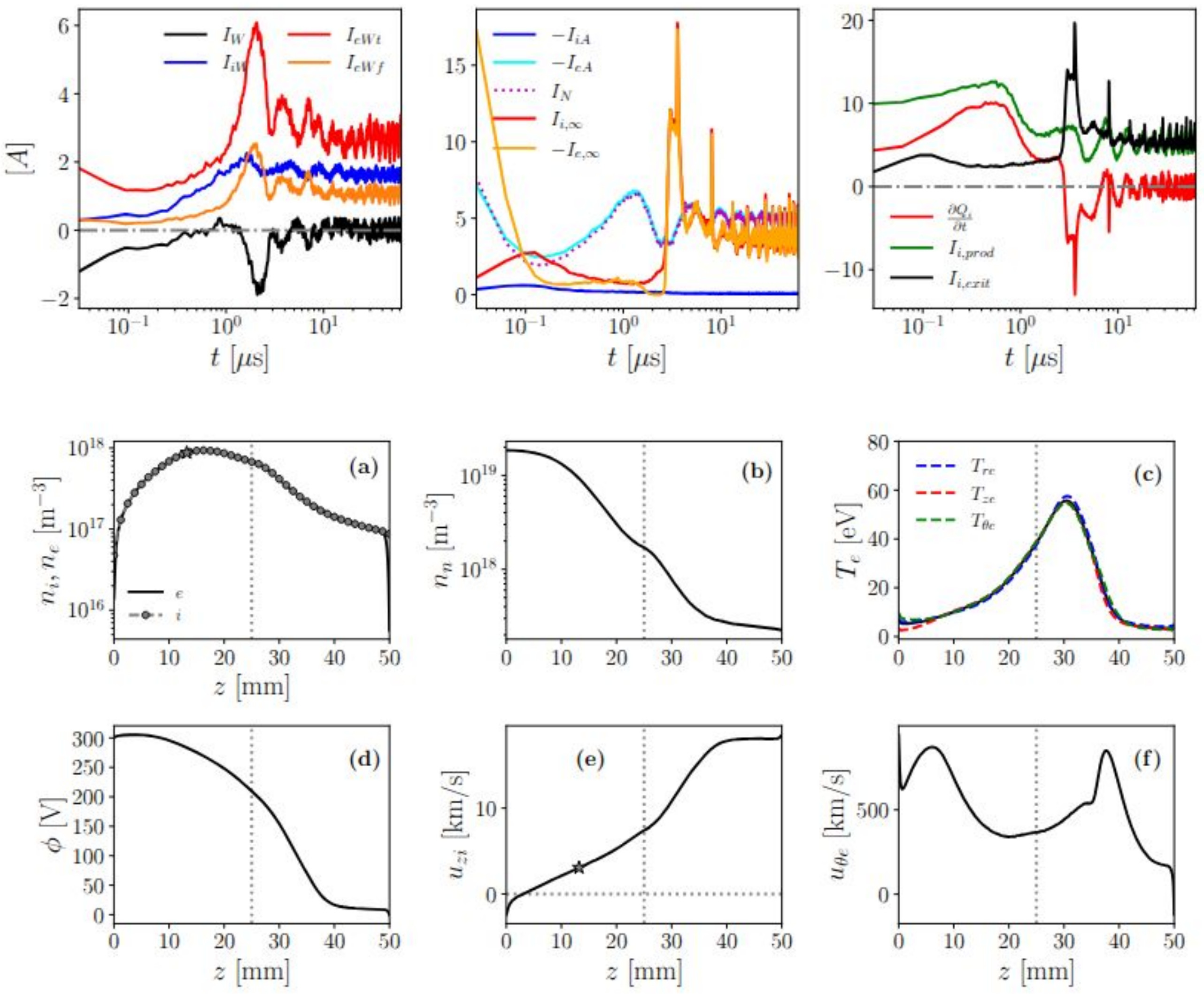
$$\tilde{u_{ri}}(z,r) = c_s(z)\left(\frac{r - r_M}{r_{W2} - r_M}\right)^3 \qquad \frac{\partial(\delta\phi_{sh})}{\partial t} = \frac{j_{rW}}{\tilde{c}}$$

Overcoming previous limitations

- Particles move radially
- Self consistent solution ensuring a current free wall
- Plume divergence effects
- Secondary electron emission

Cuerva et al. , "Towards self-consistent simulation of radial effects on an axial PIC model of the Hall discharge" IEPC, (2024)

Axial PIC simulation with wall effects

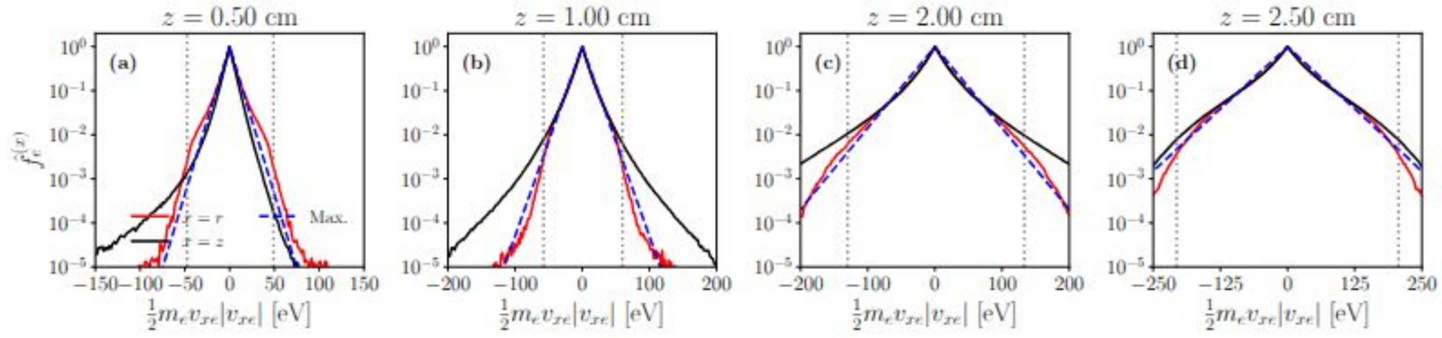


Steady-state solution

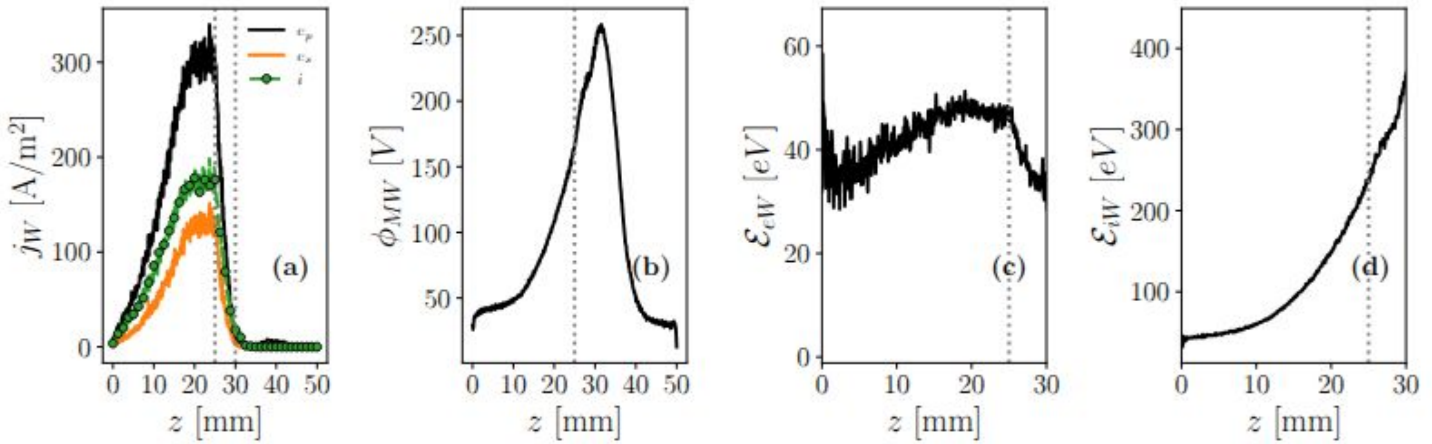
Axial profiles

Cuerva et al. , "Towards self-consistent simulation of radial effects on an axial PIC model of the Hall discharge" IEPC, (2024)

Axial PIC simulation with wall effects



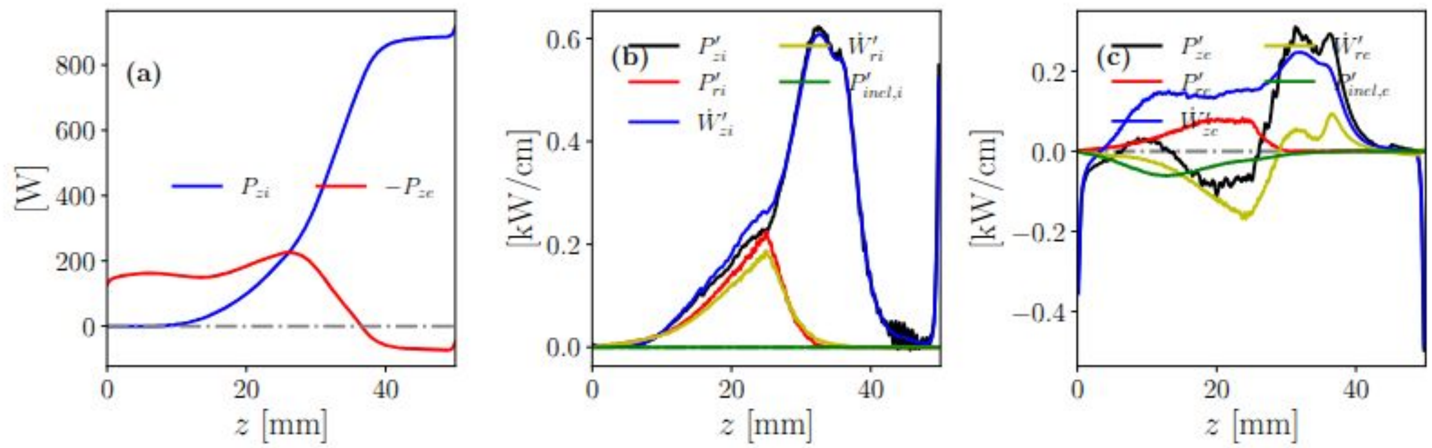
Electron VDFs



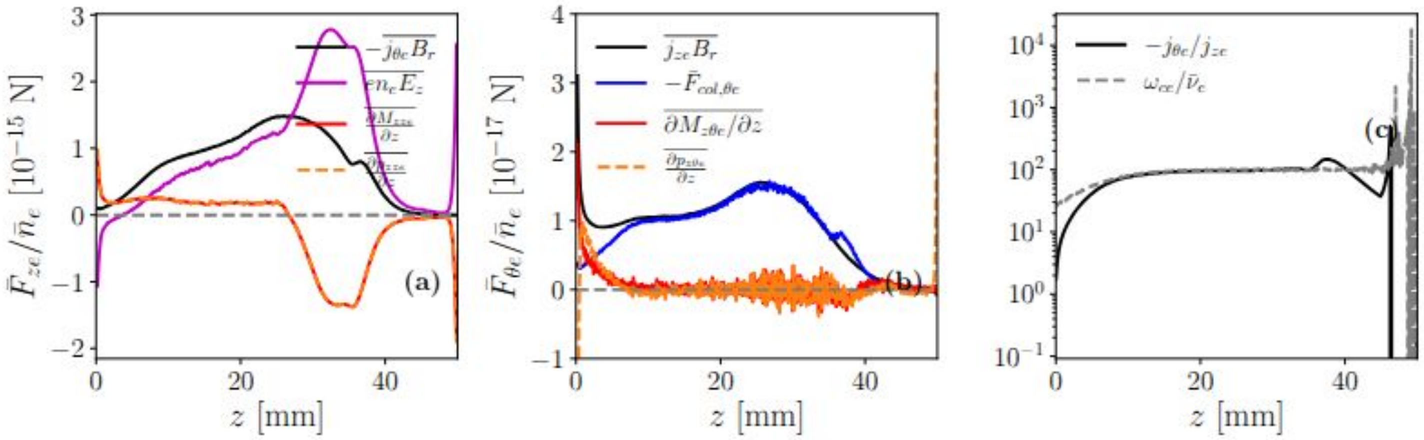
Wall profiles

Cuerva et al. , “Towards self-consistent simulation of radial effects on an axial PIC model of the Hall discharge”, IEPC, (2024)

Axial PIC simulation with wall effects



Energy balance

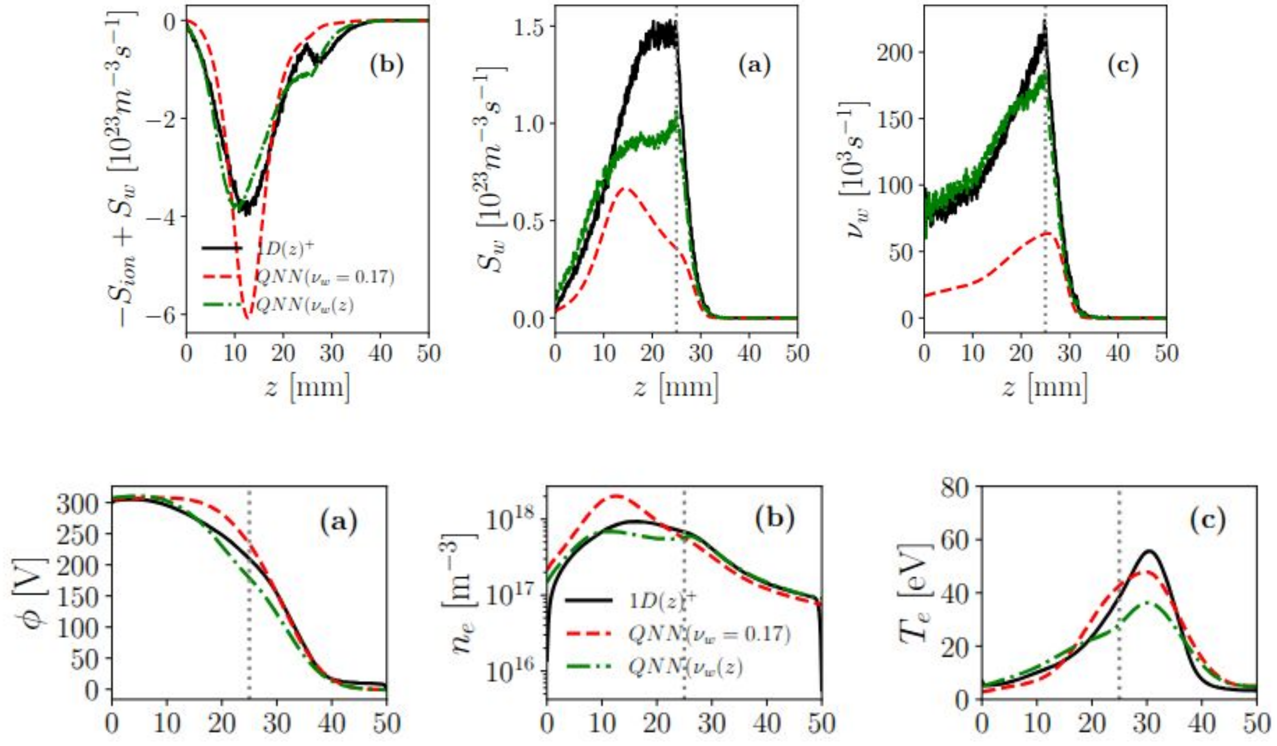


Electron momentum balance

Poli et al. , "Time-dependent axial fluid model of the Hall thruster discharge and its plume," Journal of Physics D: Applied Physics, (2023)

Axial PIC simulation with wall effects

Quasineutral electron fluid model assumptions	Kinetic results
Maxwellian electron VDF.	**Non-Maxwellian** electron VDF.
Scalar pressure.	**Pressure** is a full **tensor**. Gyrotropic + Gyroviscous
Fourier law as heat flux closure.	**Complex energy flux**.

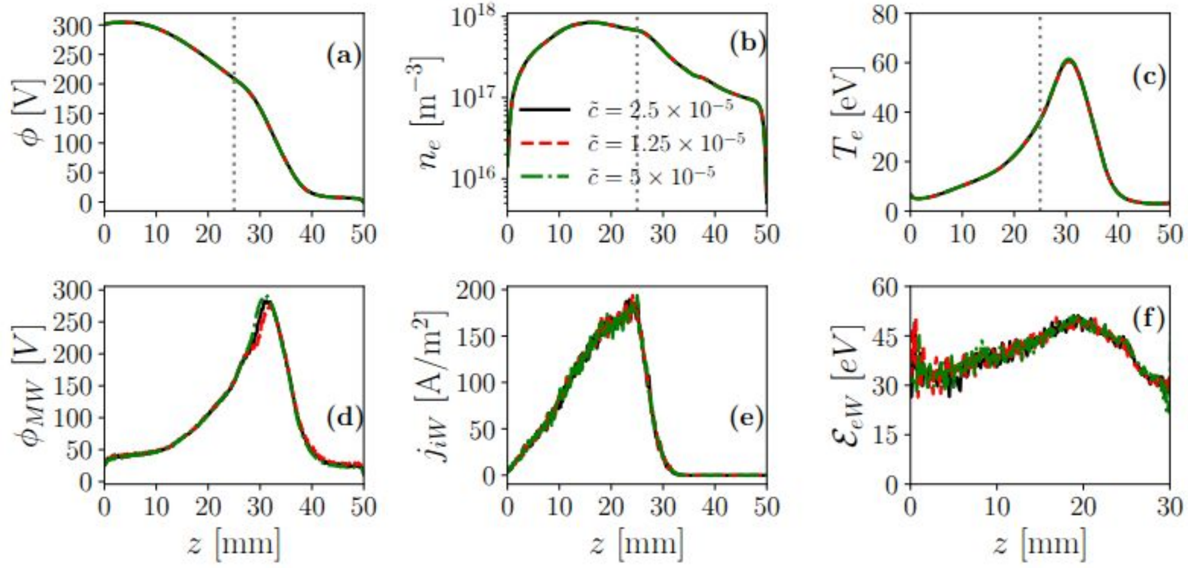


- **In fluid**, the wall collision parameter is constant along the axial direction, and the impact energy follows a 2*Te dependence.
- **PIC** results show a significantly higher effective wall collision rate compared to fluid models, with wall energy varying along the axial direction and showing no dependence on Te.

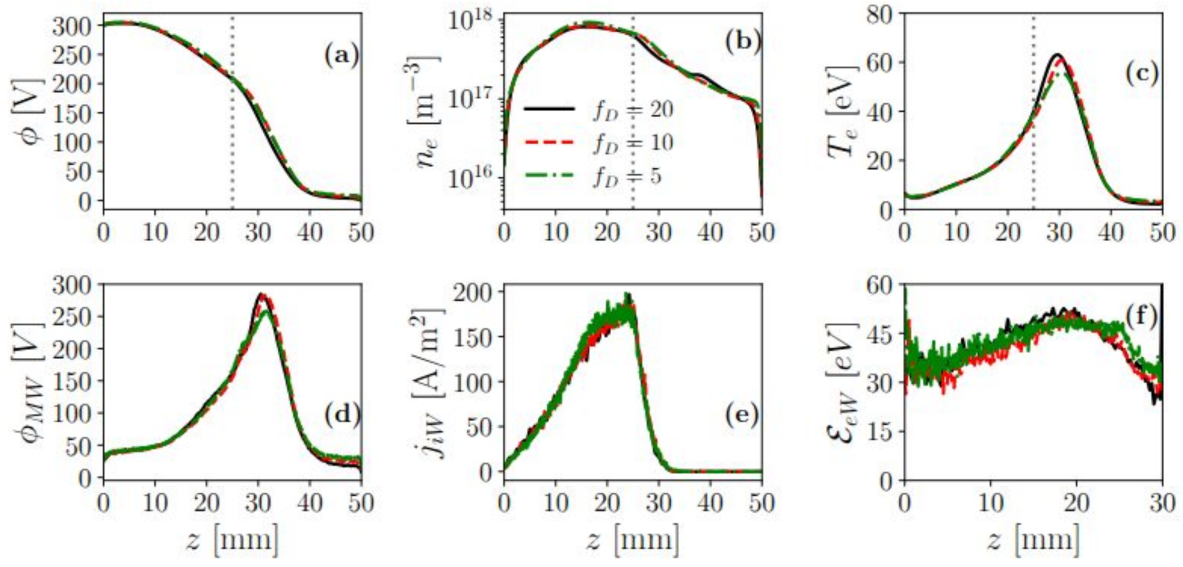
$$I_{d,PIC} \simeq 4.91A$$
$$I_{d,QNM} \simeq 4.86A$$

Axial PIC simulation with wall effects

Capacitance invariance



Augmented vacuum permittivity



ECDI: Anomalous transport

Type of instability where free energy is provided by the relative drift between ions and electrons → Driven by an ion beam streaming across a magnetic field (HET conditions)

Linear theory:

- ➢ Homogeneous equilibrium
- ➢ Propagation perpendicular to magnetic field
- ➢ Maxwellian magnetized electrons with ExB drift
- ➢ Cold unmagnetized ions

$$g(\omega_e, b_e; \omega_{ce}) = \exp(-b_e)\left[I_0(b_e) + 2\sum_{m=1}^{\infty}\frac{\omega_e^2 I_m(b_e)}{\omega_e^2 - m^2\omega_{ce}^2}\right].$$

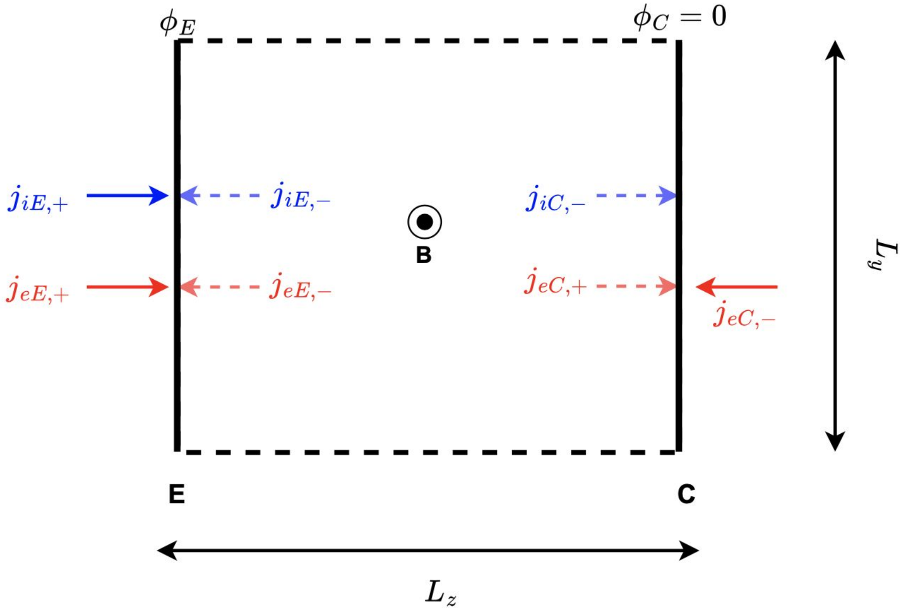
$$\frac{n_{e1}}{n_0} = [1 - g(\omega_e, b_e; \omega_{ce})]\frac{e\phi_1}{T_{e0}}$$

$$\frac{n_{i1}}{n_0} = \frac{k^2 c_s^2}{\omega_i^2}\frac{e\phi_1}{T_{e0}}$$

$$1 + k^2\lambda_{D0}^2 = \frac{k^2 c_{s0}^2}{\omega_i^2} + g(\omega_e, b_e; \omega_{ce}).$$

Solutions of the dispersion relation

1. Ion acoustic wave $g \ll k^2 c_{s0}^2/\omega_i$ $\quad \omega_{IA} = \frac{k c_{s0}}{\sqrt{1 + k^2\lambda_{D0}^2}},$
1. Berstein waves $g \gg k^2 c_{s0}^2/\omega_i$ $\quad \omega_i = \pm\frac{k c_{s0}}{\sqrt{1 + k^2\lambda_{D0}^2 - g(\omega_e)}}.$
1. Unstable acoustic mode → ECDI $\quad g > 1 + k^2\lambda_{D0}^2,$



Kinetic model

Computational properties

- Standard 2D explicit PIC algorithms
- Particle decomposition on an NVIDIA A30 GPU
- Poisson spectral solver

$$\phi = \phi_h + \phi_p$$

$$\nabla^2 \phi_h = -\frac{\rho}{\epsilon_0}$$

$$\nabla^2 \phi_p = 0$$

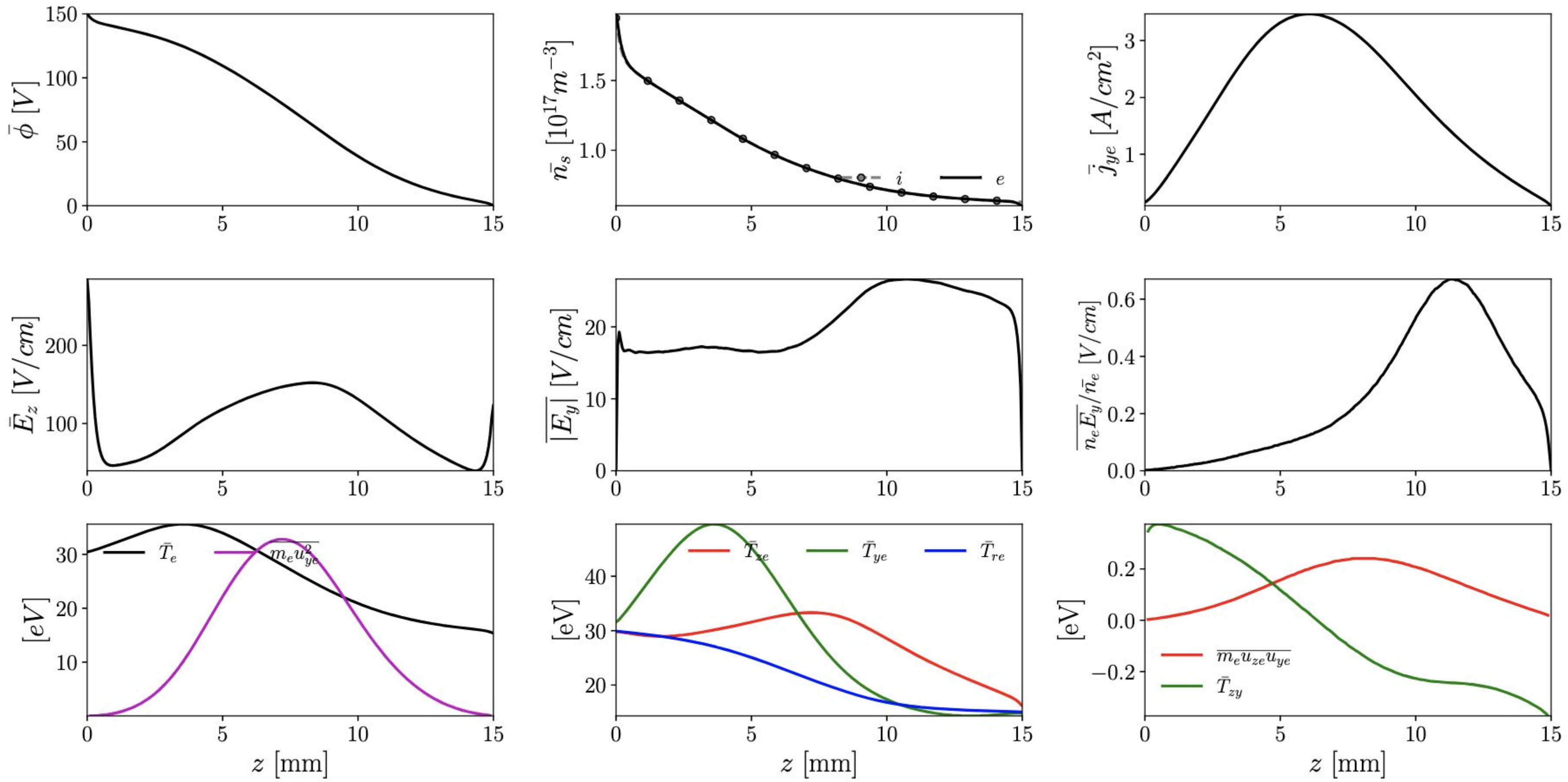
Simplified configuration of the acceleration region of Hall Thruster

- Supersonic ions
- Collisionless plasma
- Prescribed potential drop from E to C
- Prescribed Te gradient from E to C
- Constant average radial magnetic field
- Given current density of ions injected at E
- Thermal injection of electrons at E and C, based on ion density

Type	Symbol	Value
Physical parameters		
Discharge voltage	ϕ_E	$200\ V$
Radial magnetic field	B_0	$50\ G$
Ion injection current	j_{iE+}	$320\ A/m^2$
Ion initial axial velocity	$u_{zi,E+}$	$5\ km/s$
Ion emission temperature at E	T_{iE+}	$1\ eV$
Electron injection current	j_{eE+}	$58.7\ kA/m^2$
Electron injection current	j_{eC+}	$12.9\ kA/m^2$
Electron emission temperature at E	T_{eE+}	$50\ eV$
Electron emission temperature at C	T_{eC-}	$10\ eV$
Axial length	L_z	$20\ mm$
Azimuthal length	L_y	$20\ mm$
Initial state		
Macroparticles per cell	$N_{ppc,ini}$	400
Plasma density	n_0	$4 \times 10^{17}\ m^{-3}$
Electron initial temperature	$T_{e,ini}$	$30\ eV$
Computational parameters		
Time-step	Δt	$5 \times 10^{-6}\ \mu s$
Cell size	Δx	$3.9 \times 10^{-2}\ mm$
Final time	T_{final}	$25\ \mu s$
Cells in axial direction	N_z	512
Cells in azimuthal direction	N_y	512

Table 1. Input parameters for the reference simulation.

Time-average response



1D axial 'steady-state': time and azimuthally averaged variables to analyze slow electron dynamics and maintain correlated high frequency effects

- **Expected trends** in a Hall thruster **acceleration region**
- **Correlated effects** on **azimuthal electric field** variables
- Significant **anisotropy** on **electron temperature** and **electron inertia**

GPU kinetic simulations

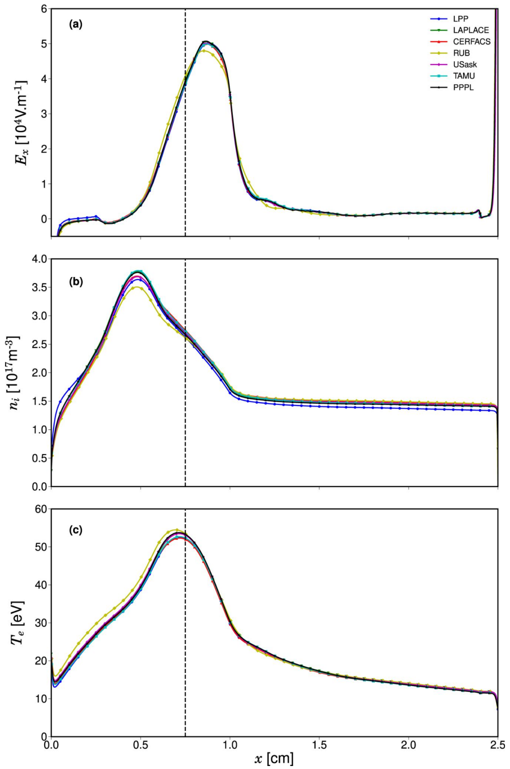
T Charoy *et al.* , "2D axial-azimuthal particle-in-cell benchmark for low-temperature partially magnetized plasmas". 2019 *Plasma Sources Sci. Technol.*

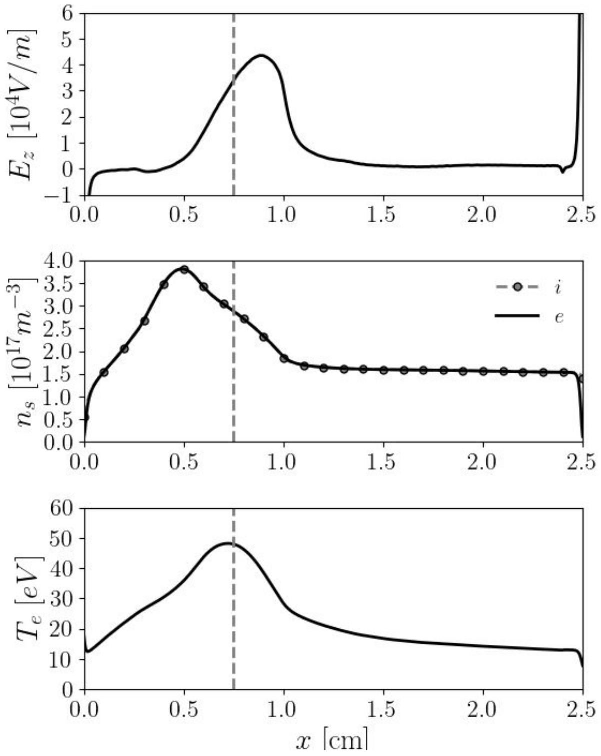
Marks, T.A., Gorodetsky, A.A. GPU-accelerated kinetic Hall thruster simulations in WarpX. J Electr Propuls 4, 34 (2025)

Simulate Case 2 of the **LANDMARK low-temperature plasma benchmark**.
Focuses on capturing **key kinetic effects in a Hall effect thruster discharges**:

- **Onset and growth of azimuthal instabilities.**
- Plasma acceleration in the axial direction

Benchmark of multiple PIC codes from the **plasma propulsion community**





Agreement with published results and validates the code for future axial-azimuthal simulations of the Hall thruster.

Performance of GPU compared to PIC codes

PICASO-G (A30)	1.88 days
PPPL (112 CPU)	2.5 days
WarpX (H100/ V100)	1.81 days/ 3.81 days