

Hybrid particle-fluid plasma modeling of Hall effect thrusters and analysis of non-conventional configurations

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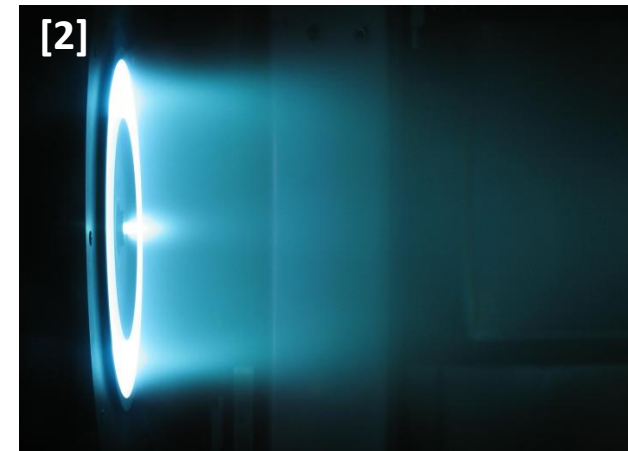
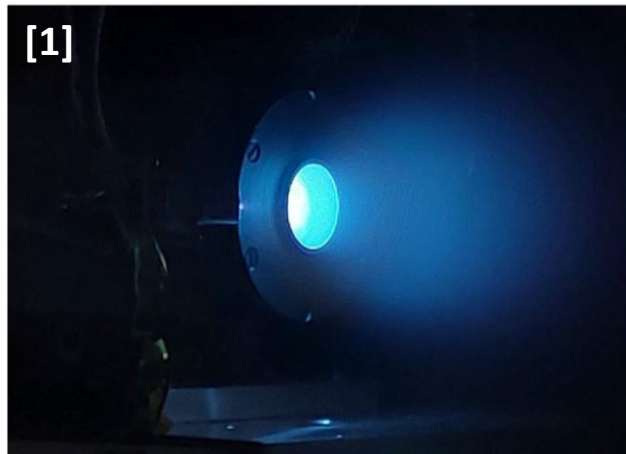
Universidad de Málaga

PhD Aerospace Engineering UC3M

Doctoral Meetings 2026

2nd and 3rd June 2026

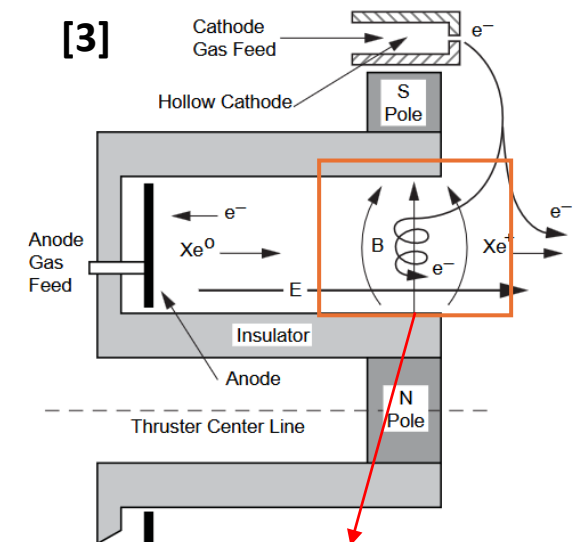
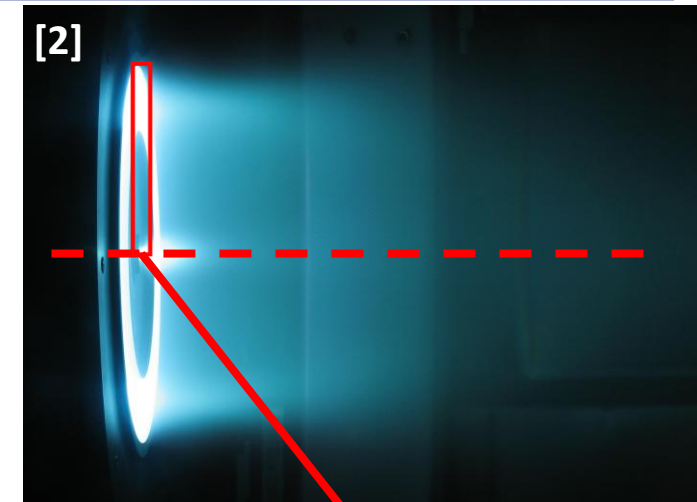
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Hybrid particle-fluid plasma modeling **of Hall effect thrusters** and analysis of non-conventional configurations

Hall effect thruster (HET)

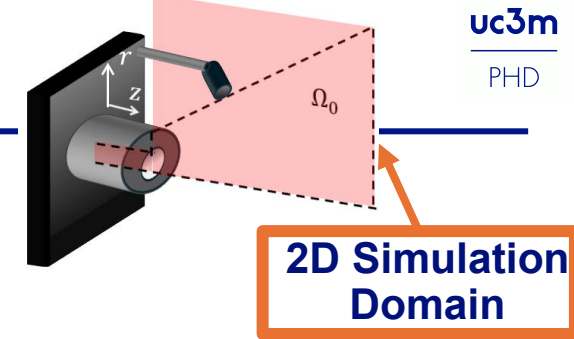
- **Leading** the sector of electric propulsion.
- **First Flight (1971):** Developed under Soviet physicist A. I. Morozov, the first operational HET was the **SPT-60**.
- **Species Roles:**
 - **Neutrals** are injected at the anode and ionized in the chamber.
 - **Ions** are accelerated downstream by the anode-cathode potential.
 - **Electrons**, injected at the cathode, ionize neutrals and neutralize the ion beam downstream.



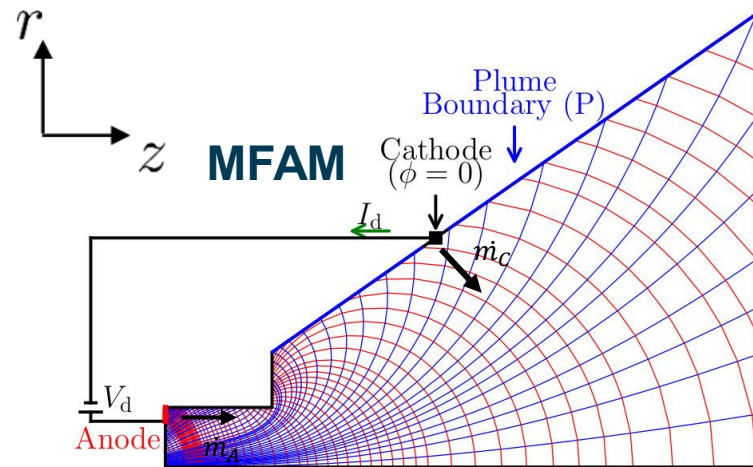
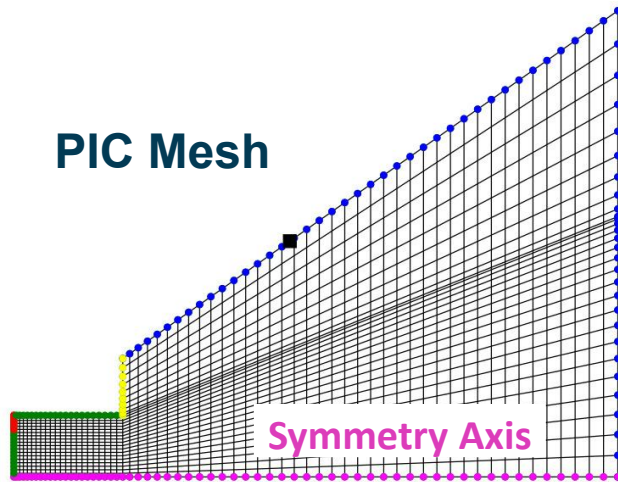
RADIAL MAGNETIC FIELD

Hybrid particle-fluid plasma modeling of Hall effect thrusters and analysis of non-conventional configurations

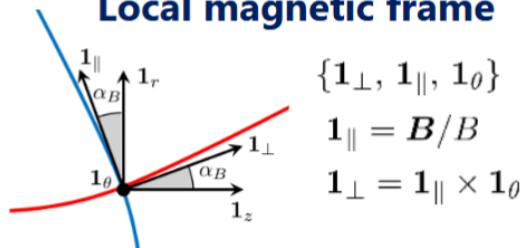
HYPHEN code (HYPHEN-MFAM)



PIC Mesh



Local magnetic frame



2D (axial-radial) hybrid PIC/fluid code

HYPHEN uses three main modules:

- **I-module:** PIC formulation for heavy species (ions and neutrals) on structured mesh.
 - Neutrals, singly, doubly and triply charged ions
 - Xe as propellant
- **E-module:** drift-diffusive fluid model for electrons on unstructured MFAM.
 - Applies quasineutrality $\rightarrow n_e = \sum Z_s n_s$ ← **Input from I-module**
 - Current continuity + Ohm's law $\rightarrow \phi$ and j_e
 - Electron energy eq. + Fourier's law $\rightarrow T_e$ and q_e
- **S-module:** interaction of plasma bulk with the thruster walls.
 - Gives boundary conditions (BCs) at each wall type: dielectric or conductive

➤ Compared to fully kinetic codes, hybrid codes offer:

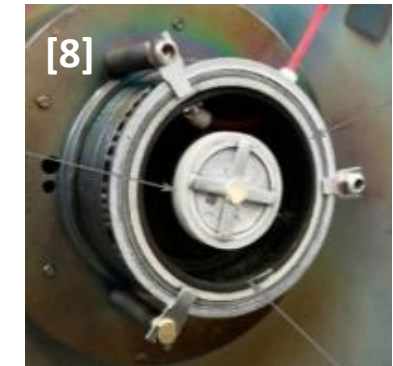
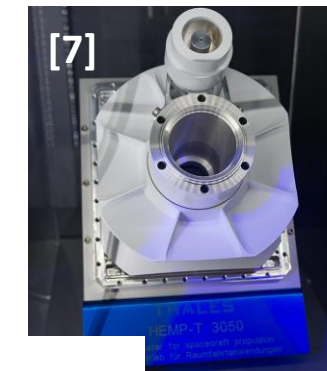
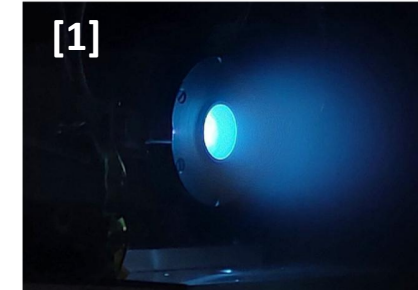
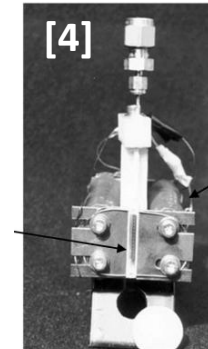
- ☐ Faster simulations of all species in a partially ionized plasma.
- ☐ Together with the simulations of the discharge chamber, it allows the study of extended radial and axial plumes, including the cathode plume.

Hybrid particle-fluid plasma modeling of Hall effect thrusters and analysis of non-conventional configurations

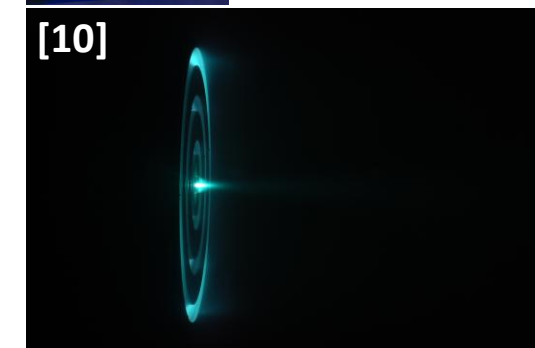
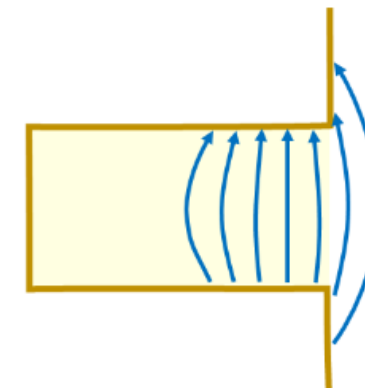
Non-conventional configurations

- Many types of unconventional setups (from channel shapes to magnetic field topologies):
 - Linear HET (LHT) [4]
 - Planar HET (PHT) [5]
 - Cylindrical HET (CHT) [1] **GOAL: WE ANALYSED THIS ONE! (LATER IN THE PRESENTATION)**
 - Diverging CHT (DCHT) [6] (also known as HEMPT [7])
 - Wall-less HET (WL-HT) [8] (**WALL LOSSES PROBLEM SOLVED!**)
 - Narrow-channel HET (NCHT) [9]
 - Nested HET (NHT) [10]
 - ...

**MOST OF THEM HAVE A
COMPLEX MAGNETIC
FIELD TOPOLOGY**

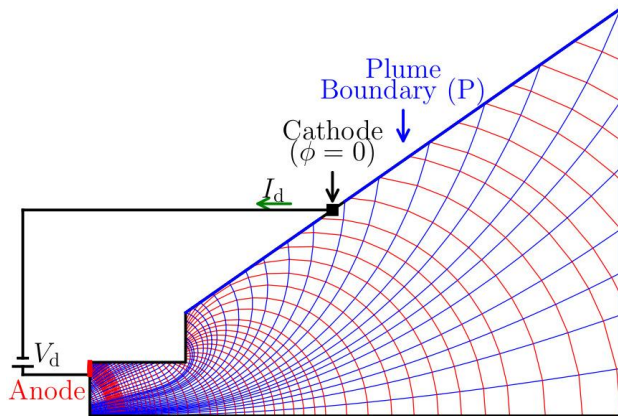


Standard

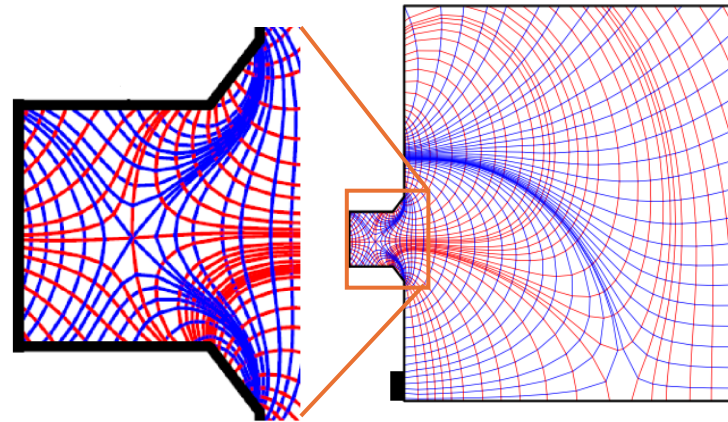


- Easier meshing approach for thruster with complex magnetic topologies and chamber geometries. It will allow:

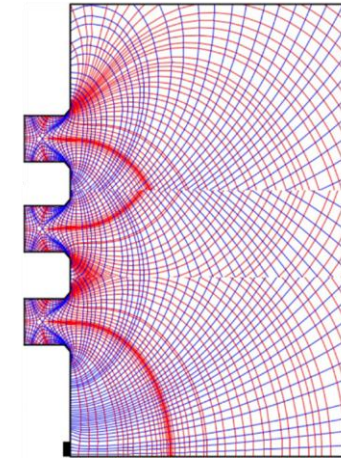
Big plume studies



Complex magnetic topologies



Complex geometries



HYPHEN-MFAM

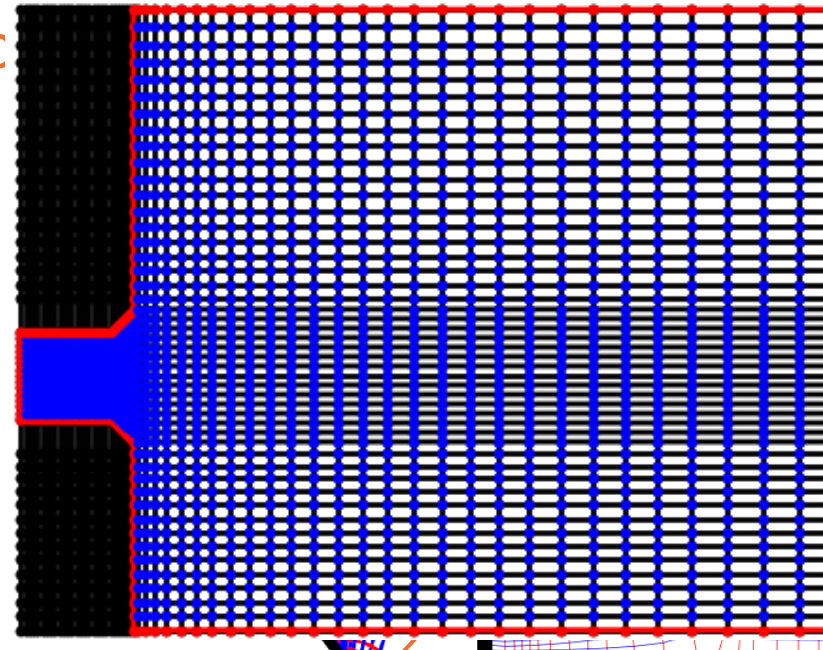
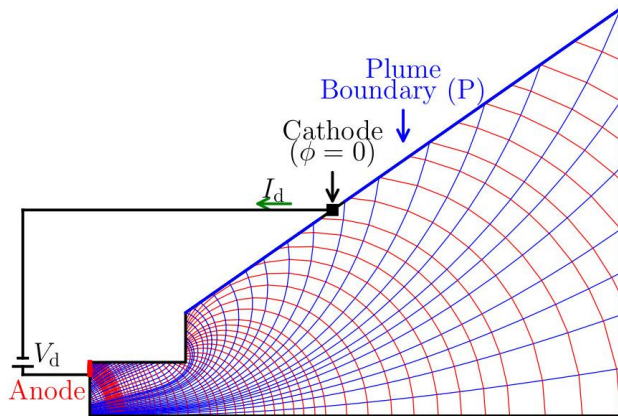
- Challenging meshes for big plume studies.
- Studies of moving the magnetic null point gives high complexity to the meshing process. It might take a **day+** to generate a proper MFAM for a given geometry.
- Very tedious mesh refinement studies.

HYPHEN-CM

- Mesh can be generated in less than 1 minute.
- Simple mesh refinement studies.
- Possibility of PIC mesh = eFIdM mesh which leads to less interpolation errors.
- Large plume meshes are not a challenge anymore.
- Applications beyond non-conventional HETs: thrusters with induced magnetic fields (RITs, High power MPDs,...).

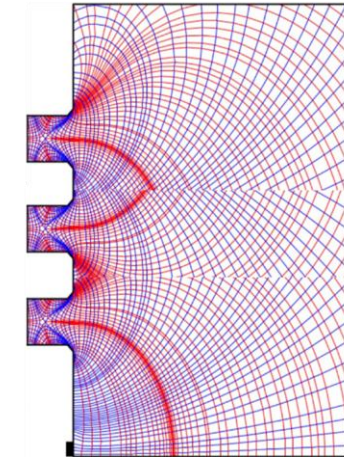
- Easier meshing approach for complex chamber geometries.

Big plume studies



magnetic topologies and

Complex geometries



HYPHEN-MFAM

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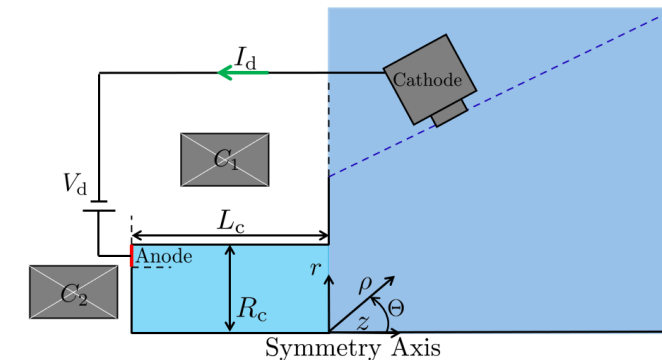
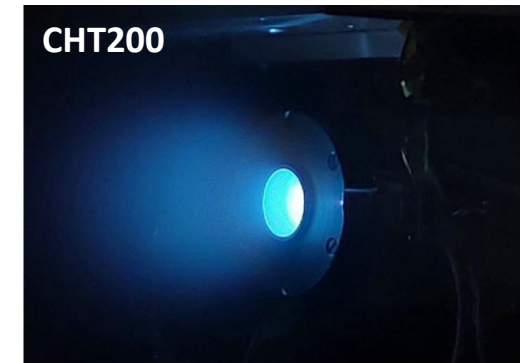
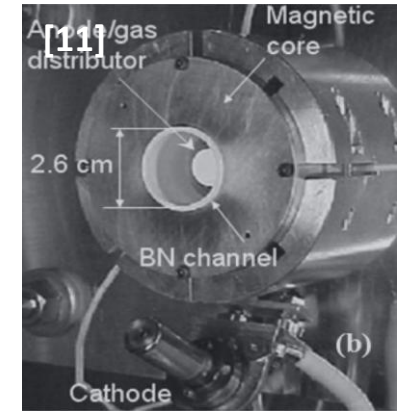
**GOAL: DEVELOP
HYPHEN-CM (LATER IN
THE PRESENTATION)**

HYPHEN-CM

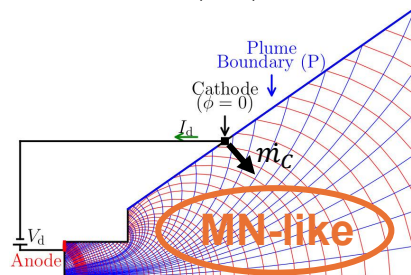
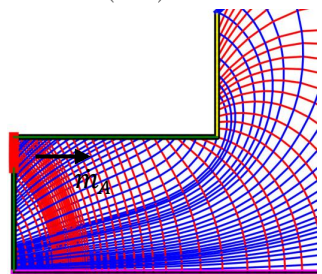
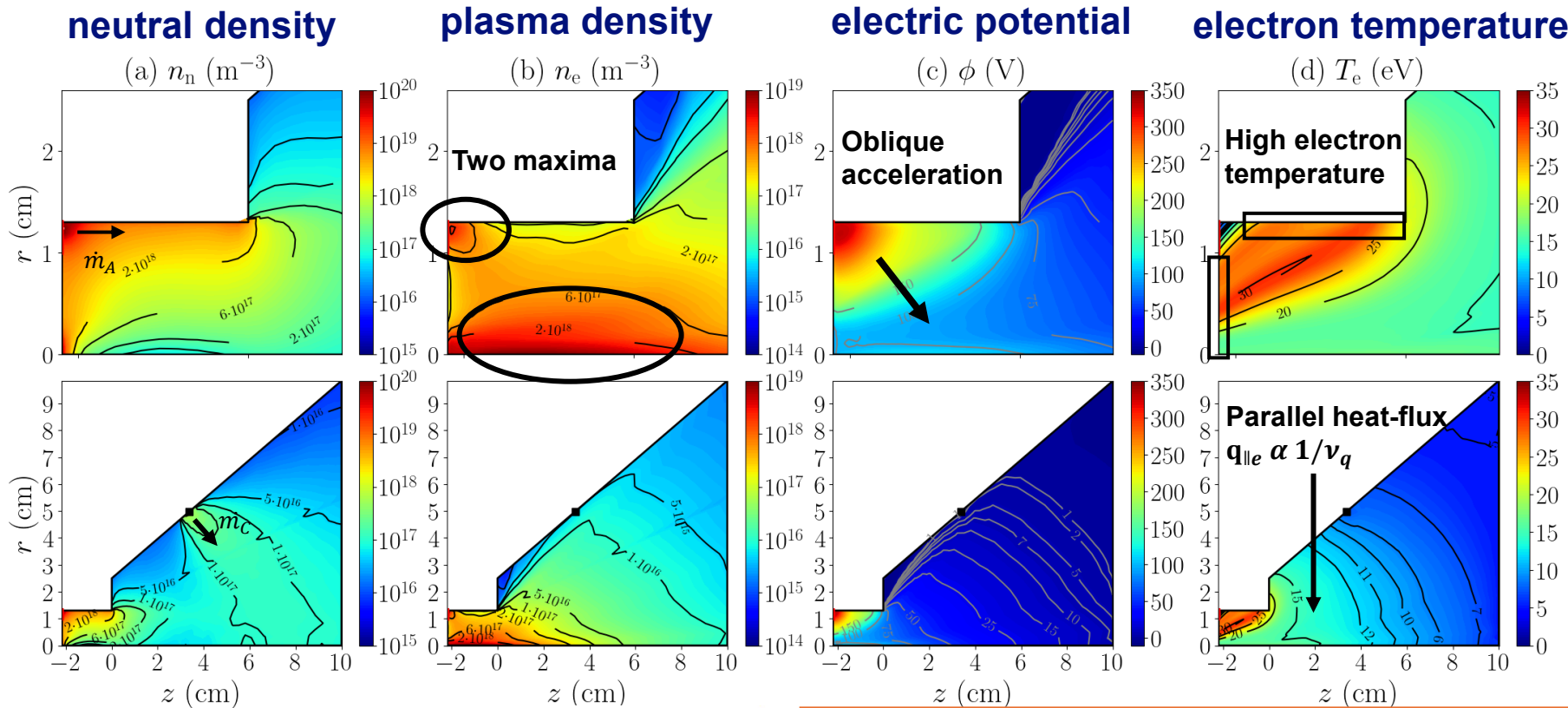
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First year of the PhD: Work on the CHT200 with HYPHEN-MFAM

- Cylindrical Hall thrusters (CHTs) emerged (two decades ago) as an efficient alternative to the traditional annular HET (AHT) for low-power applications [1].
- Tatiana Perrotin and Jaume Navarro-Cavallé developed the CHT200 in our laboratory. They observed the typical characteristics of the CHT.
 - Low thrust efficiency.
 - High propellant utilization (based on ion beam current), due to a high presence of multiply charged ions.
- In our work, the CHT200 [3] is simulated with a two-dimensional axisymmetric hybrid PIC/fluid code, with partial validation against experimental data. Focus on:
 - Identify key features of the plasma discharge.
 - Observe the effect of multiply charged ions (up to triply charged ions). Previously done up to doubly charged ions [4,5].
 - Analyse main sources of inefficiency.



Simulation results: 2D plasma maps (I)



Comparing to a AHT:

- ☐ Two plasma density peaks.
- ☐ Rear annular anode/injector region: main source of Xe^+ ions.
- ☐ Geometric convergence creates a plasma density peak on axis.

Similar solution for n_e , ϕ and T_e for all Ops.

CHT200 features:

- n_n peaks near the anode wall (injection + recombination) and decreases downstream (ionization + gas expansion).
- n_e is high inside the channel, with two clear maxima: (i) close to the anode/injection and (ii) on the axis.
- ϕ is concentrated near the anode, generating an oblique electric field, enabling ~ 200 V acceleration before exiting the channel.
- T_e rises near the anode to ~ 31 eV. High electron temperature at the lateral and back plate (losses). In the near plume, T_e decreases along the parallel magnetic field lines (MN-like).

Comparing to a AHT:

Simulation results: efficiencies

Comparing to a AHT:

- ❑ **LOW** thrust efficiency **9–12%**.

$$\eta = \frac{F^2}{2\dot{m}P} \equiv \eta_{\text{ene}}\eta_{\text{div}}\eta_{\text{disp}}$$

OP	P (W)	η	η_{disp}	η_{div}	η_{ene}	$\frac{P_{\text{inel}}}{P}$	$\frac{P_{\text{eD}}}{P}$	$\frac{P_{\text{iD}}}{P}$	$\frac{P_{\text{A}}}{P}$	$\frac{F_{\text{i1}}}{F}$	$\frac{F_{\text{i2}}}{F}$	$\frac{F_{\text{i3}}}{F}$
1	309	0.090	0.69	0.42	0.31	0.04	0.44	0.09	0.12	0.78	0.20	0.02
2	323	0.110	0.67	0.44	0.37	0.04	0.40	0.09	0.10	0.76	0.21	0.03
3	386	0.120	0.72	0.44	0.38	0.04	0.41	0.07	0.10	0.73	0.23	0.04

Simulation results: efficiencies

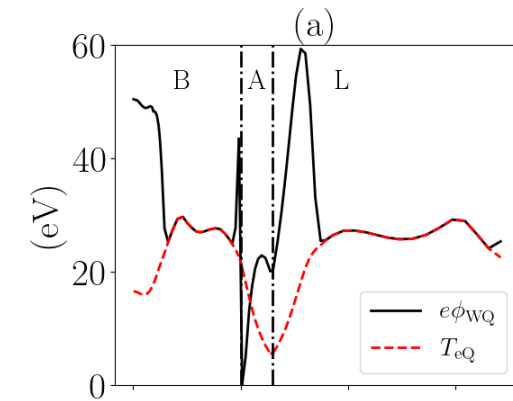
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- ❑ **LOW** thrust efficiency **9–12%**.
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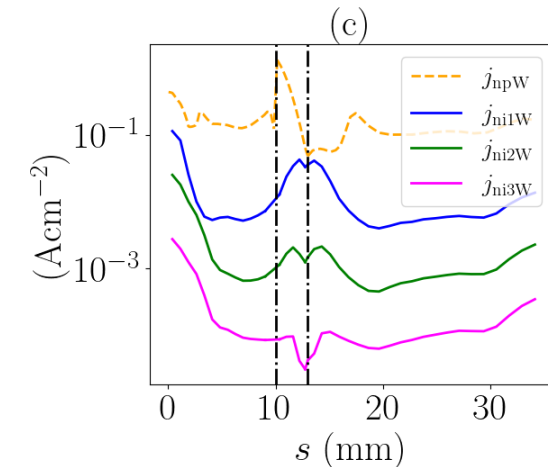
$$\eta = \frac{F^2}{2\dot{m}P} \equiv \eta_{\text{ene}}\eta_{\text{div}}\eta_{\text{disp}}$$

Wall-losses

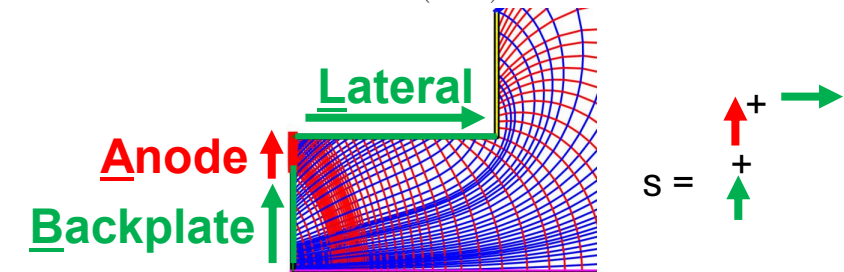
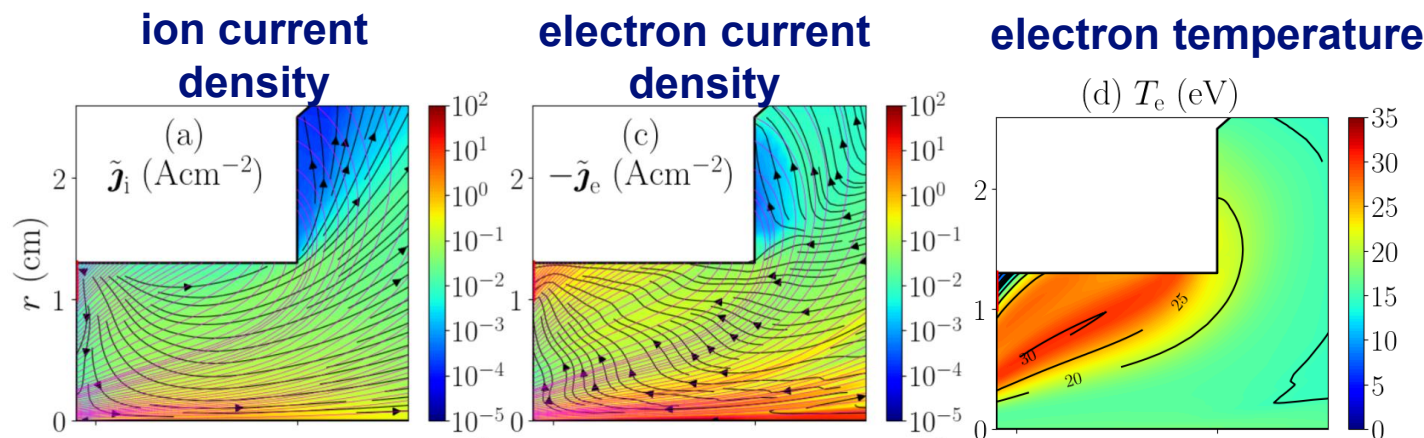
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RED:
electron
temperature



**Current
densities**



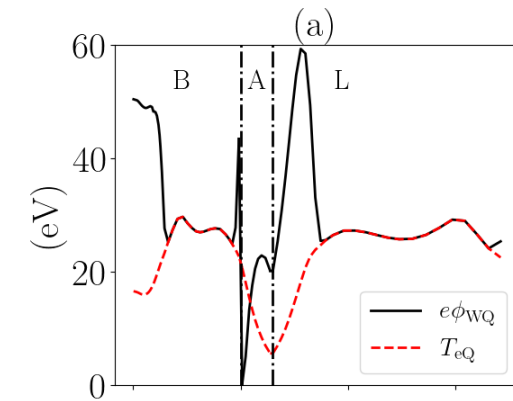
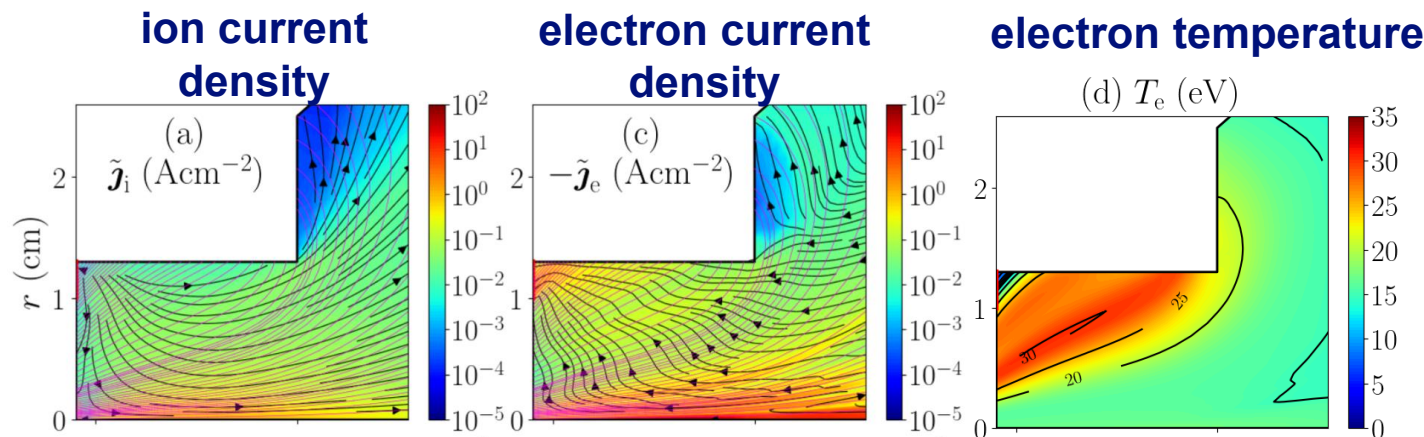
Simulation results: efficiencies

Comparing to a AHT:

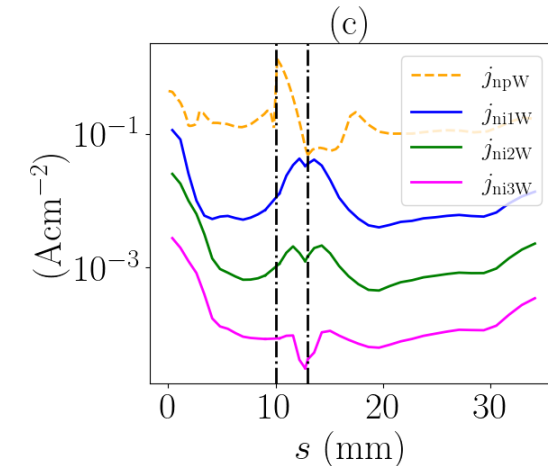
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- ❑ **LOW** beam energy efficiency, **31–38%**, due to high power losses.

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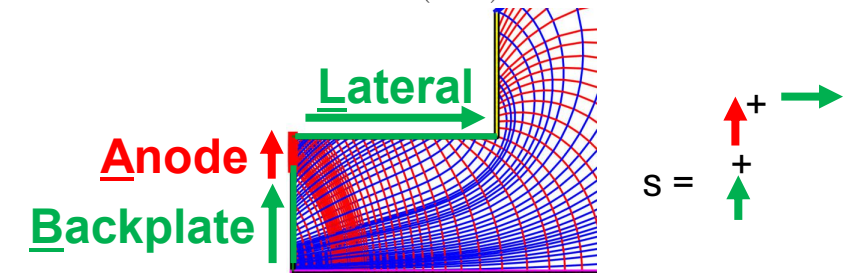
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RED:
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**Current
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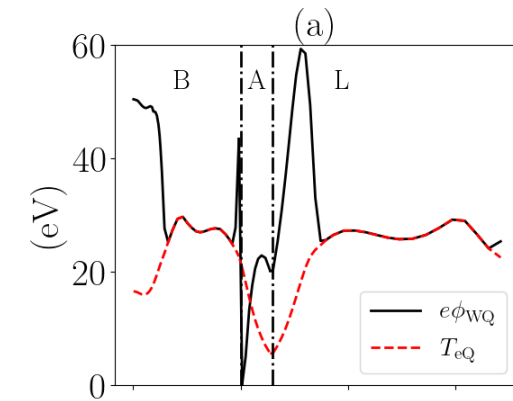
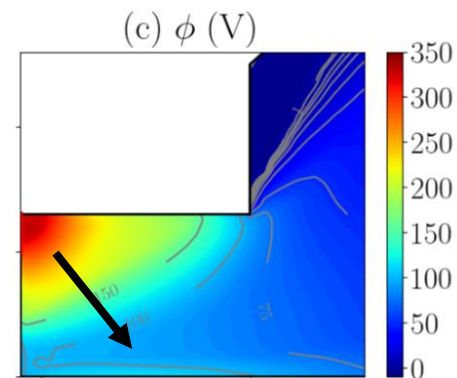
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- ❑ **LOW** beam energy efficiency, **31–38%**, due to high power losses.
- ❑ **STRONG** radial plume divergence efficiency **42–44% (around 50°)**.

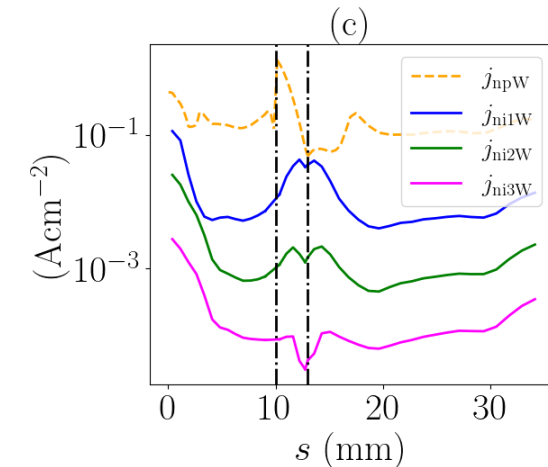
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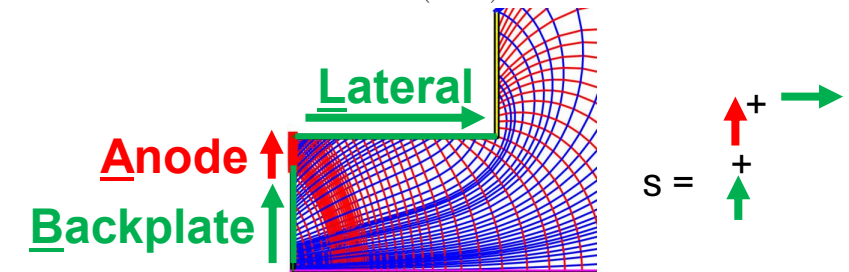
electric potential



RED:
electron
temperature



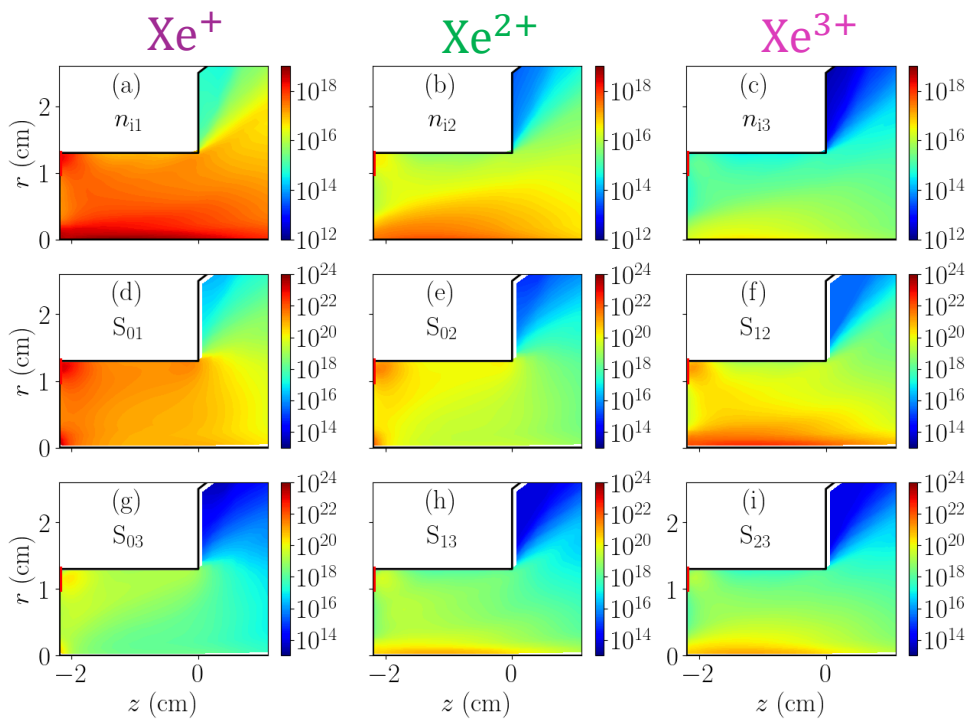
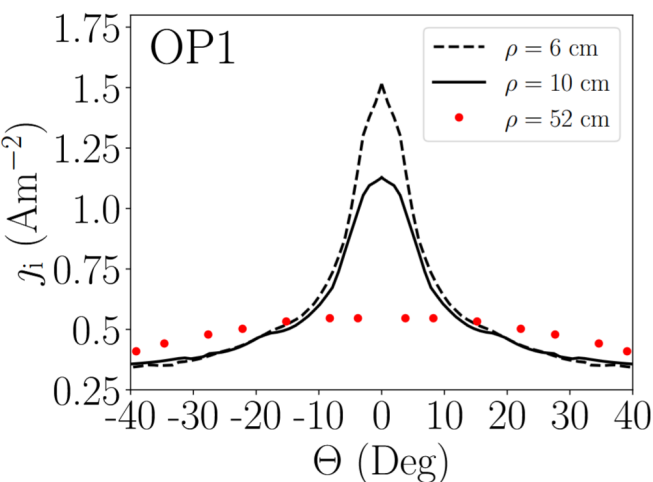
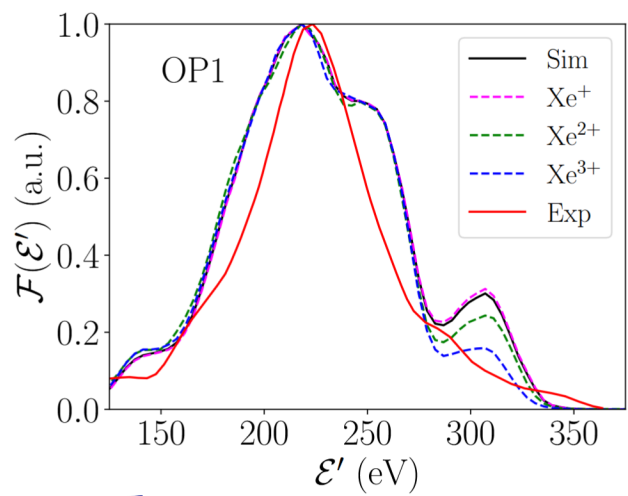
**Current
densities**



More in the paper

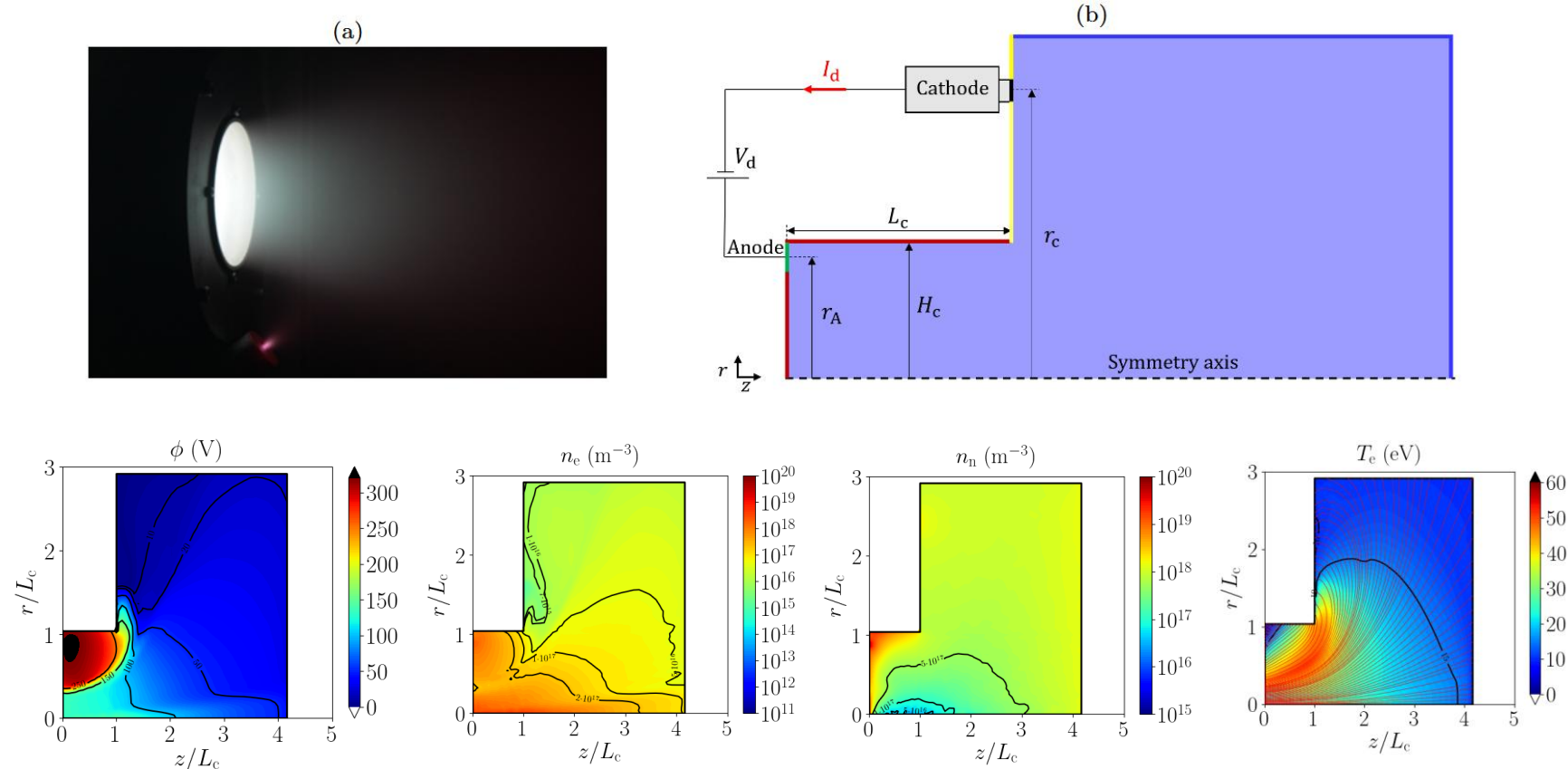
- **This presentation is a reduced version of a full paper submitted to a peer reviewed journal (PSST).**
 - Validation against experimental data.
 - Analysis of the other OPs.
 - Effect of the multiply-charged ions.
 - Brief comparison with a 2D full-PIC simulation of a CHT.
 - Discussion on the electron temperatura in the plume.

OP	V_d (V)	$\dot{m}_A$ (mg s ⁻¹)	Experimental			Simulation					
			F (mN)	I_d (A)	$I_{i\infty}$ (A)	α_{t1} (%)	α_{t2} (%)	F (mN)	I_d (A)	I_{iP} (A)	I_{prod} (A)
1	300	0.40	5.04	1.005	0.360	4.10	1.30	5.14	1.01	0.34	0.64
2	300	0.45	5.96	1.060	0.430	3.85	1.08	6.13	1.06	0.39	0.74
3	350	0.45	6.89	1.085	0.437	3.75	0.98	7.02	1.09	0.40	0.76



Next steps around the CHT

- The journal paper on the CHT200 is currently under peer review at *Plasma Sources Science and Technology* (PSST).
- We will continue the work carried out by Jesus Perales on the CHT of the *Korean Advanced Institute of Technology* (KCHT-50).



DEVELOPMENT OF HYPHEN-CM AND CURRENT STATUS

HYPHEN-CM APPROACH I: Direct method

How does the MFAM solve the continuity + momentum equation?

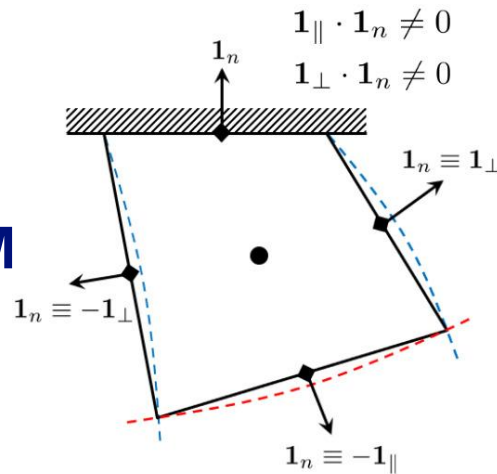
- Solve for: ϕ, j_e
- Known: n_e, T_e, j_i
- Current continuity:

$$\nabla \cdot j = \nabla \cdot (j_e + j_i) = 0 \xrightarrow[\text{MFAM}]{\text{FVM on a unstructured}}$$

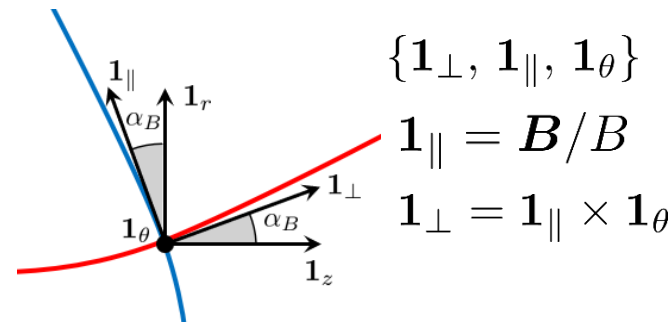
For a given element in a volume V and m faces:

$$\int_V \nabla \cdot j dV = \oint_{S \in V} j \cdot n dS = \sum_m (j_e + j_i) \cdot \mathbf{1}_n A|_m = 0$$

HYPHEN-MFAM



Local magnetic frame



$$j_{||e} = \sigma_{||e} \left[\frac{T_e}{e} \frac{\partial \ln n_e}{\partial \mathbf{1}_{||}} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_{||}} - \frac{\partial \phi}{\partial \mathbf{1}_{||}} \right] - j_{||c}$$

$$j_{\perp e} = \sigma_{\perp e} \left[\frac{T_e}{e} \frac{\partial \ln n_e}{\partial \mathbf{1}_{\perp}} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_{\perp}} - \frac{\partial \phi}{\partial \mathbf{1}_{\perp}} \right] - \frac{j_{\perp c} - \chi_t j_{\theta c}}{1 + \chi \chi_t}$$

$$j_{\theta e} = \chi_t j_{\perp e} - \frac{\chi_t}{\chi} j_{\theta c}$$

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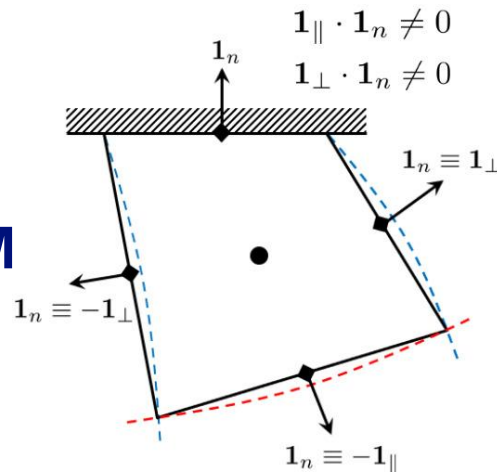
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FVM on a unstructured
MFAM

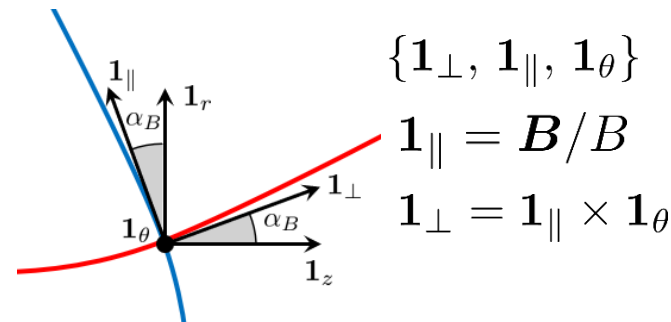
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HYPHEN-MFAM



Local magnetic frame



$$\{\mathbf{1}_\perp, \mathbf{1}_\parallel, \mathbf{1}_\theta\}$$

$$\mathbf{1}_\parallel = \mathbf{B}/B$$

$$\mathbf{1}_\perp = \mathbf{1}_\parallel \times \mathbf{1}_\theta$$

$$j_{\parallel e} = \sigma_{\parallel e} \left[\frac{T_e}{e} \frac{\partial \ln n_e}{\partial \mathbf{1}_\parallel} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_\parallel} - \frac{\partial \phi}{\partial \mathbf{1}_\parallel} \right] - j_{\parallel c}$$

$$j_{\perp e} = \sigma_{\perp e} \left[\frac{T_e}{e} \frac{\partial \ln n_e}{\partial \mathbf{1}_\perp} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_\perp} - \frac{\partial \phi}{\partial \mathbf{1}_\perp} \right] - \frac{j_{\perp c} - \chi_t j_{\theta c}}{1 + \chi \chi_t}$$

$$j_{\theta e} = \chi_t j_{\perp e} - \frac{\chi_t}{\chi} j_{\theta c}$$

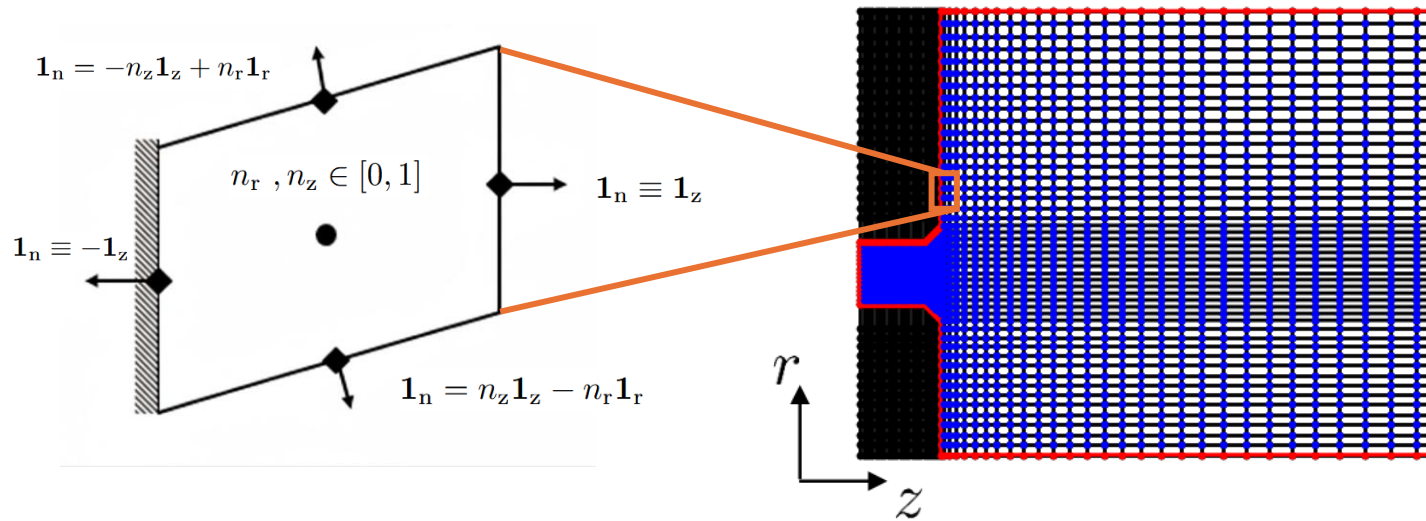
$$\sigma_{\perp e} \ll \sigma_{\parallel e}$$

➤ The MFAM helps limit numerical diffusion caused by the **strongly anisotropic transport of magnetized electrons:**

- MFAM aligns the computational mesh with the magnetic field lines, choosing the natural basis for electron transport (parallel and perpendicular to $\mathbf{B}$).
- In this basis, the conductivity tensor becomes diagonal, decoupling the parallel and perpendicular currents and significantly improving the numerical conditioning of the electron-fluid equations.

HYPHEN-CM APPROACH I: Direct method

HYPHEN-CM



The natural basis for HYPHEN-CM is to solve the electron current in axial and radial directions.



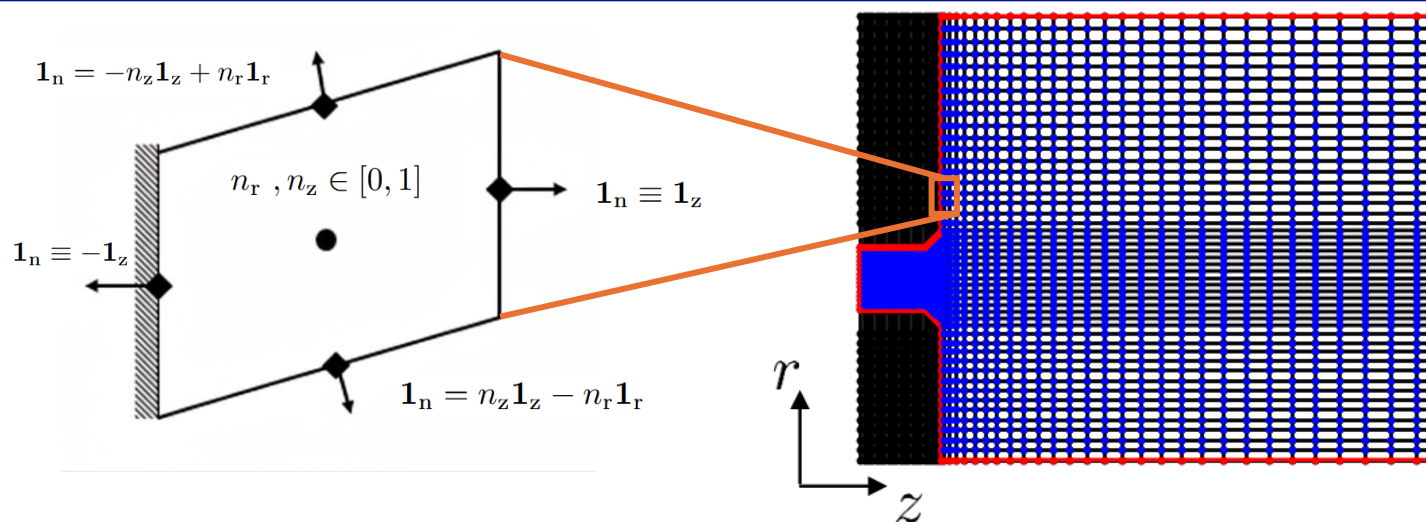
Conductivity tensor is not diagonal anymore.



We do not limit the numerical diffusion

HYPHEN-CM APPROACH I: Direct method

HYPHEN-CM



The natural basis for HYPHEN-CM is to solve the electron current in axial and radial directions.



Conductivity tensor is not diagonal anymore.



We do not limit the numerical diffusion

For all the faces in the domain:

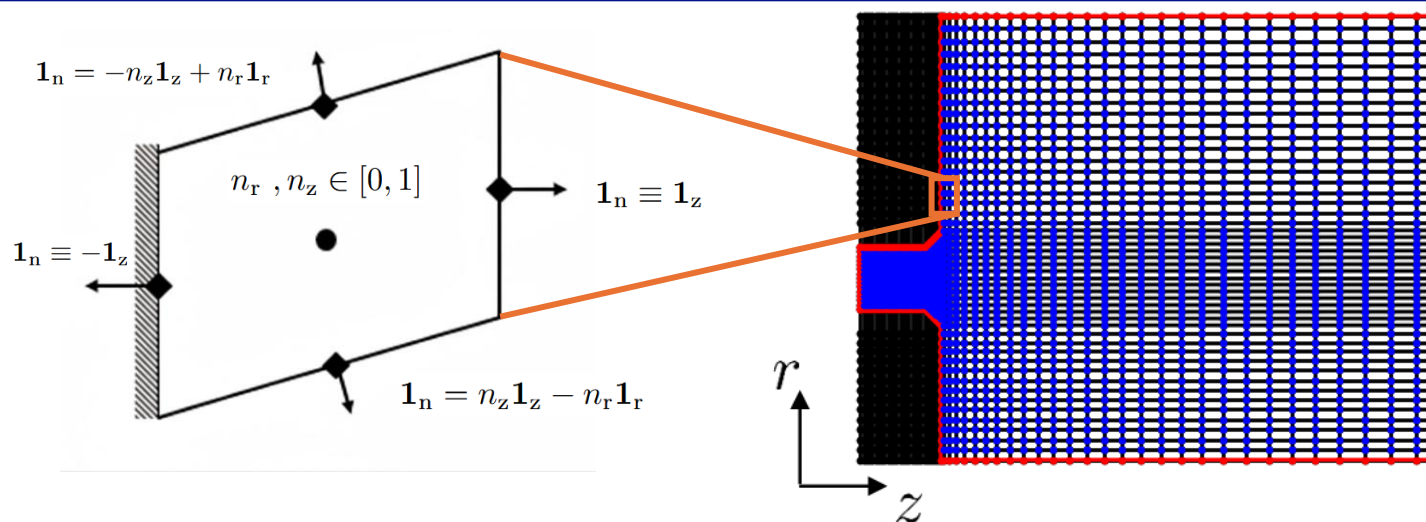
$$\chi = \frac{eB}{m_e \nu_e} ; \chi_z = \frac{eB_z}{m_e \nu_e} ; \chi_r = \frac{eB_r}{m_e \nu_e} ; \chi^2 = \chi_r^2 + \chi_z^2 ; \chi_t = \frac{eB}{m_e \nu_e + \alpha_t eB}$$

$$j_{re} = \frac{1}{\chi^2 + \chi\chi_t^{-1}} \left\{ (\chi_r^2 + \chi\chi_t^{-1}) \left[\sigma_e \left(\frac{T_e}{e} \frac{\partial \ln(n_e)}{\partial \mathbf{1}_r} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_r} - \frac{\partial \phi}{\partial \mathbf{1}_r} \right) - j_{rc} \right] + \chi_r \chi_z \left[\sigma_e \left(\frac{T_e}{e} \frac{\partial \ln(n_e)}{\partial \mathbf{1}_z} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_z} - \frac{\partial \phi}{\partial \mathbf{1}_z} \right) - j_{zc} \right] + \chi_z j_{\theta c} \right\}$$

$$j_{ze} = \frac{1}{\chi^2 + \chi\chi_t^{-1}} \left\{ \chi_r \chi_z \left[\sigma_e \left(\frac{T_e}{e} \frac{\partial \ln(n_e)}{\partial \mathbf{1}_r} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_r} - \frac{\partial \phi}{\partial \mathbf{1}_r} \right) - j_{rc} \right] + (\chi_z^2 + \chi\chi_t^{-1}) \left[\sigma_e \left(\frac{T_e}{e} \frac{\partial \ln(n_e)}{\partial \mathbf{1}_z} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_z} - \frac{\partial \phi}{\partial \mathbf{1}_z} \right) - j_{zc} \right] - \chi_r j_{\theta c} \right\}$$

HYPHEN-CM APPROACH I: Direct method

HYPHEN-CM



The natural basis for HYPHEN-CM is to solve the electron current in axial and radial directions.



Conductivity tensor is not diagonal anymore.



We do not limit the numerical diffusion

For all the faces in the domain:

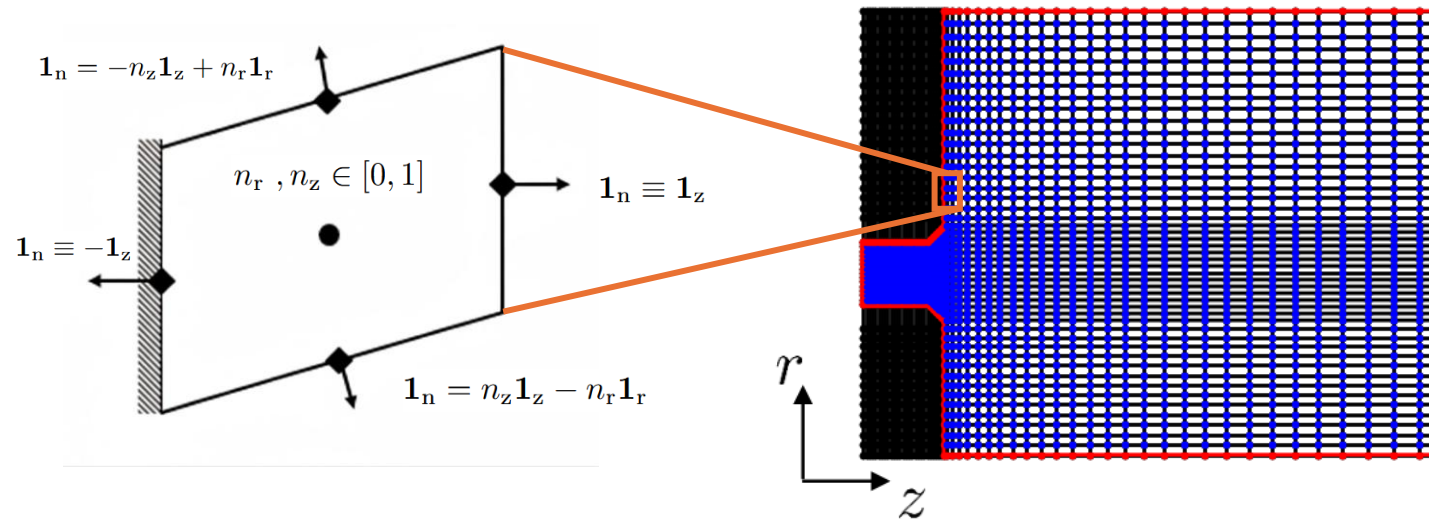
$$\chi = \frac{eB}{m_e \nu_e} ; \chi_z = \frac{eB_z}{m_e \nu_e} ; \chi_r = \frac{eB_r}{m_e \nu_e} ; \chi^2 = \chi_r^2 + \chi_z^2 ; \chi_t = \frac{eB}{m_e \nu_e + \alpha_t eB}$$

$$j_{re} = \frac{1}{\chi^2 + \chi \chi_t^{-1}} \left\{ (\chi_r^2 + \chi \chi_t^{-1}) \left[\sigma_e \left(\frac{T_e}{e} \frac{\partial \ln(n_e)}{\partial \mathbf{1}_r} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_r} - \frac{\partial \phi}{\partial \mathbf{1}_r} \right) - j_{rc} \right] + \chi_r \chi_z \left[\sigma_e \left(\frac{T_e}{e} \frac{\partial \ln(n_e)}{\partial \mathbf{1}_z} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_z} - \frac{\partial \phi}{\partial \mathbf{1}_z} \right) - j_{zc} \right] + \chi_z j_{\theta c} \right\}$$

$$j_{ze} = \frac{1}{\chi^2 + \chi \chi_t^{-1}} \left\{ \chi_r \chi_z \left[\sigma_e \left(\frac{T_e}{e} \frac{\partial \ln(n_e)}{\partial \mathbf{1}_r} - \frac{\partial \phi}{\partial \mathbf{1}_r} \right) - j_{rc} \right] + \left[\sigma_e \left(\frac{T_e}{e} \frac{\partial \ln(n_e)}{\partial \mathbf{1}_z} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_z} - \frac{\partial \phi}{\partial \mathbf{1}_z} \right) - j_{zc} \right] - \chi_r j_{\theta c} \right\} \gg j_{re}$$

HYPHEN-CM APPROACH I: Direct method

HYPHEN-CM



The natural basis for HYPHEN-CM is to solve the electron current in axial and radial directions.



Conductivity tensor is not diagonal anymore.



We do not limit the numerical diffusion

For all the faces in the domain:

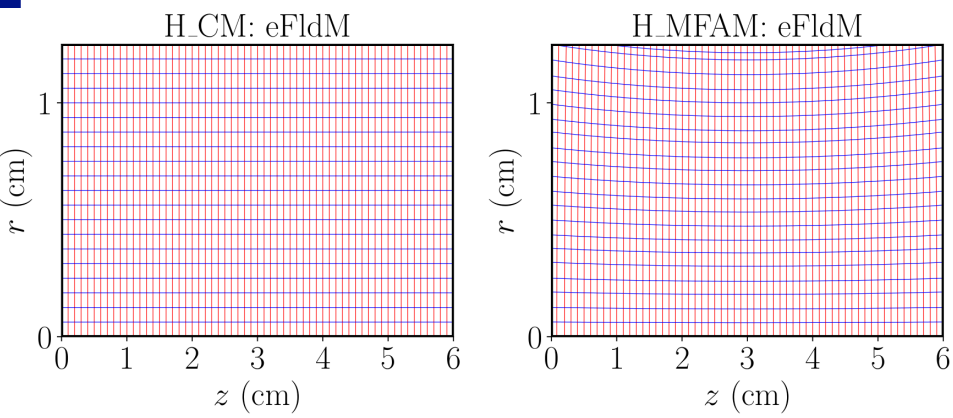
We can limit numerical diffusion by refining the mesh

$$+ \chi_z^2 ; \quad \chi_t = \frac{eB}{m_e \nu_e + \alpha_t eB}$$

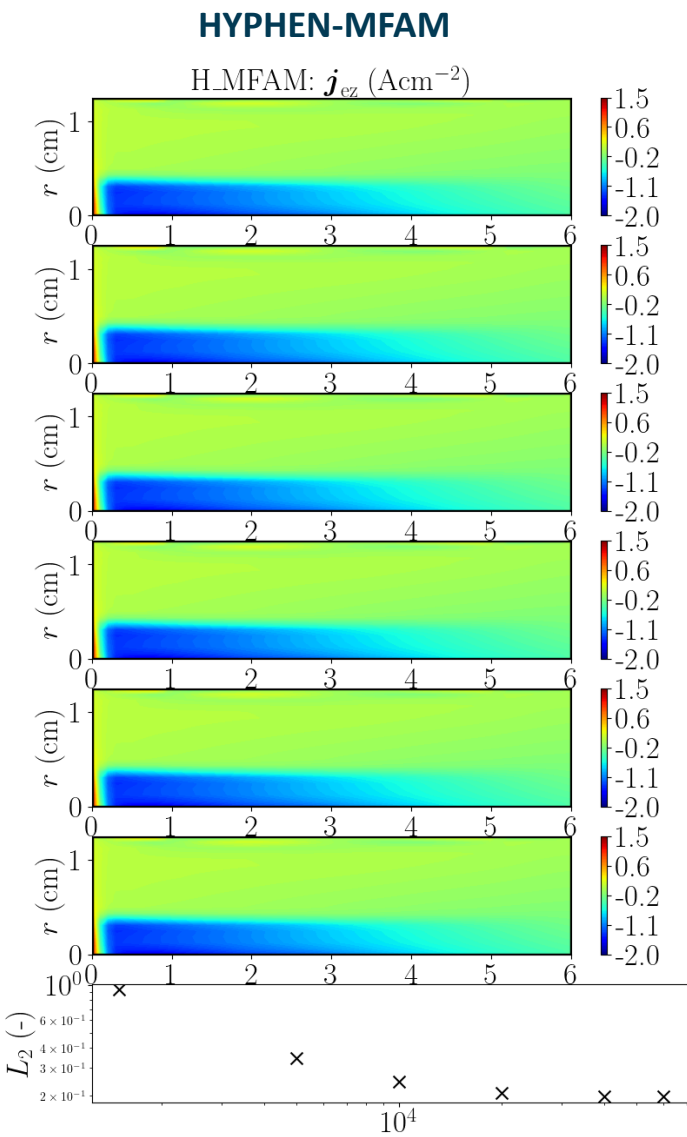
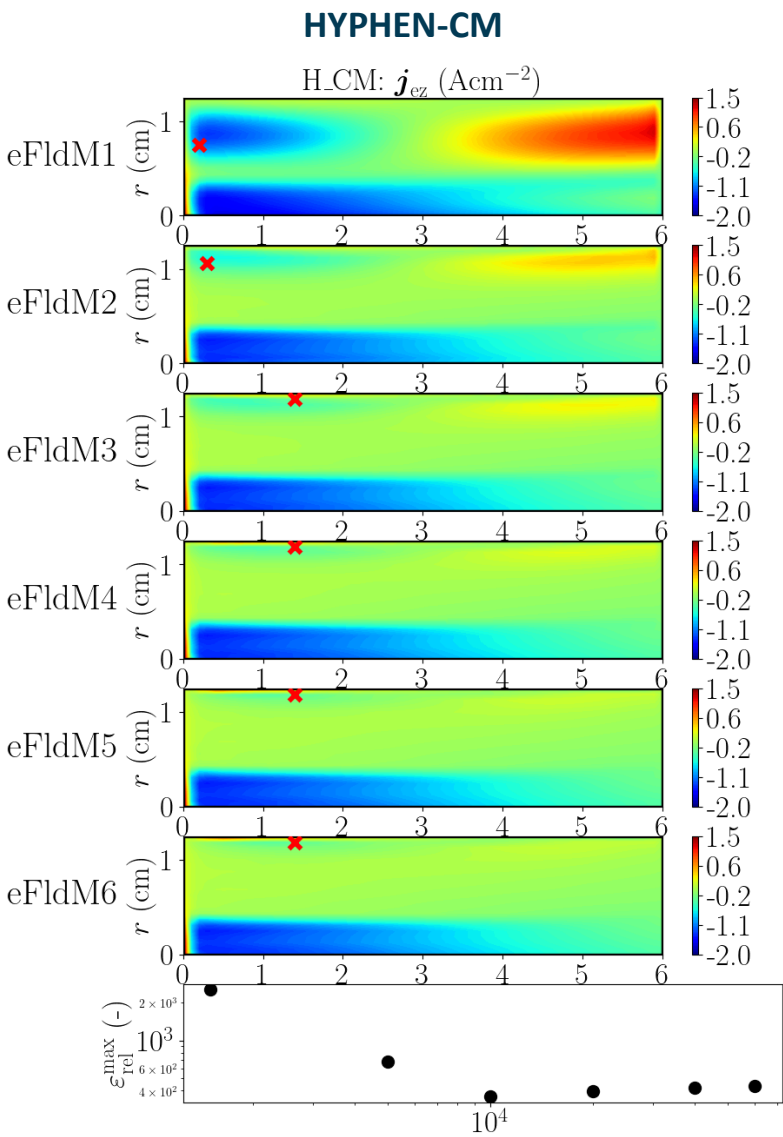
$$j_{re} = \frac{1}{\chi^2 + \chi \chi_t^{-1}} \left\{ (\chi_r^2 + \chi \chi_t^{-1}) \left[\sigma_e \left(\frac{T_e}{e} \frac{\partial \ln(n_e)}{\partial \mathbf{1}_r} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_r} - \frac{\partial \phi}{\partial \mathbf{1}_r} \right) - j_{rc} \right] + \chi_r \chi_z \left[\sigma_e \left(\frac{T_e}{e} \frac{\partial \ln(n_e)}{\partial \mathbf{1}_z} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_z} - \frac{\partial \phi}{\partial \mathbf{1}_z} \right) - j_{zc} \right] + \chi_z j_{\theta c} \right\}$$

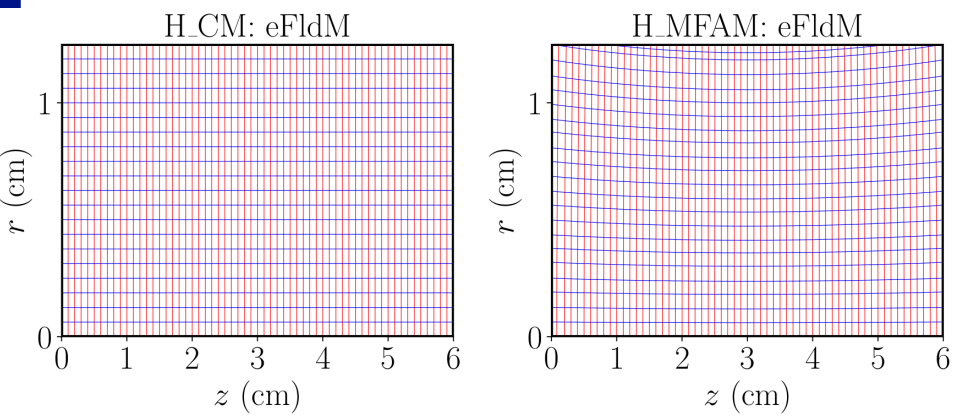
$$j_{ze} = \frac{1}{\chi^2 + \chi \chi_t^{-1}} \left\{ \chi_r \chi_z \left[\sigma_e \left(\frac{T_e}{e} \frac{\partial \ln(n_e)}{\partial \mathbf{1}_r} - \left[\text{Green Box} \right] + \left[\text{Blue Box} \right] \right) - j_{zc} \right] - \chi_r j_{\theta c} \right\} \gg j_{re}$$

HYPHEN-CM APPROACH I: Direct method



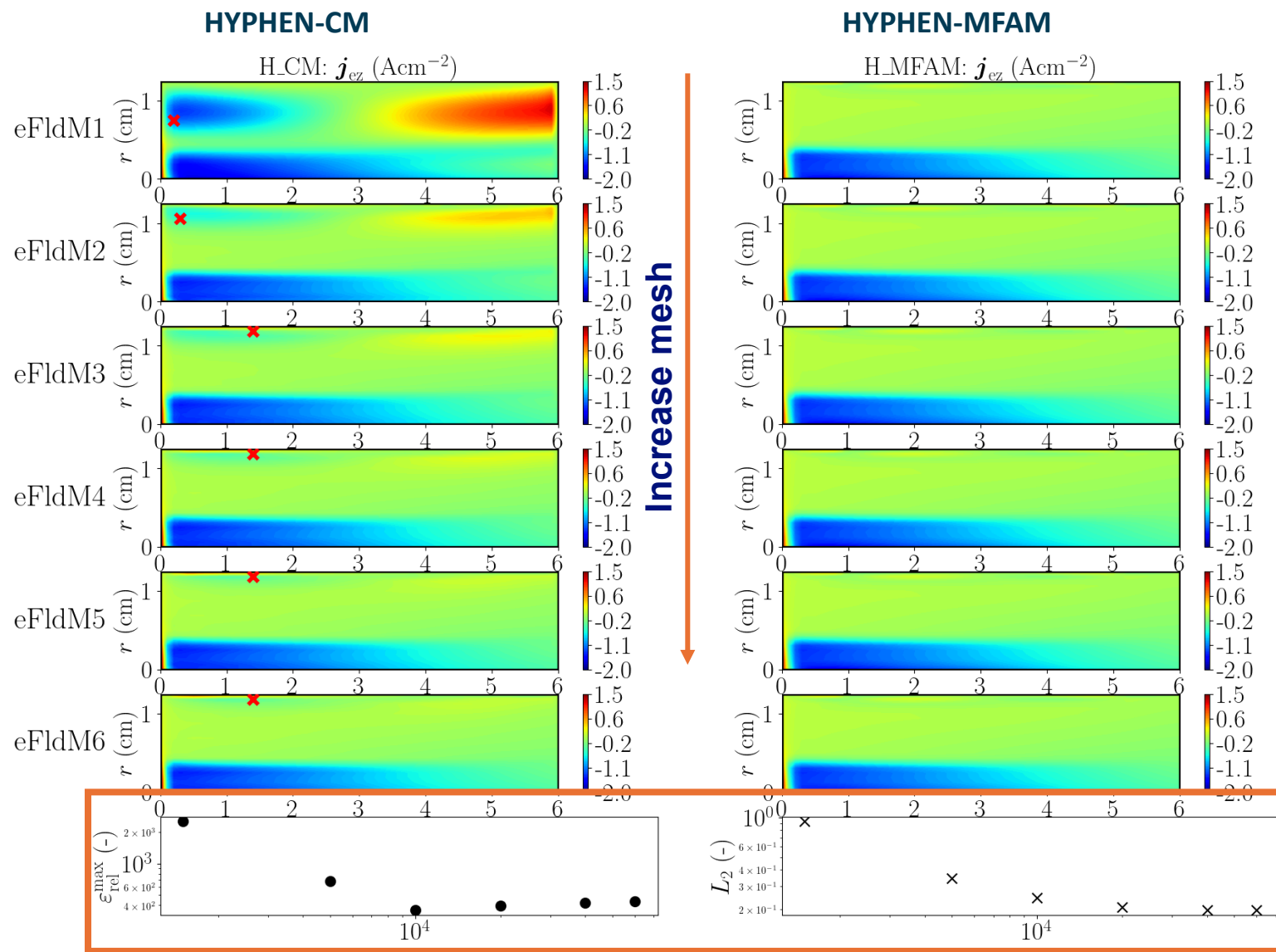
Case	PIC nodes	eFldM nodes	Simulation Time (h)
eFldM1	1281	1500	8.7
eFldM2	1281	5000	9.5
eFldM3	1281	10000	10.2
eFldM4	1281	20000	13.4
eFldM5	1281	40000	16.4
eFldM6	1281	60000	19.1
MFAM	1281	1384	9.4

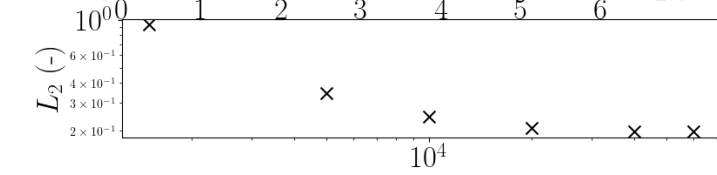
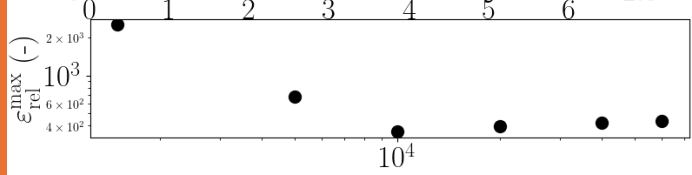




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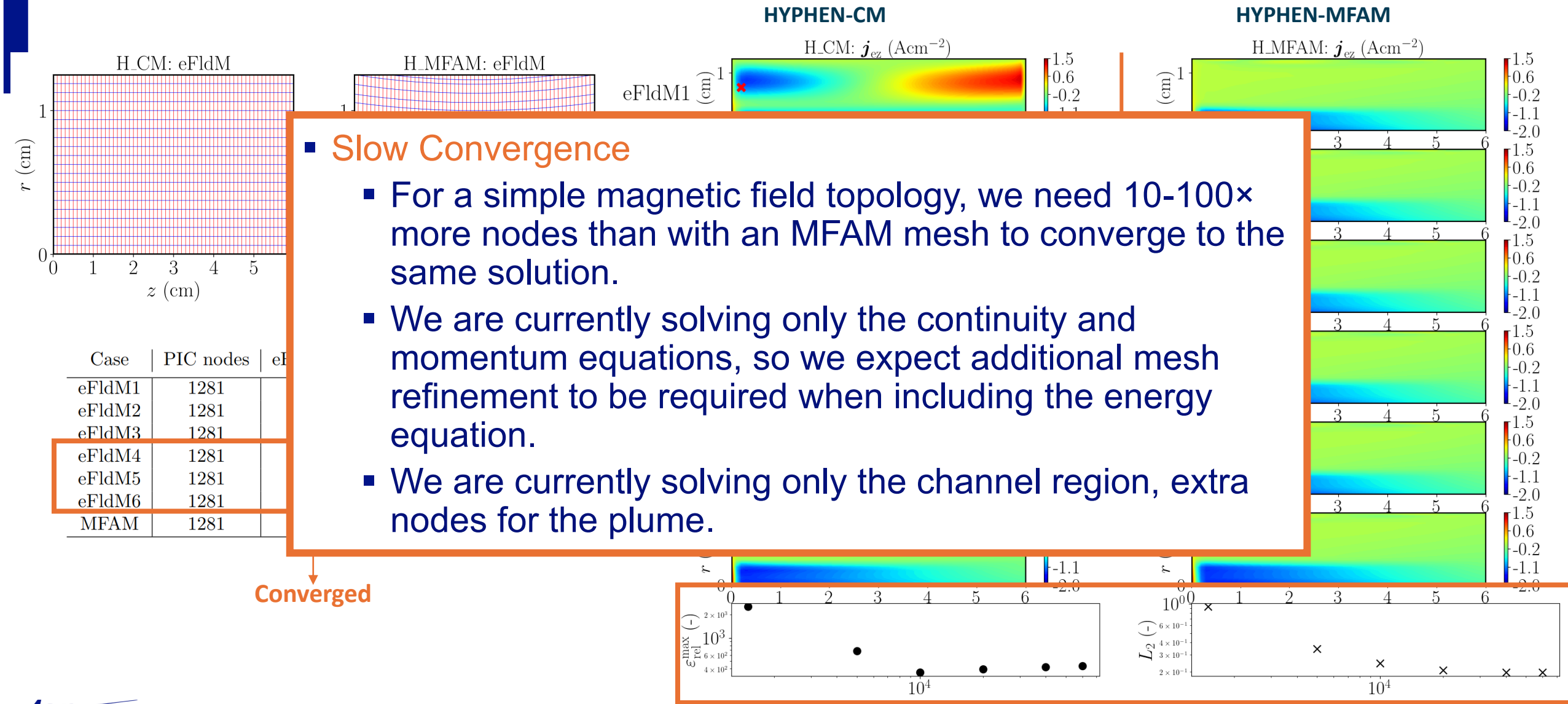
Converged





Increase mesh

HYPHEN-CM APPROACH I: Direct method



Slow Convergence

- For a simple magnetic field topology, we need 10-100× more nodes than with an MFAM mesh to converge to the same solution.
- We are currently solving only the continuity and momentum equations, so we expect additional mesh refinement to be required when including the energy equation.
- We are currently solving only the channel region, extra nodes for the plume.

Case	PIC nodes	eF
eFldM1	1281	
eFldM2	1281	
eFldM3	1281	
eFldM4	1281	
eFldM5	1281	
eFldM6	1281	
MFAM	1281	

HYPHEN-CM APPROACH II: Hagelaar's iterative method

INSTITUTE OF PHYSICS PUBLISHING
Plasma Sources Sci. Technol. 16 (2007) S57–S66
doi:10.1088/0963-0252/16/1/S06

[13] Modelling electron transport in magnetized low-temperature discharge plasmas

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Abstract

Magnetic fields are sometimes used to confine the plasma in low-pressure low-temperature gas discharges, for example in magnetron discharges, Hall-effect-thruster discharges, electron-cyclotron-resonance discharges and helicon discharges. We discuss how these magnetized discharges can be modelled by two-dimensional self-consistent models based on electron fluid equations. The magnetized electron flux is described by an anisotropic drift-diffusion equation, where the electron mobility is much smaller perpendicular to the magnetic field than parallel to it. The electric potential is calculated either from Poisson's equation or from the electron equations, assuming quasineutrality. Although these models involve many assumptions, they are appropriate to study the main effects of the magnetic field on the charged particle transport and space charge electric fields in realistic two-dimensional discharge configurations. We demonstrate by new results that these models reproduce known phenomena such as the establishment of the Boltzmann relation along magnetic field lines, the penetration of perpendicular applied electric fields into the plasma bulk and the decrease in magnetic confinement by short-circuit wall currents. We also present an original method to prevent numerical errors arising from the extreme anisotropy of the electron mobility, which tend to invalidate model results from standard numerical methods.

1. Introduction

The use of steady, external magnetic fields makes it possible to create high-density low-temperature plasmas in gas discharges at very low gas pressure. There are different types of such magnetized discharges, where the magnetic field plays different roles. In general, the magnetic field confines the plasma and limits charge particle loss to the walls, which allows magnetron discharges to operate at relatively low dc or rf voltage. More specifically, in Hall-effect thrusters (HETs), the magnetic field allows perpendicular applied electric fields to penetrate inside the plasma bulk, heat the bulk electrons and accelerate the ions. In electron-cyclotron-resonance (ECR) discharges and helicon discharges, the electrons are heated by magnetized wave modes. Throughout the present paper we will simply refer to these discharges as magnetized discharges; this is not to be confused with fusion-related discharges.

The magnetic field complicates the physics and hence the numerical modelling of magnetized discharges. Standard discharge modelling techniques, such as particle-in-cell (PIC), are often cumbersome for these discharges, and low-order physical approximations are necessary to avoid technical complications and limit computation times. On the other hand, even the lowest order approximations show rich and interesting physics. There is great interest in developing relatively simple numerical models, not to be (mis)used as predictive simulation tools, but simply to help identify and understand the physical principles of magnetized discharges. In fact, the most basic principles of magnetized discharges are known from early analytical work for idealized configurations and can be found in classical texts, but numerical models are necessary to investigate the role of these basic principles in realistic multi-dimensional discharge configurations.

0963-0252/07/010057+10\$30.00 © 2007 IOP Publishing Ltd Printed in the UK

S57

$$j_{re}^{k+1} = \left[-\frac{\sigma_e}{1 + \chi_z \chi'_z} \frac{\partial \Phi}{\partial r} \right]^{k+1} + \left\{ \frac{\sigma_e}{1 + \chi_z \chi'_z} \frac{1}{e} \frac{\partial T_e}{\partial r} \left[1 - \ln \left(\frac{n_e}{n_{e0}} \right) \right] \right\}^{k+1} + \frac{\chi_z \chi'_r}{1 + \chi_z \chi'_z} j_{ze}^k + \left\{ \frac{\chi'_z}{1 + \chi_z \chi'_z} j_{\theta c} - \frac{j_{rc}}{1 + \chi_z \chi'_z} \right\}^{k+1}$$

$$j_{ze}^{k+1} = \left[-\frac{\sigma_e}{1 + \chi_r \chi'_r} \frac{\partial \Phi}{\partial z} \right]^{k+1} + \left\{ \frac{\sigma_e}{1 + \chi_r \chi'_r} \frac{1}{e} \frac{\partial T_e}{\partial z} \left[1 - \ln \left(\frac{n_e}{n_{e0}} \right) \right] \right\}^{k+1} + \frac{\chi_r \chi'_z}{1 + \chi_r \chi'_r} j_{re}^k + \left[-\frac{\chi'_r}{1 + \chi_r \chi'_r} j_{\theta c} - \frac{j_{zc}}{1 + \chi_r \chi'_r} \right]^{k+1}$$

$$j_{\theta e} = j_{re} \chi'_z - j_{ze} \chi'_r - j_{\theta c} \frac{\chi_t}{\chi}$$

Stop when residual for both R and Z are smaller than a tolerance

HYPHEN-CM APPROACH II: Hagelaar's iterative method

INSTITUTE OF PHYSICS PUBLISHING

Plasma Sources Sci. Technol. 16 (2007) S57–S66

PLASMA SOURCES SCIENCE AND TECHNOLOGY

doi:10.1088/0963-0252/16/1/S06

[13] Modelling electron transport in magnetized low-temperature discharge plasmas

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S57

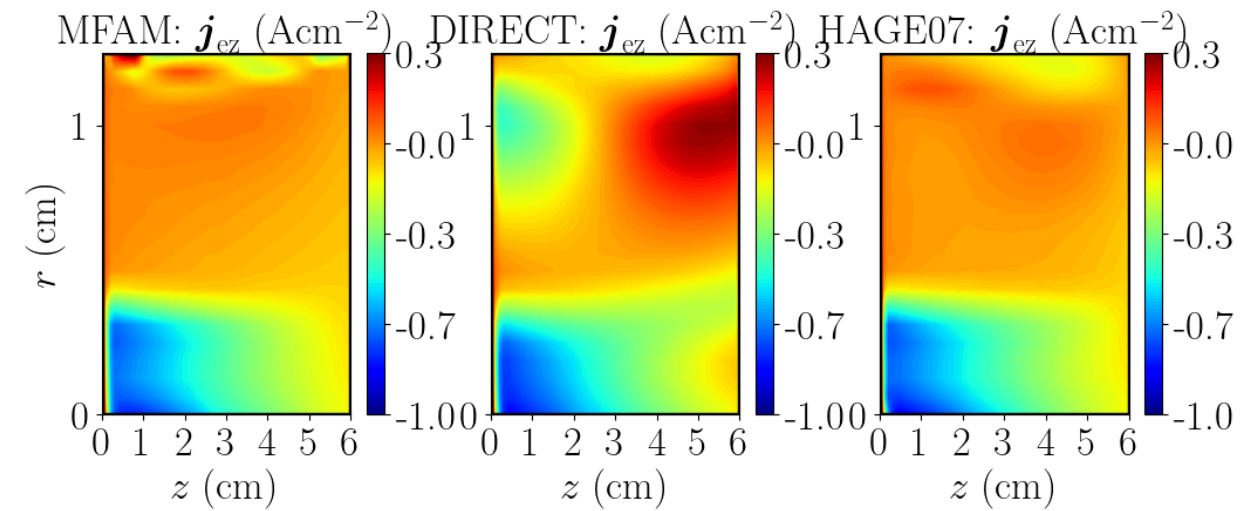
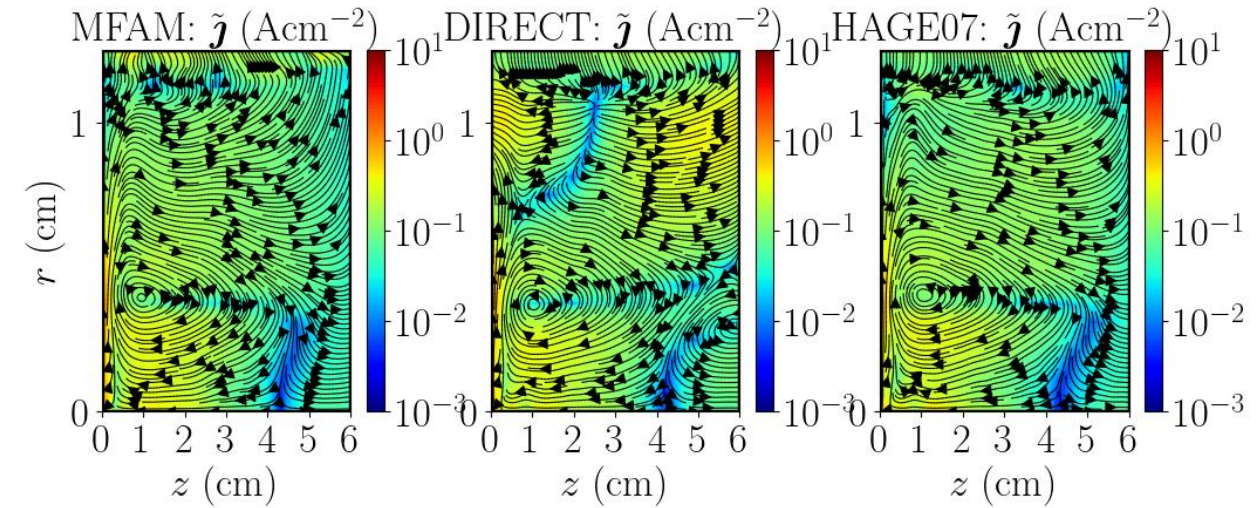
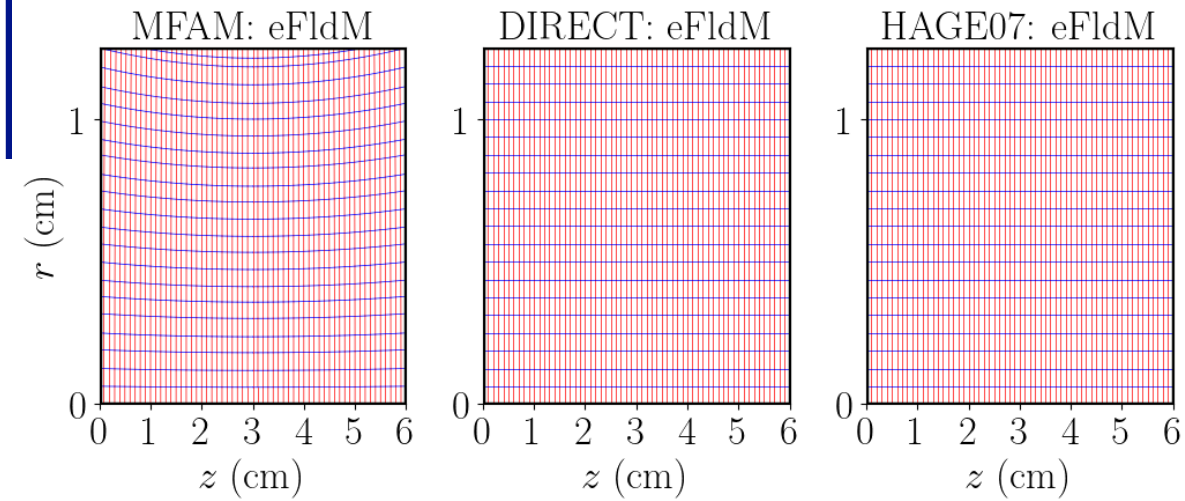
$$j_{re}^{k+1} = \left[-\frac{\sigma_e}{1 + \chi_z \chi'_z} \frac{\partial \Phi}{\partial r} \right]^{k+1} + \left\{ \frac{\sigma_e}{1 + \chi_z \chi'_z} \frac{1}{e} \frac{\partial T_e}{\partial r} \left[1 - \ln \left(\frac{n_e}{n_{e0}} \right) \right] \right\}^{k+1} \\ + \frac{\chi_z \chi'_r}{1 + \chi_z \chi'_z} j_{ze}^k + \left\{ \frac{\chi'_z}{1 + \chi_z \chi'_z} j_{\theta c} - \frac{j_{rc}}{1 + \chi_z \chi'_z} \right\}^{k+1}$$

Rearrange equations such that we are summing terms of the same order + use an iterative scheme.

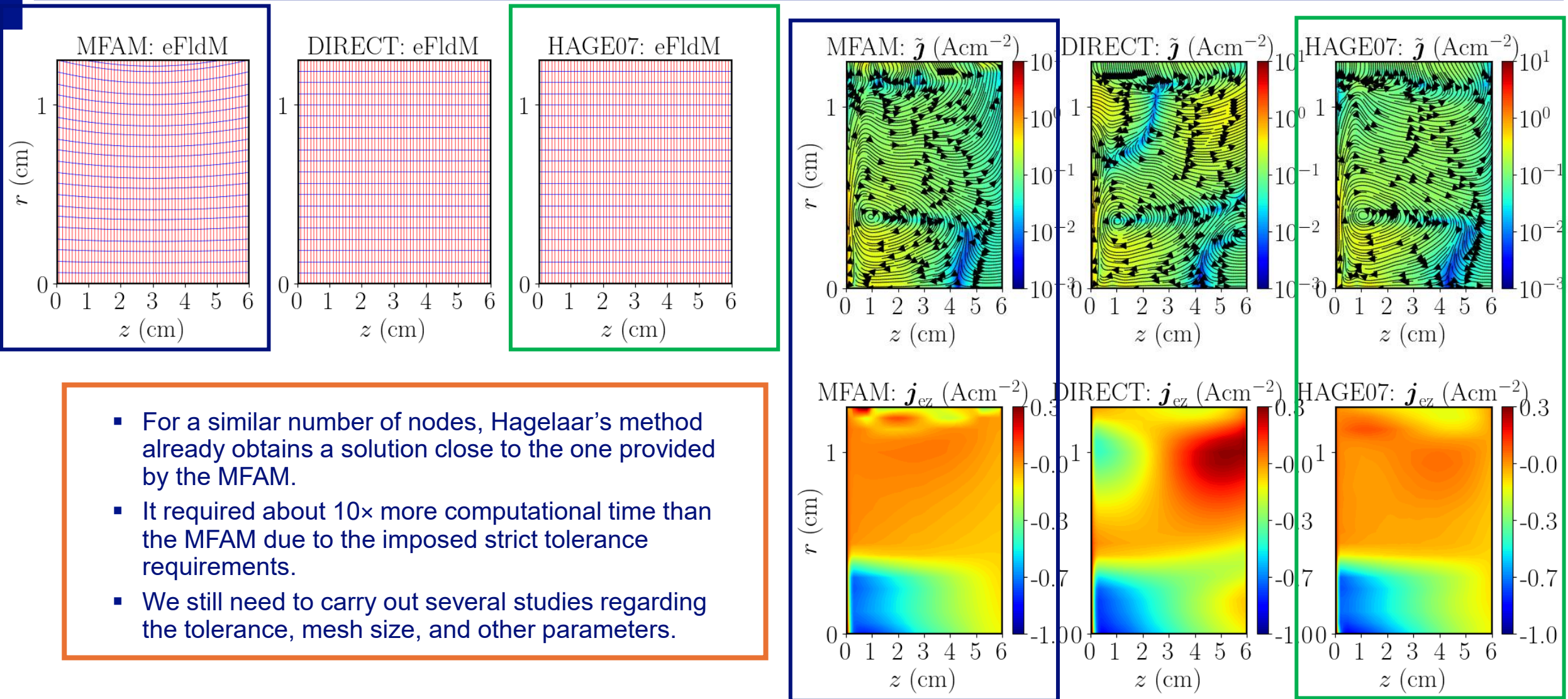
$$j_{ze}^{k+1} = \left[-\frac{\sigma_e}{1 + \chi_r \chi'_r} \frac{\partial \Phi}{\partial z} \right]^{k+1} + \left\{ \frac{\sigma_e}{1 + \chi_r \chi'_r} \frac{1}{e} \frac{\partial T_e}{\partial z} \left[1 - \ln \left(\frac{n_e}{n_{e0}} \right) \right] \right\}^{k+1} \\ + \frac{\chi_r \chi'_z}{1 + \chi_r \chi'_r} j_{re}^k + \left[-\frac{\chi'_r}{1 + \chi_r \chi'_r} j_{\theta c} - \frac{j_{zc}}{1 + \chi_r \chi'_r} \right]^{k+1}$$

$$j_{\theta e} = j_{re} \chi'_z - j_{ze} \chi'_r - j_{\theta c} \frac{\chi_t}{\chi}$$

Stop when residual for both R and Z are smaller than a tolerance

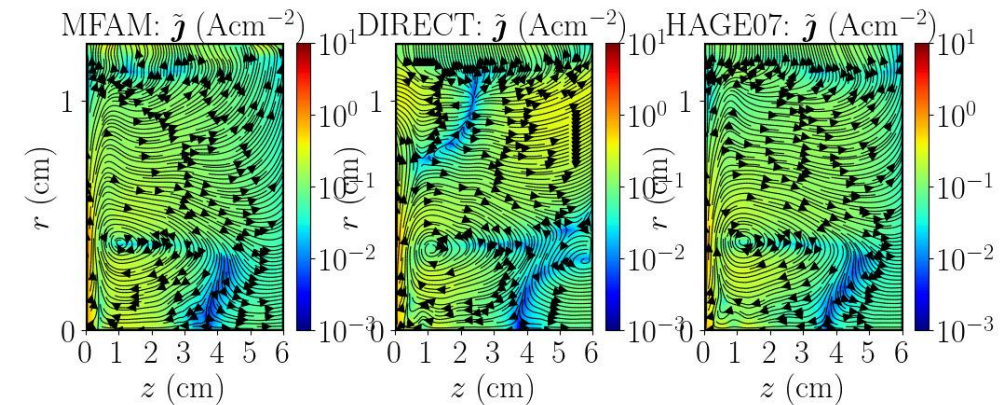
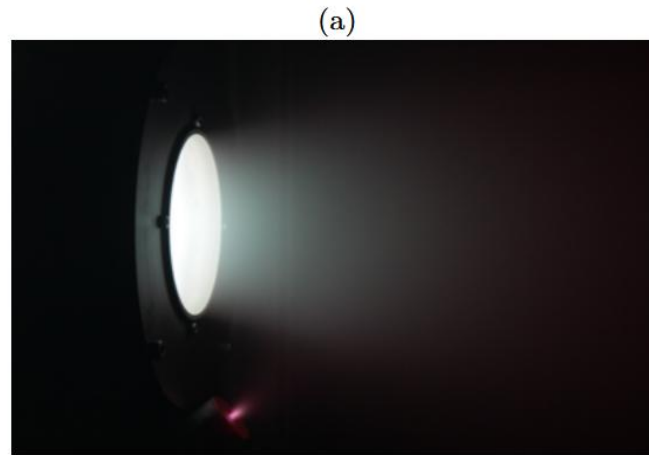


PRELIMINARY results



Next Steps

- Hopefully, finish the review process and publish the work on the CHT200.
- Advance the analysis of the KAIST CHT.
- Continue the development of HYPHEN-CM.
- Use HYPHEN-CM to simulate a non-conventional Hall thruster configuration.



➤ Journal papers:

- ❑ **Under revision at PSST:** Fernández-Tena, I., Perrotin, T., Navarro-Cavallé, J., Zhou, J., Ahedo, E., & Dominguez-Vázquez, A. Analysis of a low-power cylindrical Hall thruster with 2D hybrid plasma simulations.

➤ Conference papers:

- ❑ Fernández-Tena, I., Zhou, J., Ahedo, E., Perrotin, T., Navarro-Cavallé, J., & Dominguez-Vázquez, A. (2025, September). Experimentally-informed 2D hybrid simulations of a low-power cylindrical Hall thruster. In *39th International Electric Propulsion Conference, no. IEPC-2025-185. London, UK, September* (pp. 14-19).
- ❑ Fernández-Tena, I., Perrotin, T., Navarro-Cavallé, J., Zhou, J., Ahedo, E., & Dominguez-Vázquez, A. (2026, May). 2D hybrid plasma simulations of the CHT200 cylindrical Hall thruster. In *10th Space Propulsion Conference, no. SPC-2026-402. Bari, Italy, May* (pp. 17-21).

➤ Workshops:

- ❑ 2nd Electric Propulsion Summer School, *ESA ESTEC, Noordwijk (The Netherlands) , April 1-5, 2025.*
- ❑ 11th European Space Propulsion System Simulation (ESPSS) Workshop, *ESA ESTEC, Noordwijk (The Netherlands) , October 1-2, 2025.*
- ❑ One of the organiser of the Exchange Workshop 5: “Reference management and note taking best practices”

- [1] - T. Perrotin, P. Fajardo, and J. Navarro-Cavallé, "Direct thrust measurements of a 200 W cylindrical Hall thruster," in 38th International Electric Propulsion Conference, no. IEPC-2024-747. Toulouse, France, June 23-28: Electric Rocket Propulsion Society, 2024.
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- [9] - Kronhaus I, Linossier A (2018) Experimental characterization of the narrow channel Hall thruster. Plasma Sources Sci Technol 27:124005 [https:// doi. org/ 10. 1088/ 1361- 6595/ aaec65](https://doi.org/10.1088/1361-6595/aaec65).
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- [11] - Raites Y, Fisch N, Ertmer K and Burlingame C 2000 A Study of Cylindrical Hall Thruster for Low Power Space Applications 36th Joint Propulsion Conference, Huntsville, AL AIAA 2000-3421 .
- [12] - Perales-Díaz, J., Ahedo, E., Kim, Y., & Choe, W. (2025). 2D numerical analysis of the KCHT-50 cylindrical Hall thruster.
- [13] - Hagelaar, G. J. M. (2007). Modelling electron transport in magnetized low-temperature discharge plasmas. *Plasma Sources Science and Technology*, 16(1), S57-S66.

Thank you! Questions?



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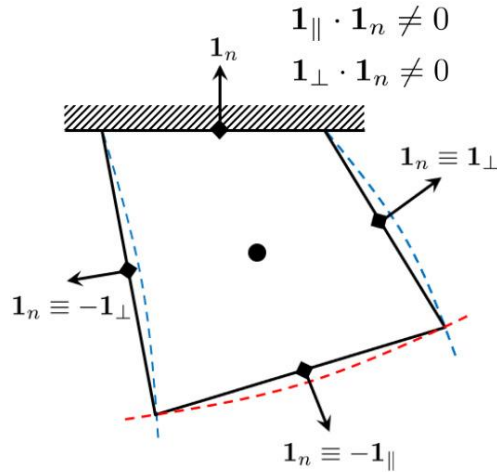
EP² uc3m

This work has been supported by the R&D project PID2022-140035OB-I00 (HEEP) funded by MCIN/AEI/10.13039/501100011033 and by “ERDF A way of making Europe”.

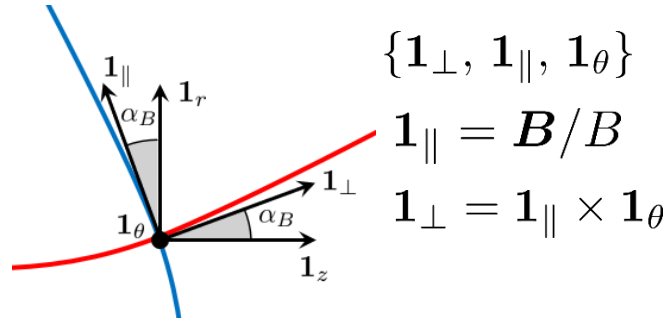
Extra slides

HYPHEN-CM APPROACH I: Direct method

HYPHEN-MFAM



Local magnetic frame

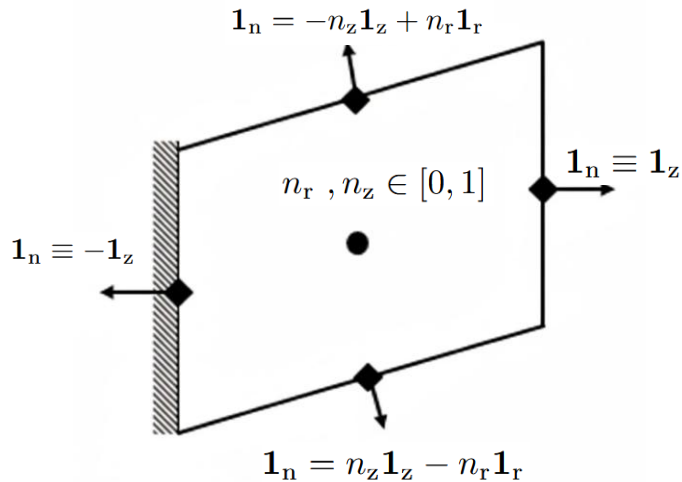


$$j_{\parallel e} = \sigma_{\parallel e} \left[\frac{T_e}{e} \frac{\partial \ln n_e}{\partial \mathbf{1}_{\parallel}} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_{\parallel}} - \frac{\partial \phi}{\partial \mathbf{1}_{\parallel}} \right] - j_{\parallel c}$$

$$j_{\perp e} = \sigma_{\perp e} \left[\frac{T_e}{e} \frac{\partial \ln n_e}{\partial \mathbf{1}_{\perp}} + \frac{1}{e} \frac{\partial T_e}{\partial \mathbf{1}_{\perp}} - \frac{\partial \phi}{\partial \mathbf{1}_{\perp}} \right] - \frac{j_{\perp c} - \chi_t j_{\theta c}}{1 + \chi \chi_t}$$

$$j_{\theta e} = \chi_t j_{\perp e} - \frac{\chi_t}{\chi} j_{\theta c}$$

HYPHEN-CM



Fastest method to implement: Obtain parallel and perpendicular gradients from z and r gradients

$$\frac{\partial}{\partial \mathbf{1}_{\parallel}} = \cos(\alpha_B) \frac{\partial}{\partial \mathbf{1}_z} + \sin(\alpha_B) \frac{\partial}{\partial \mathbf{1}_r}$$

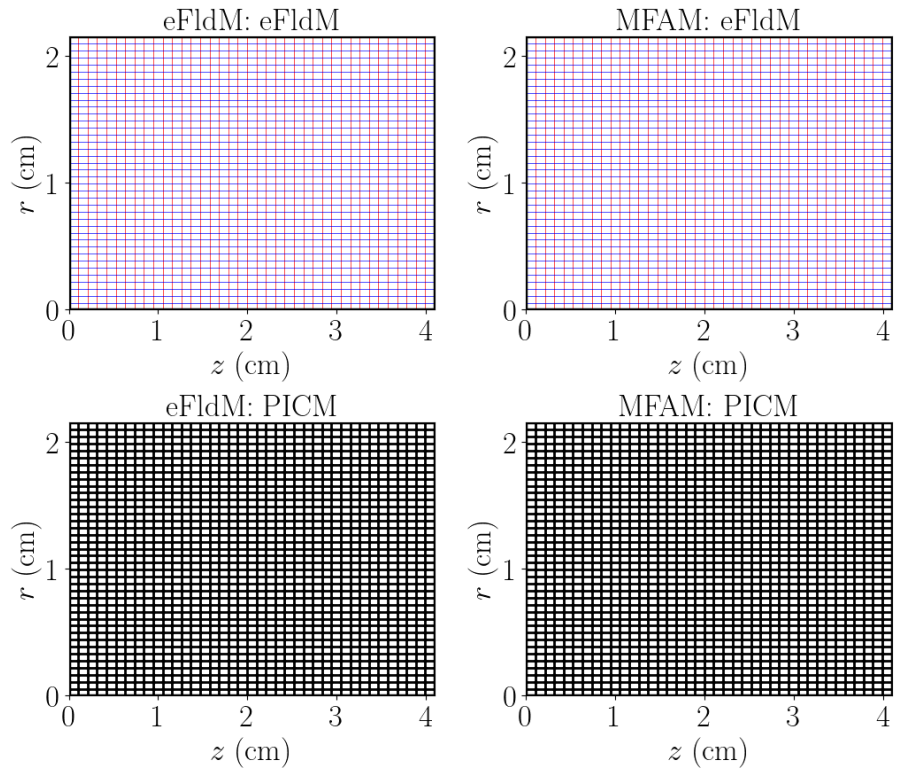
$$\frac{\partial}{\partial \mathbf{1}_{\perp}} = \sin(\alpha_B) \frac{\partial}{\partial \mathbf{1}_z} - \cos(\alpha_B) \frac{\partial}{\partial \mathbf{1}_r}$$

For ALL faces solve parallel and perpendicular electron currents

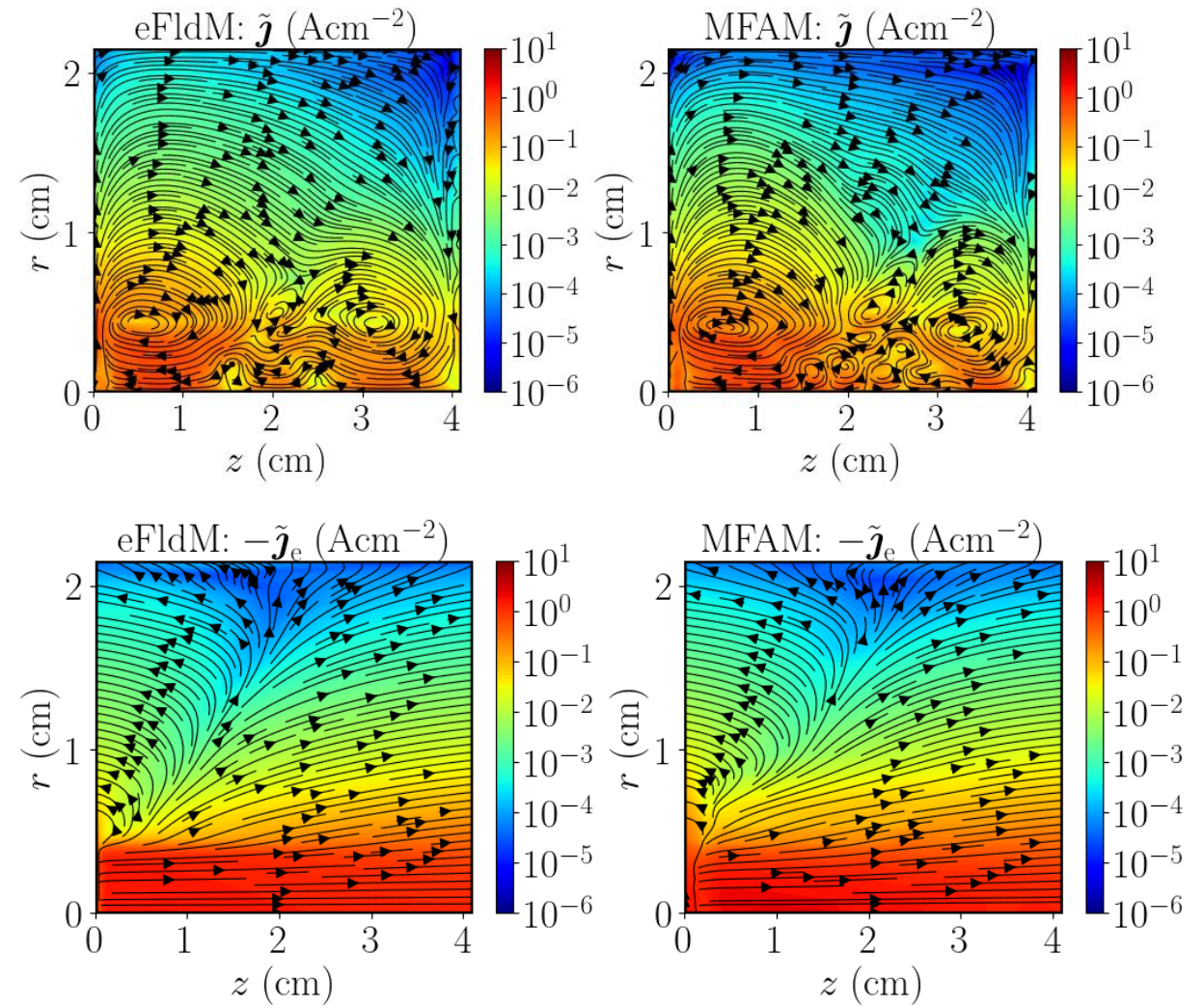
WE ARE MIXING DIRECTIONS → NUMERICAL DIFFUSION NOT AVOIDED

Hybrid particle-fluid plasma modeling of Hall effect thrusters and analysis of non-conventional configurations

IMPLEMENTATION TEST: CASE (I)

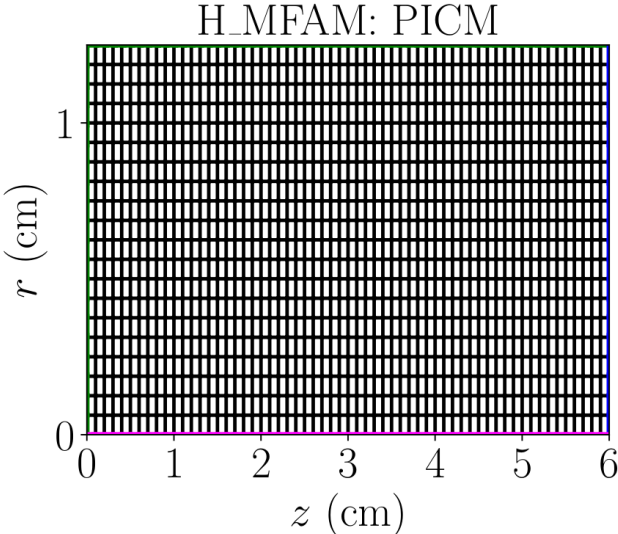
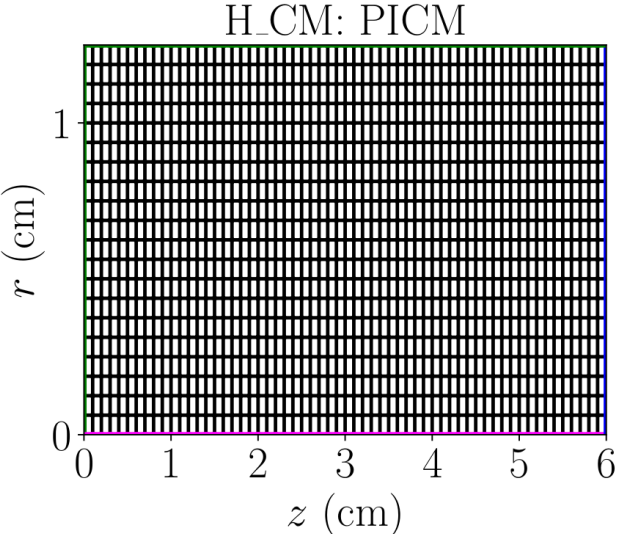
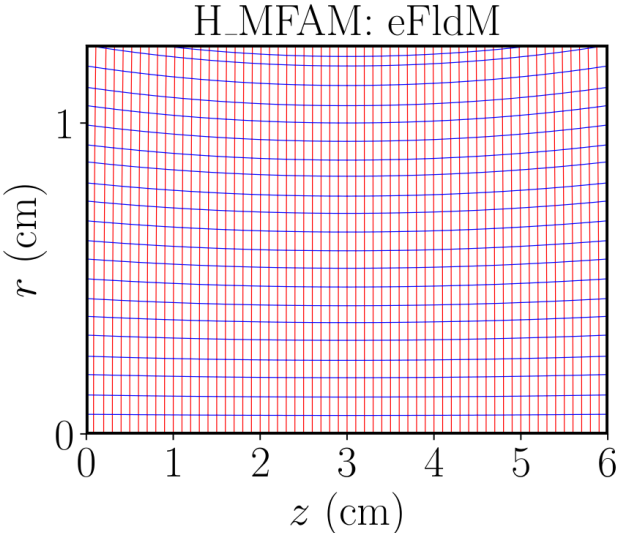
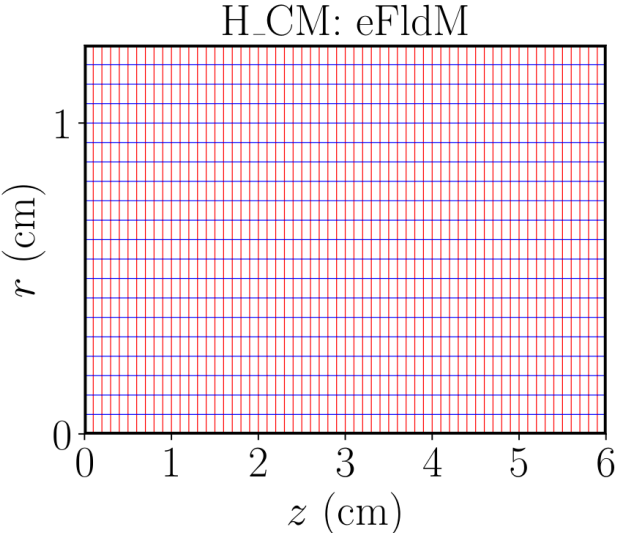


Case	N_r	N_z	Δt (ns)	Δx (mm)	N_{steps}	Time (μs)
eFldM1	21	21	10	2.050	60000	600
eFldM2	41	41	5.0	1.025	120000	600
eFldM3	61	61	3.3	0.680	150000	600
eFldM4	81	81	2.5	0.512	240000	600
MFAM1	21	21	10	2.050	60000	600
MFAM2	41	41	5.0	1.025	120000	600
MFAM3	61	61	3.3	0.680	150000	600
MFAM4	81	81	2.5	0.512	240000	600



Hybrid particle-fluid plasma modeling of Hall effect
thrusters and analysis of non-conventional
configurations

Convergence Study: CASE (II)

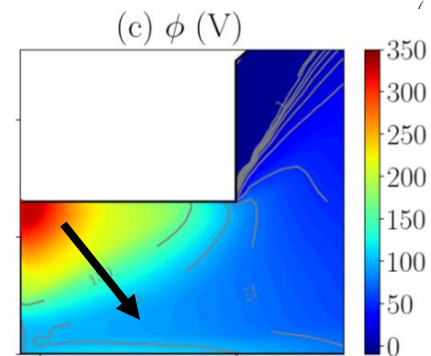
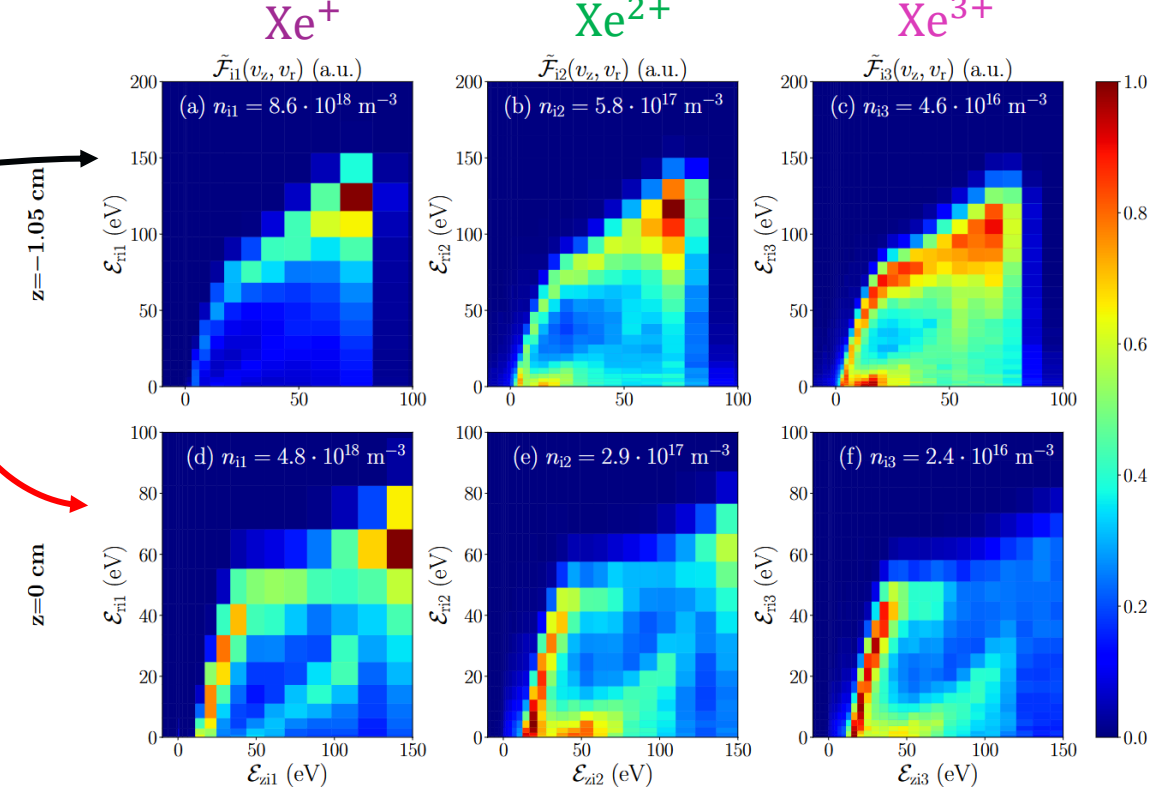
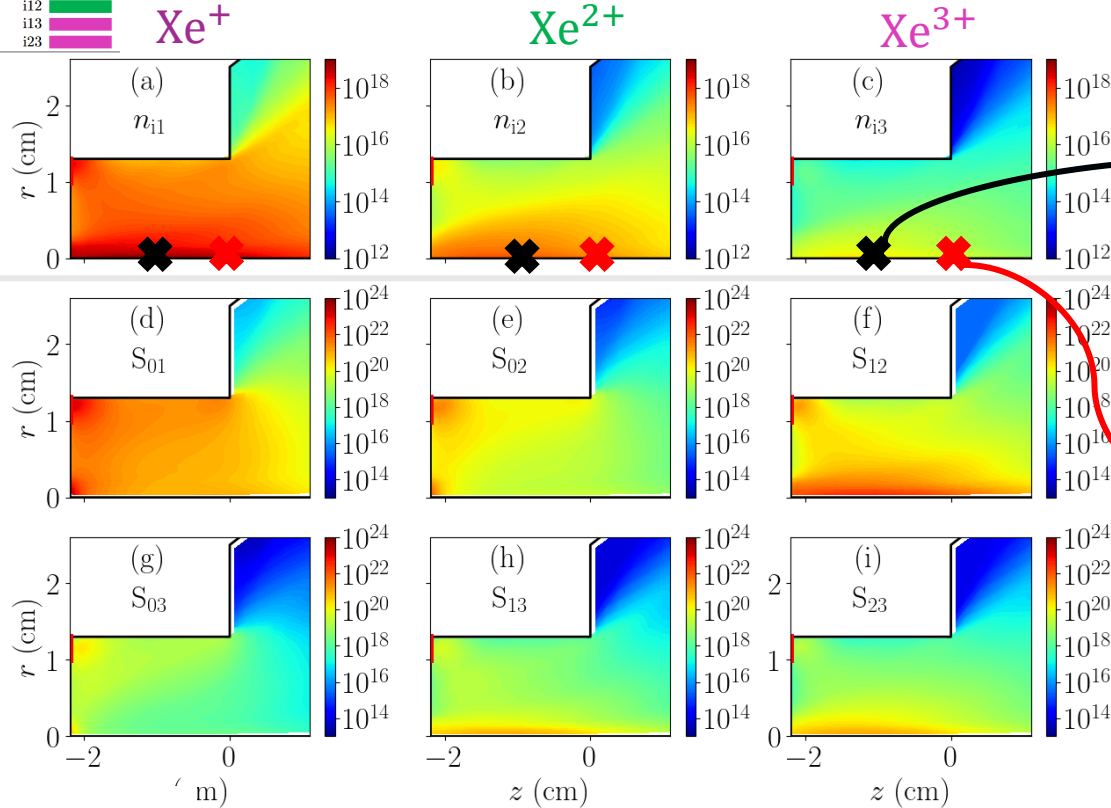


Case	PIC nodes	eFldM nodes	Simulation Time (h)
eFldM1	1281	1500	8.7
eFldM2	1281	5000	9.5
eFldM3	1281	10000	10.2
eFldM4	1281	20000	13.4
eFldM5	1281	40000	16.4
eFldM6	1281	60000	19.1
MFAM	1281	1384	9.4

Simulation results: 2D plasma maps (II)

Reaction	Nomenclature
$e + \text{Xe} \rightarrow 2e + \text{Xe}^+$	i01
$e + \text{Xe} \rightarrow 3e + \text{Xe}^{2+}$	i02
$e + \text{Xe} \rightarrow 4e + \text{Xe}^{3+}$	i03
$e + \text{Xe}^+ \rightarrow 2e + \text{Xe}^{2+}$	i12
$e + \text{Xe}^+ \rightarrow 3e + \text{Xe}^{3+}$	i13
$e + \text{Xe}^{2+} \rightarrow 2e + \text{Xe}^{3+}$	i23

Ion densities (m^{-3})



Comparing to a AHT:

- ☐ Xe^+ mainly generated near anode/injector and walls.
- ☐ Xe^+ ions accelerate toward axis, creating strong central density peak.
- ☐ Xe^{2+} shows two peaks: near anode and on axis. Xe^{2+} peak caused by Xe^+ ionization + ion convergence.
- ☐ High Xe^{3+} ionization around the axis from $\text{Xe}^+ / \text{Xe}^{2+}$.
- ☐ Multiply charged ions inherit parent ion energy.
- ☐ Mechanism is characteristic of cylindrical Hall thrusters and it explains high fraction of $\text{Xe}^{2+} / \text{Xe}^{3+}$ ions and propellant efficiencies above 100%.
- ☐ All ion species accelerate axially and radially toward plume exit.

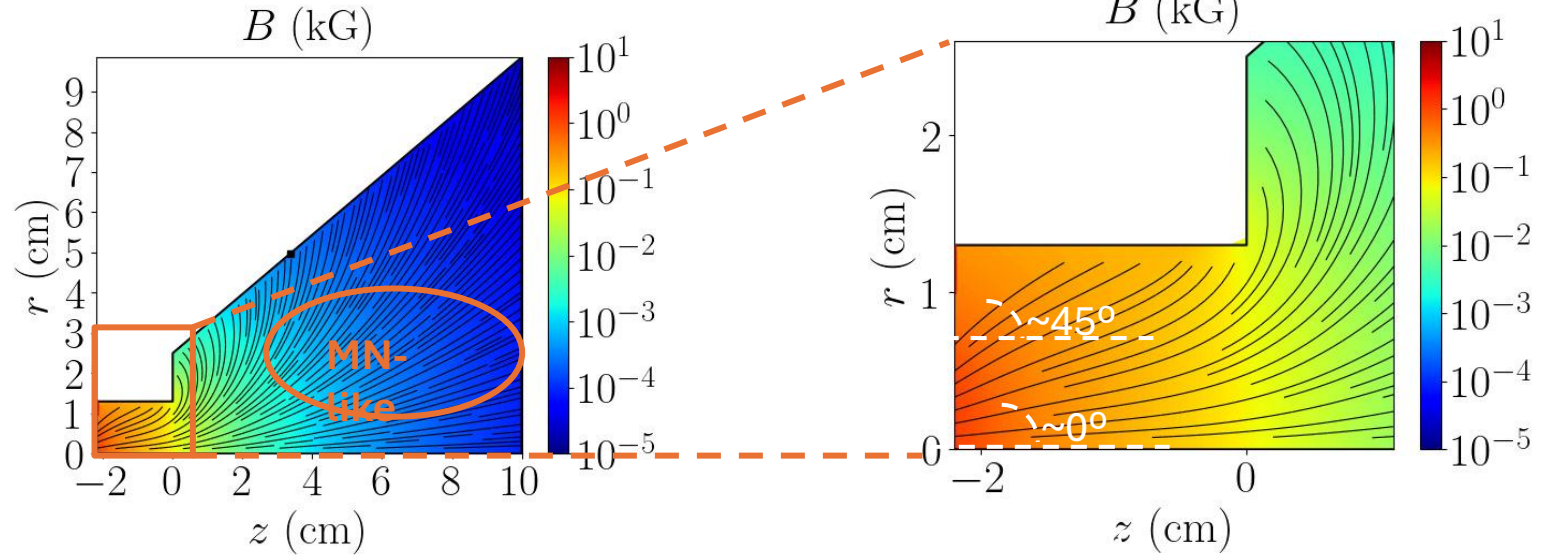
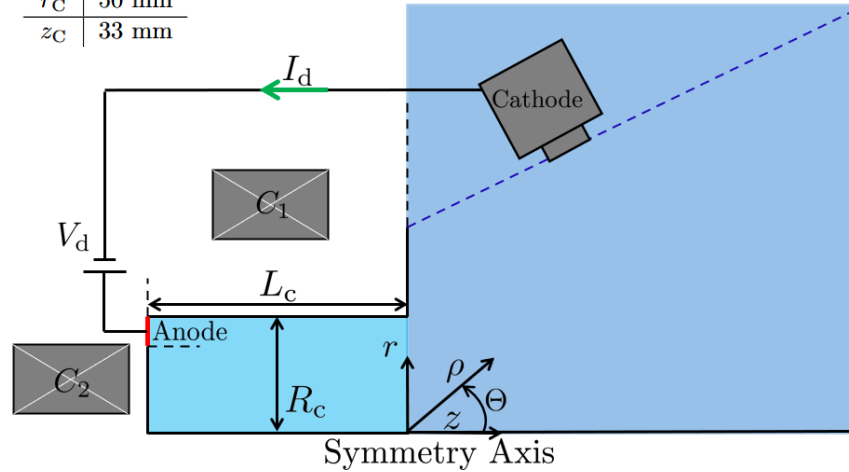
$$n_{ik} = \int_{-\infty}^{\infty} dv_r \int_{-\infty}^{\infty} dv_z \tilde{\mathcal{F}}_{ik}(v_z, v_r)$$

$$\mathcal{E}_{zik} = m_i v_{zik} |v_{zik}| / (2Z_{ik})$$

$$\mathcal{E}_{rik} = m_i v_{rik}^2 / (2Z_{ik})$$

Simulation setup for the CHT200 (I)

L_c	22 mm
H_c	13 mm
r_A	10 mm
r_C	50 mm
z_C	33 mm



➤ Some general features:

- ❑ Anode at the rear corner of the backplate with 3 mm of extension
- ❑ Cathode located at the near plume
- ❑ Neutrals are injected through the anode and the cathode with $\dot{m}_A$ and $\dot{m}_C$, respectively.

➤ Magnetic field: No induced magnetic-field and known *a priori*.

- ❑ Maximum at $(z, r) = (-L_c, 0)$ of 1150 G
- ❑ Magnetic nozzle (MN) like shape in the near plume (T_e decays along $1_{\parallel}$ direction)

Simulation setup for the CHT200 (I)

For all OPs Xe
as propellant

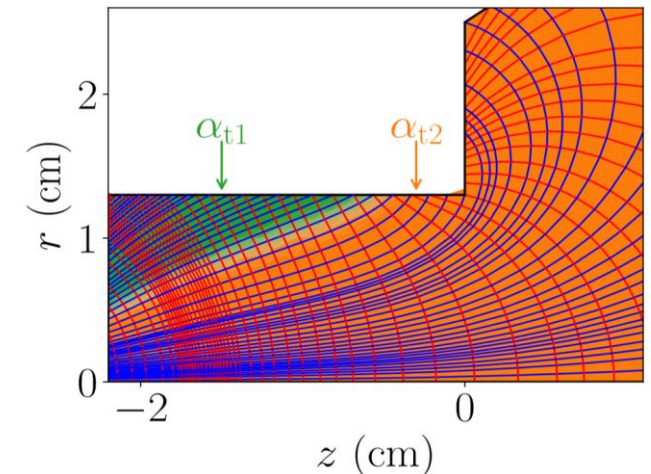
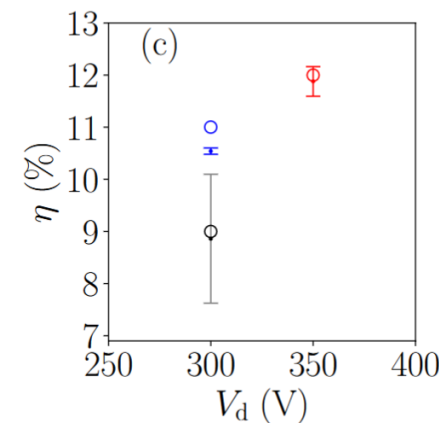
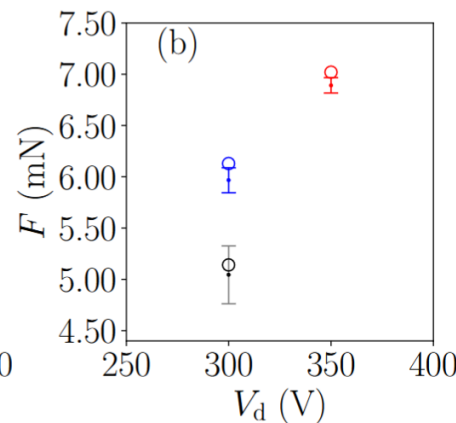
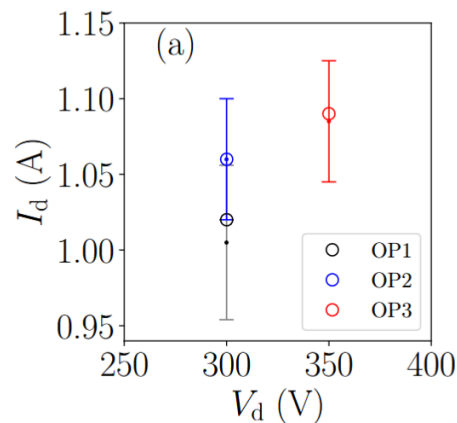


- Three operational points (OPs) defined by pairs of $(V_d, \dot{m}_A)$.

$\dot{m}_C = 0.08 \text{ mgs}^{-1}$											
OP	V_d (V)	$\dot{m}_A$ (mg s ⁻¹)	Experimental			Simulation					
			F (mN)	I_d (A)	$I_{i\infty}$ (A)	α_{t1} (%)	α_{t2} (%)	F (mN)	I_d (A)	I_{iP} (A)	I_{prod} (A)
1	300	0.40	5.04	1.005	0.360	4.10	1.30	5.14	1.01	0.34	0.64
2	300	0.45	5.96	1.060	0.430	3.85	1.08	6.13	1.06	0.39	0.74
3	350	0.45	6.89	1.085	0.437	3.75	0.98	7.02	1.09	0.40	0.76

- Fitting electron fluid model parameters.

- ❑ Phenomenological model for electron turbulence: $\nu_t = \alpha_t \omega_{ce}$.
- ❑ $\alpha_t \rightarrow$ Two-parameter (α_{t1} , α_{t2}) “step-in” function fitted to reproduce measurements of (I_d, F, η) with errors below 5%.
- ❑ Similar approach by Perales et al. for the KAIST-CHT [5].

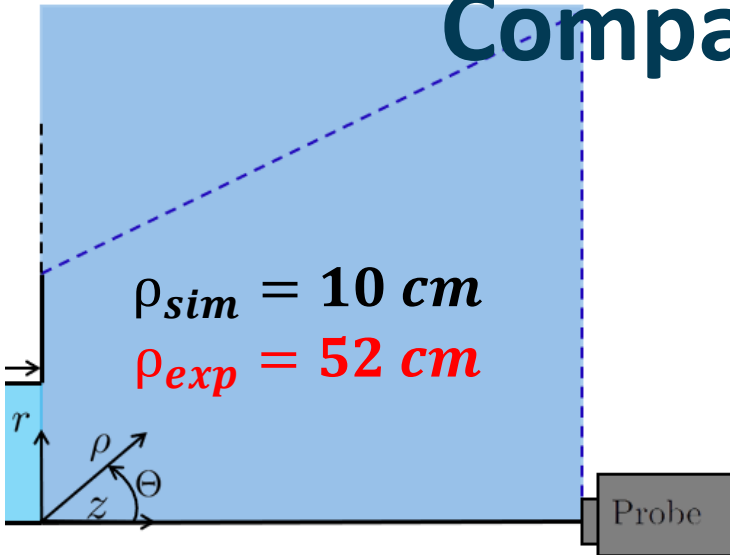


[5] - Perales-Díaz J, Ahedo E, Kim Y and Choe W 2025 2D numerical analysis of the discharge of a cylindrical Hall thruster 39th International Electric Propulsion Conference, London, UK IEPC-2025-356

Hybrid particle-fluid plasma modeling of Hall effect thrusters and analysis of non-conventional configurations

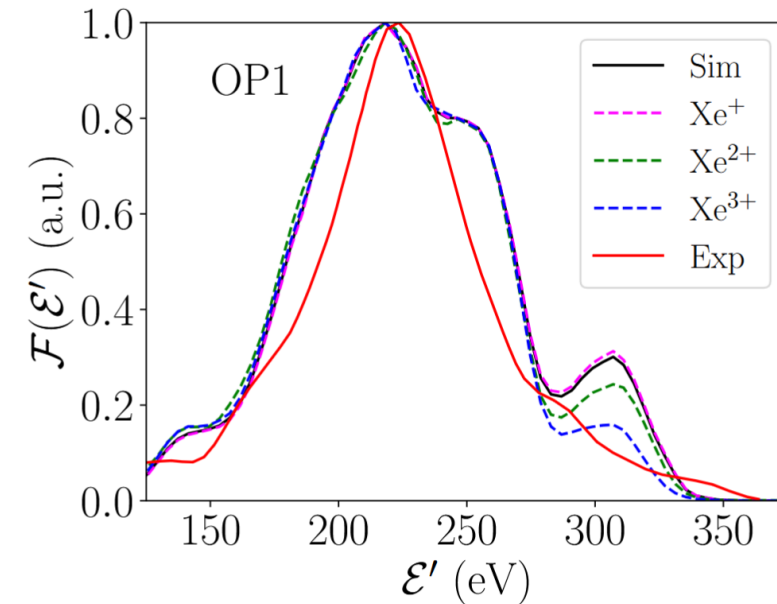
Comparison with experiments: RPA (I)

Larger acceleration in the simulations



$$j_{zi} = \int_0^\infty \mathcal{F}(\mathcal{E}') d\mathcal{E}'$$

$$\mathcal{E}' = m_i v_z^2 / 2Z$$



$\mathcal{E}'_{i:sim} = 215 \text{ eV}$
 $\mathcal{E}'_{i:exp} = 220 \text{ eV}$
 $\langle \mathcal{E}' \rangle_{sim} = 216 \text{ eV}$
 $\langle \mathcal{E}' \rangle_{exp} = 194 \text{ eV}$

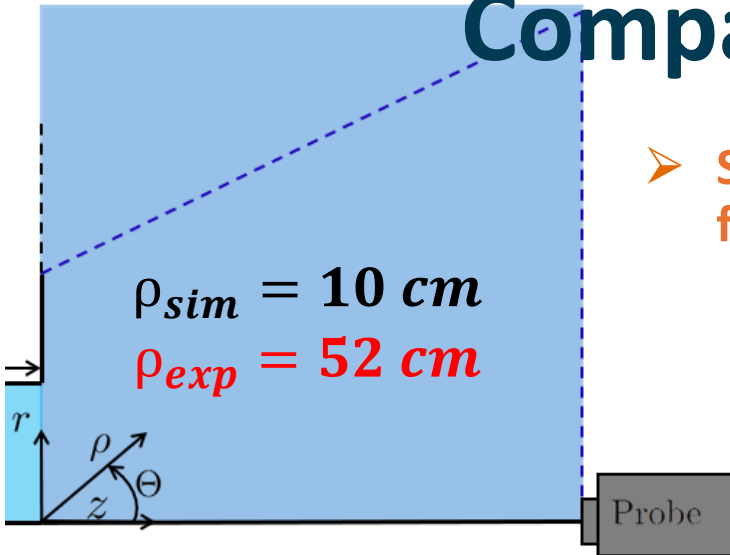
➤ Comparison:

- ❑ Close agreement in the most probable and mean axial kinetic energy.
- ❑ Ions collected by the RPA are generated approximately in the same region.
- ❑ Differences might come from:
 - ❖ 1- Different locations between simulations and experiments (limited by the numerical errors of big plume MFAMs).
 - ❖ 2- Complexity of the local plasma at the entrance of the RPA [6].
 - ❖ 3- Misalignments of the experimental plasma plume with the geometric axis of the thruster (HYPHEN assumes perfect axisymmetry).

[6] - F. Cichocki, J. Navarro-Cavallé, A. Modesti, and G. Ramírez Vázquez, "Magnetic nozzle and rpa simulations vs. experiments for a helicon plasma thruster plume," Frontiers in Physics, vol. 10, 2022.

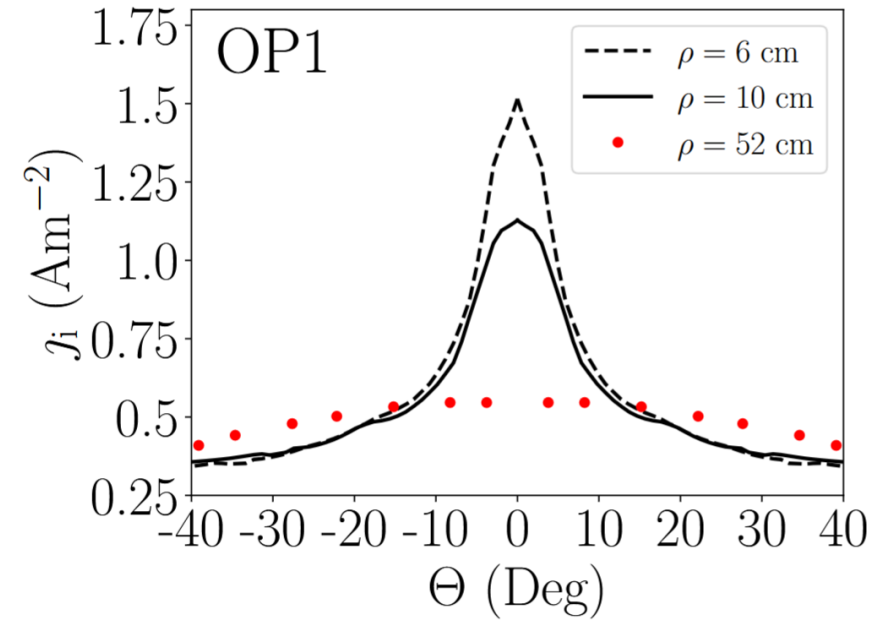
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Comparison with experiments: FC (II)



➤ Spherical expansions with factors of:

$$\begin{aligned} &\square \left(\frac{6}{52}\right)^2 \\ &\square \left(\frac{10}{52}\right)^2 \end{aligned} \quad j_i = j_i \cdot \hat{\rho}$$



➤ Comparison:

- Peak profile in the simulations curves.
- The peak decreases from $\rho = 6 \text{ cm}$ to $\rho = 10$ → radial diffusion of ions, which could explain the absence of the peak at $\rho = 52 \text{ cm}$.
- **Differences might come from:** Different locations between simulations and experiments (limited by the numerical errors of big plume MFAMs):

$$I_{i\infty} = \pi \rho^2 \int_{-\pi/2}^{\pi/2} j_i(\Theta) |\sin(\Theta)| d\Theta \quad \rightarrow \quad \text{Current vanishes at } \Theta \rightarrow 0 \text{ mit. Relative errors on the ion beam current below 10\% for all OPs}$$

Simulation results: 2D plasma maps (II)

Considered Ionization Reactions

Reaction	Nomenclature	
$e + \text{Xe} \rightarrow 2e + \text{Xe}^+$	i01	
$e + \text{Xe} \rightarrow 3e + \text{Xe}^{2+}$	i02	
$e + \text{Xe} \rightarrow 4e + \text{Xe}^{3+}$	i03	
$e + \text{Xe}^+ \rightarrow 2e + \text{Xe}^{2+}$	i12	
$e + \text{Xe}^+ \rightarrow 3e + \text{Xe}^{3+}$	i13	
$e + \text{Xe}^{2+} \rightarrow 2e + \text{Xe}^{3+}$	i23	

Xe^+

Xe^{2+}

Xe^{3+}

Efficiencies

$$\eta'_u = \frac{m_i I_{iP}}{e \dot{m}} = \frac{\eta_u}{\eta_{ch}} \quad \eta = \frac{F^2}{2 \dot{m} P} \equiv \eta_{ene} \eta_{div} \eta_{disp}$$

$$\eta_{ene} = \frac{P_P}{P}, \quad \eta_{div} = \frac{P_{zP}}{P_P}, \quad \eta_{disp} = \frac{F^2}{2 \dot{m} P_{zP}},$$

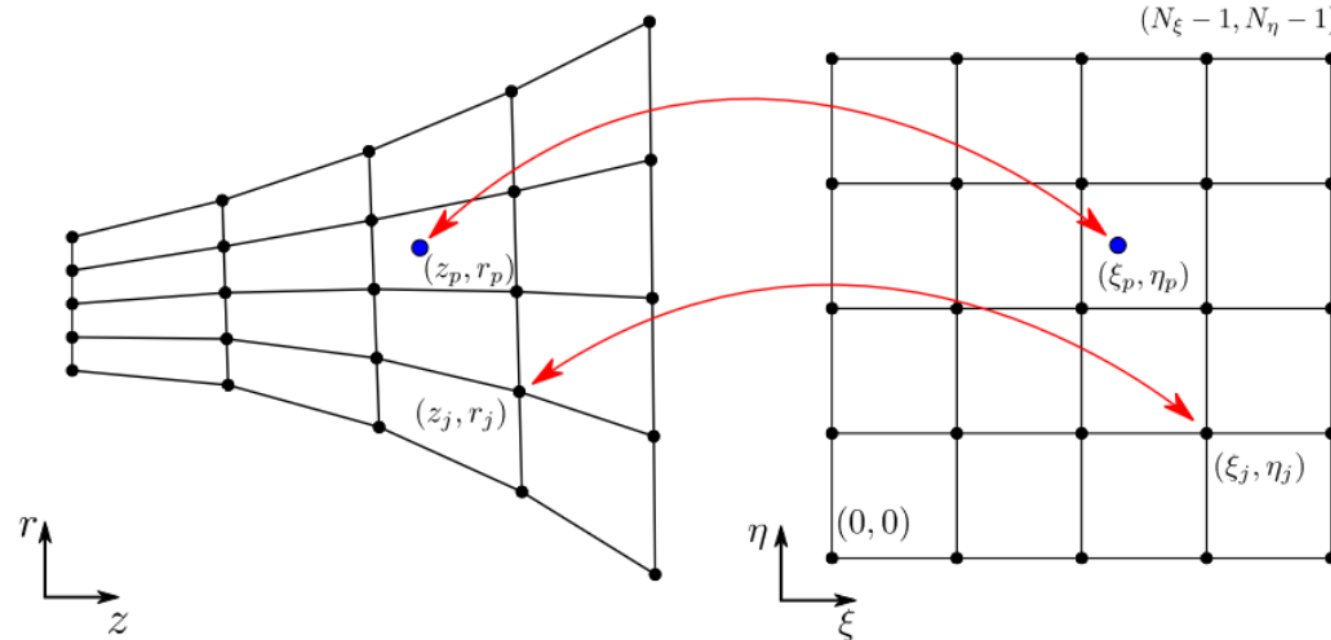
$$\eta_u = \frac{\dot{m}_{iP}}{\dot{m}}, \quad \eta_{cur} = \frac{I_{iP}}{I_d}, \quad \eta_{ch} = \frac{e \dot{m}_{iP}}{m_i I_{iP}},$$

MFAM (II)

We have to evaluate the gradients, which are given in (z,r) such that:

$$\frac{\partial \Phi}{\partial \mathbf{1}_r} \quad \text{and} \quad \frac{\partial \Phi}{\partial \mathbf{1}_z}$$

- 1- For a structure mesh $\rightarrow$ Gradients can be done with FDM.
- 2- For a non-structure mesh (like an expansion in the plume) $\rightarrow$ Compute the gradients in the computational mesh (ξ, η) .



Domínguez-Vázquez, A. (2019). Axisymmetric simulation codes for Hall effect thrusters and plasma plumes.

Let's assume that $z = z(\xi, \eta)$ and $r = r(\xi, \eta)$

$$\frac{\partial \Phi}{\partial z} = \frac{\partial \Phi}{\partial \xi} \frac{\partial \xi}{\partial z} + \frac{\partial \Phi}{\partial \eta} \frac{\partial \eta}{\partial z} \quad \text{and} \quad \frac{\partial \Phi}{\partial r} = \frac{\partial \Phi}{\partial \xi} \frac{\partial \xi}{\partial r} + \frac{\partial \Phi}{\partial \eta} \frac{\partial \eta}{\partial r}$$

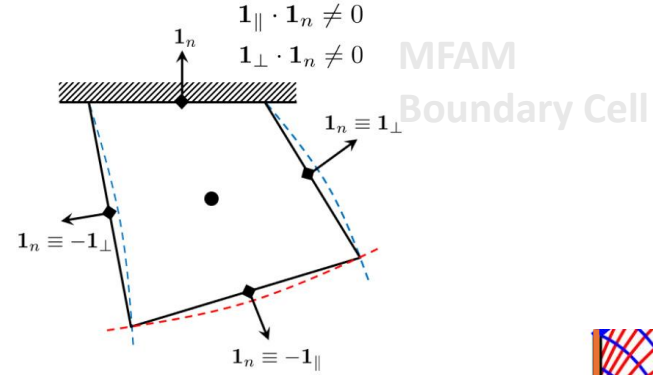
$$\begin{bmatrix} \frac{\partial \Phi}{\partial z} \\ \frac{\partial \Phi}{\partial r} \end{bmatrix} = \begin{pmatrix} \frac{\partial \xi}{\partial z} & \frac{\partial \eta}{\partial z} \\ \frac{\partial \xi}{\partial r} & \frac{\partial \eta}{\partial r} \end{pmatrix} \begin{bmatrix} \frac{\partial \Phi}{\partial \xi} \\ \frac{\partial \Phi}{\partial \eta} \end{bmatrix}$$

Objective

Compute

$$(J^{-1})^T = \begin{pmatrix} \frac{\partial \xi}{\partial z} & \frac{\partial \eta}{\partial z} \\ \frac{\partial \xi}{\partial r} & \frac{\partial \eta}{\partial r} \end{pmatrix}$$

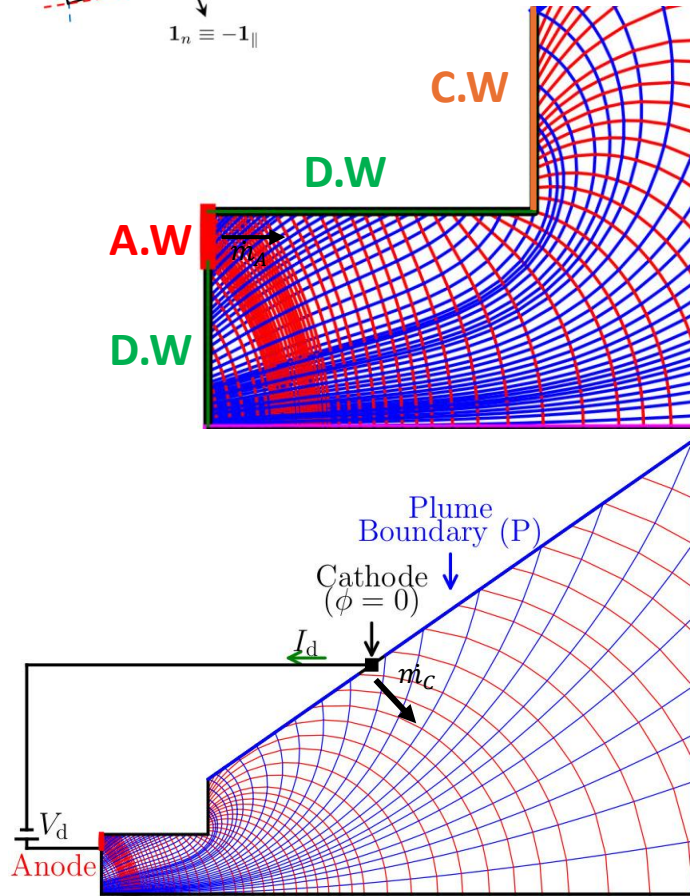
Hybrid particle-fluid plasma modeling of Hall effect thrusters and analysis of non-conventional configurations



The HYPHEN code (II)

Two wall (W) types : dielectric or conductive [4]

- **Dielectric wall:** Local null current condition imposes $j_{ne} = -j_{ni}$ and provides $\Delta\phi_{WQ} = \phi_Q - \phi_W$.
- **Conductive wall:** Floating wall at unknown ϕ_W such that : $I_W = 0$.
- **Anode wall:** $\phi_W = \phi_A = V_d$.
- **Plume boundary:** GPC model [5] which ensures that the collected net current in P is zero. Two possibilities for the energy equation at P:
 - ☐ non-homogeneous Neumann condition on T_e .
 - ☐ Dirichlet condition on T_e .

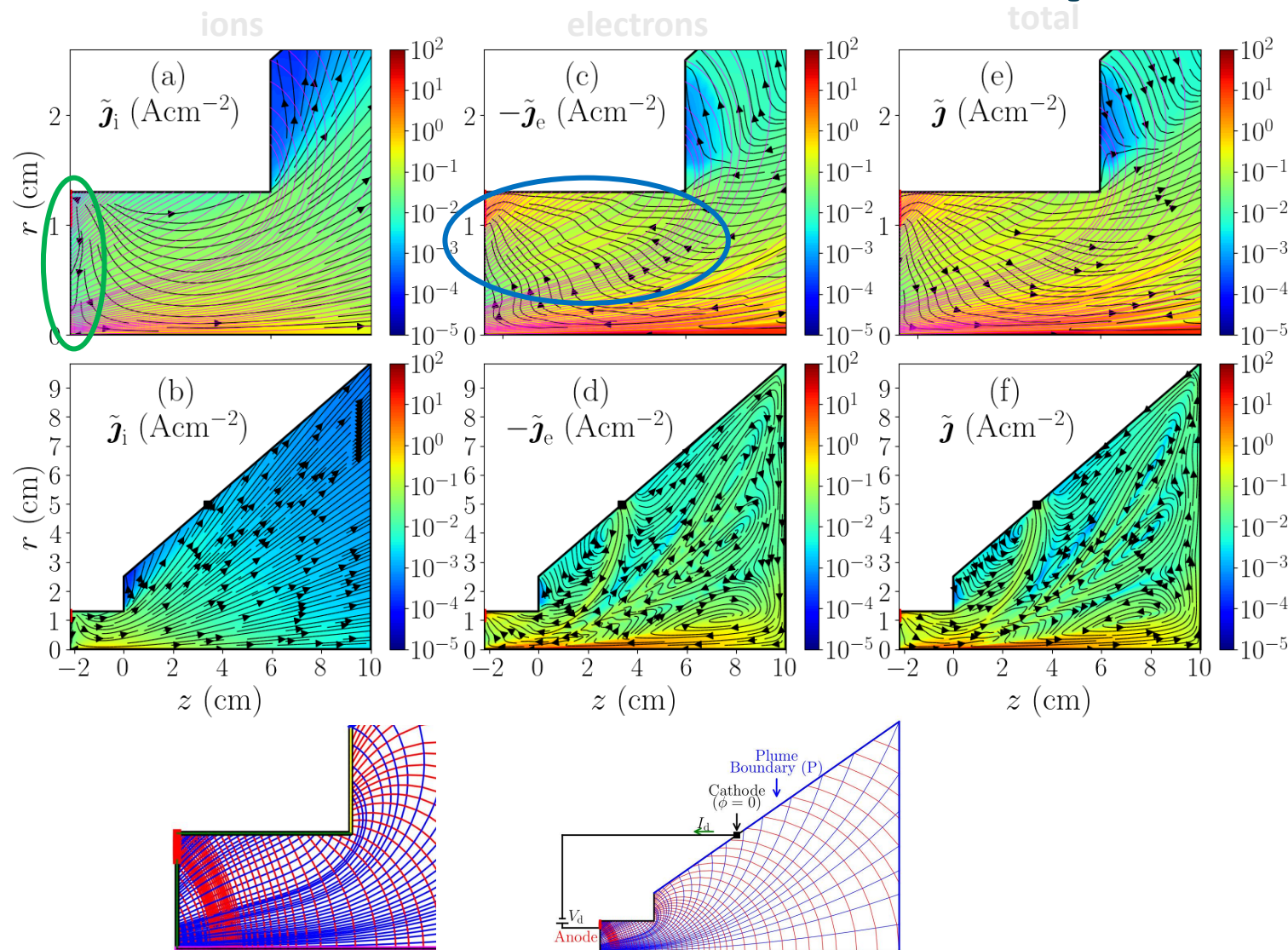


[4] - J. Perales-Díaz, A. Domínguez-Vázquez, P. Fajardo, E. Ahedo, F. Faraji, M. Reza, and T. Andreussi, "Hybrid plasma simulations of a magnetically shielded Hall thruster," Journal of Applied Physics, vol. 131, no. 10, p. 103302, 2022.

[5] - A. Domínguez-Vázquez, J. Zhou, A. Sevillano-González, and E. Ahedo, "Modelling of a Hall thruster plume: influence of plume boundary conditions and size, and cathode location," Plasma Sources Science and Technology, vol. 34, no. 8, p.

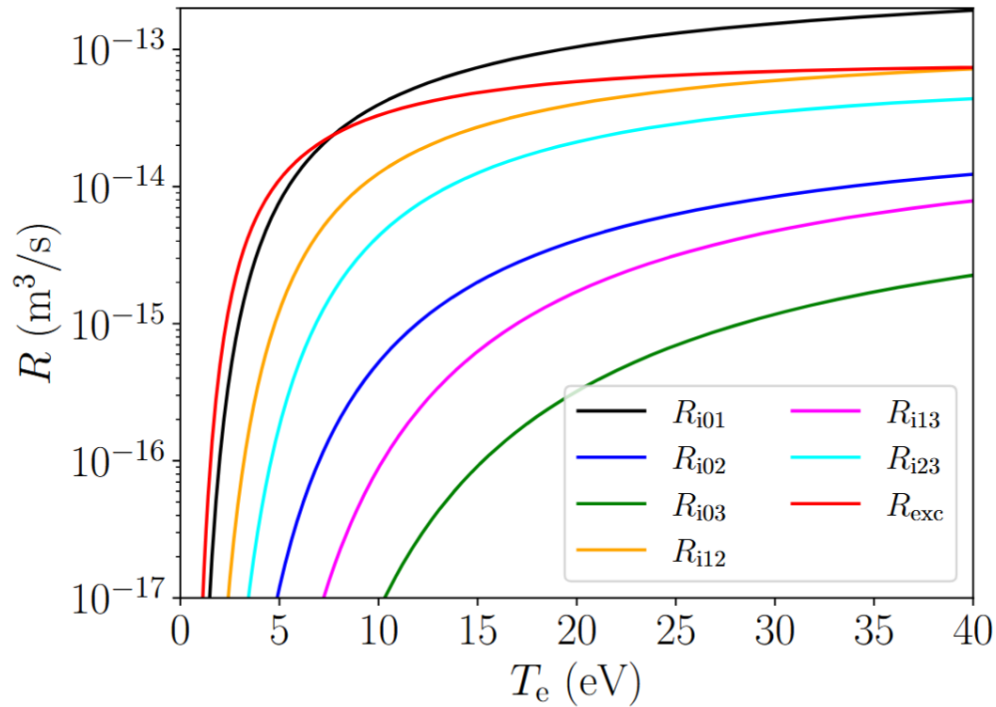
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Simulation results: 2D plasma maps (III)



- Ion current density (j_i) does not follow B -lines, showing high values along the axis and streamlines toward the backplate, leading to **high recombination**. j_i expands outward into the plume.
- Electron current density (j_e) streams from the cathode to the anode, **deviating across isolines**, with a strong peak at the axis due to high n_e .
- Two distinct j_e beams form: one reaching the anode, another propagating downstream to neutralize the plume.
- Total current density (j) is dominated by electrons in the plume, forming loops and ensuring $I_P = 0$ at the boundary (GPC model).

Appendix A



Considered Ionization Reactions

Reaction	Nomenclature	ε_k (eV)
$e + \text{Xe} \rightarrow 2e + \text{Xe}^+$	i01	12.1
$e + \text{Xe} \rightarrow 3e + \text{Xe}^{2+}$	i02	33.3
$e + \text{Xe} \rightarrow 4e + \text{Xe}^{3+}$	i03	65.4
$e + \text{Xe}^+ \rightarrow 2e + \text{Xe}^{2+}$	i12	21.1
$e + \text{Xe}^+ \rightarrow 3e + \text{Xe}^{3+}$	i13	53.3
$e + \text{Xe}^{2+} \rightarrow 2e + \text{Xe}^{3+}$	i23	32.1

$$S_e = n_e \sum_k n_k \Delta Z_k R_k(T_e)$$

$$P_{\text{inel}} = P_{i01} + P_{i02} + P_{i03} + P_{i12} + P_{i13} + P_{i23} + P_{\text{exc}}$$

$$P_k = \int_{\Omega_p} n_e n_k \varepsilon_k R_k(T_e) d\Omega$$

$$P_{\text{exc}} = \int_{\Omega_p} n_e n_n \sum_l \varepsilon_l R_l(T_e) d\Omega$$

Refs for reaction rates and energy thresholds:

[A] - "Biagi database," <https://www.lxcat.net/Biagi>, accessed: 2019-10-23.

[B] - R. Rejoub, B. Lindsday, and R. Stebbings, "Determination of the absolute partial and total cross sections for electron-impact ionization of the rare gases," *Physical Review A*, vol. 65, p. 042713, 2002.

[C] - R. C. Wetzel, F. A. Baiocchi, T. R. Hayes, and R. S. Freund, "Absolute cross sections for electron-impact ionization of the rare-gas atoms by the fast-neutral-beam method," *Physical Review A*, vol. 35, no. 2, p. 559, 1987.

[D] - C. Achenbach, A. Muller, E. Salzborn, and R. Becker, "Single ionisation of multiply charged xenon ions by electron impact," *Journal of Physics B: Atomic and Molecular Physics*, vol. 17, no. 7, p. 1405, 1984.

[E] - M. Hayashi, "Bibliography of electron and photon cross sections with atoms and molecules published in the 20th century. Xenon," National Institute for Fusion Science, Tech. Rep., 2003.

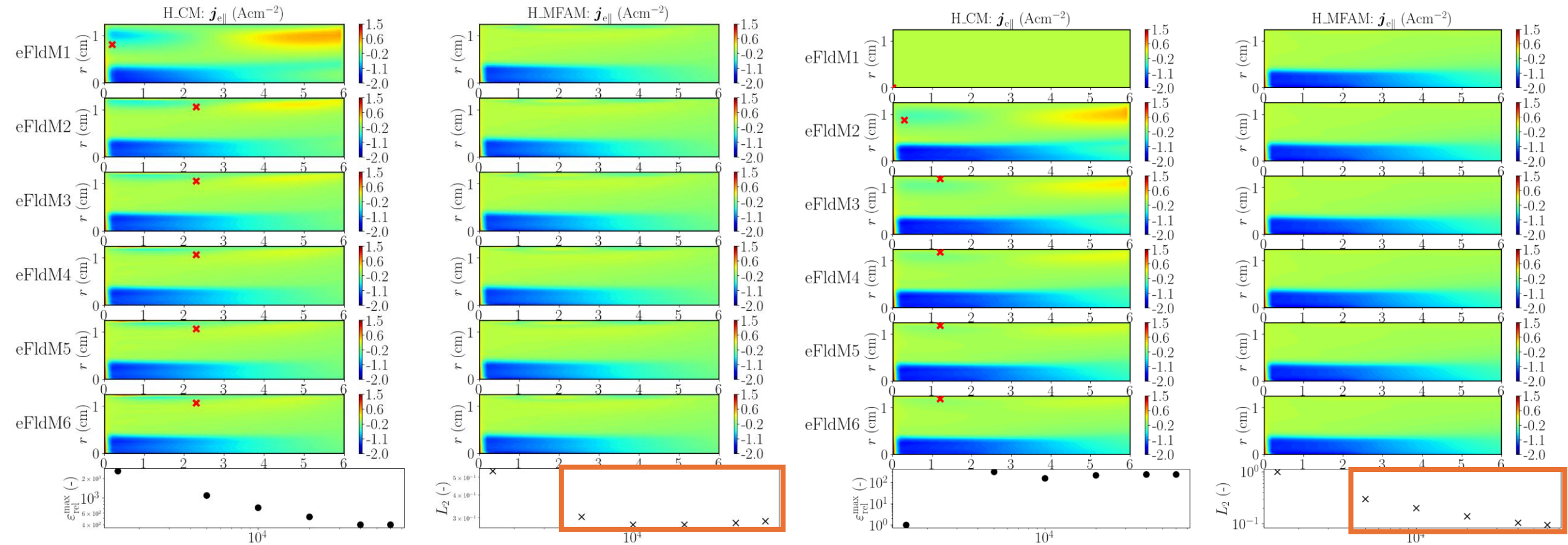
[F] - "Nifs database," <http://dpc.nifs.ac.jp/DB/IEEJ/>, accessed: 2019-10-23.

[G] - B. D. Smith, I. D. Boyd, and H. Kamhawi, "Influence of triply-charged ions and ionization cross-sections in a hybrid-pic model of a Hall thruster discharge," in *AIAA/ASME/SAE/ASEE Joint Propulsion Conference*, no. GRC-E-DAA-TN16340, 2014.

Convergence Study CASE (I)

0.5 * |B|

2.0 * |B|



Slower convergence for higher magnetic field

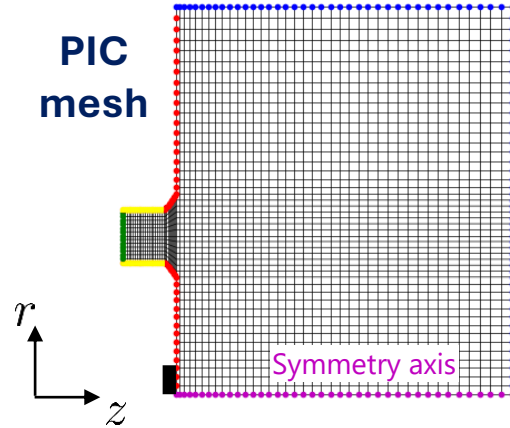
Hybrid particle-fluid plasma modeling of Hall effect thrusters and analysis of non-conventional configurations

Backup slides

Contents

- Introduction
- The HYPHEN code
- Simulation setup for the CHT200
- Simulations results
- Conclusions

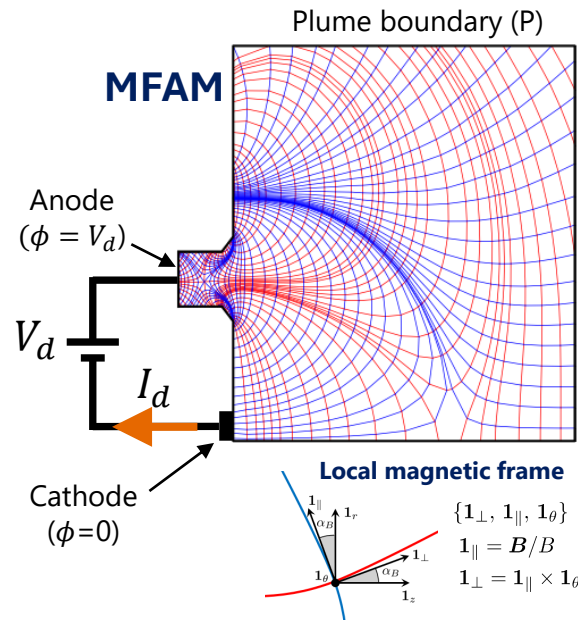
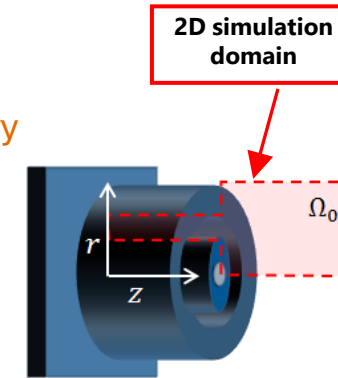
The HYPHEN code (I)



2D (axial-radial) hybrid PIC/fluid code

I-module: PIC formulation for multiple heavy species

- ☐ Structured mesh
- ☐ Neutrals, singly and doubly-charged ions
- ☐ Noise reduction by population control

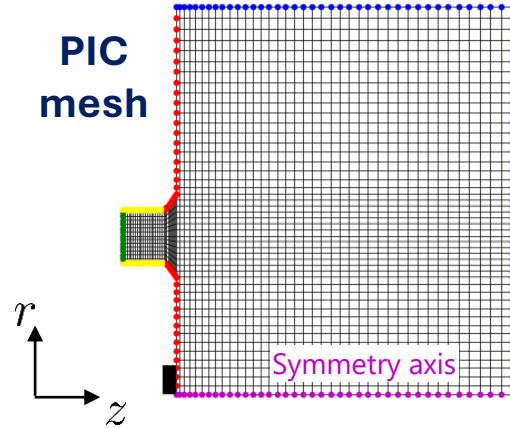


E-module: magnetized inertialess fluid model

- ☐ $n_e = \sum Z_s n_s$ → Quasineutrality
- ☐ $\nabla \cdot \mathbf{j}_e = -\nabla \cdot \mathbf{j}_i$ → Current continuity
- ☐ $\mathbf{j}_e = \bar{\sigma}_e \cdot \left[-\nabla \phi + \frac{1}{en_e} \nabla(n_e T_e) - \frac{1}{en_e} \mathbf{F}_{ch} \right]$ → Ohm's law for electron current
- ☐ $\nabla \cdot \mathbf{P}_e'' = -\frac{3}{2} \frac{\partial(n_e T_e)}{\partial t} - \nabla \phi \cdot \mathbf{j}_e + P_{inel}'''$ → Electron energy equation
- ☐ $\mathbf{P}_e'' = \frac{5}{2} n_e T_e \mathbf{u}_e + \mathbf{q}_e$; $\mathbf{q}_e = -\bar{K}_e \cdot \nabla T_e$
- ☐ ϕ and T_e at MFAM cell centers and boundary faces centers
- ☐ Cathode at $\phi = 0$; anode wall at discharge voltage $\phi = V_d$
- ☐ Discharge current I_d collected at the anode is injected through the cathode

Hybrid particle-fluid plasma modeling of Hall effect thrusters and analysis of non-conventional configurations

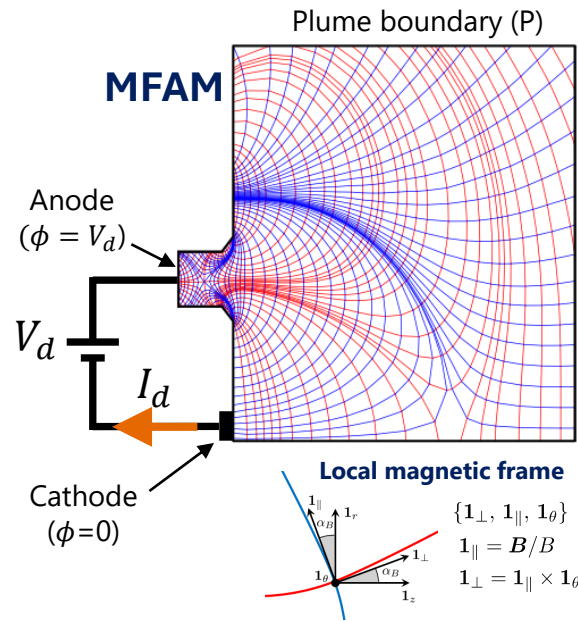
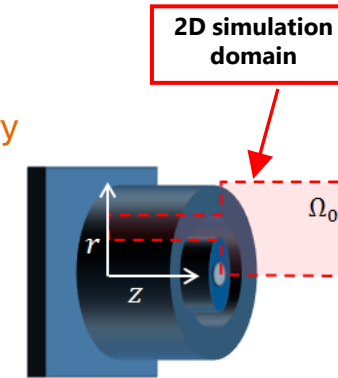
The HYPHEN code (I)



2D (axial-radial) hybrid PIC/fluid code

I-module: PIC formulation for multiple heavy species

- ❑ Structured mesh
- ❑ Dedicated list for each species



E-module: magnetized inertialess fluid model

- ❑ $n_e = \sum Z_s n_s$ → Quasineutrality
- ❑ $\nabla \cdot \mathbf{j}_e = -\nabla \cdot \mathbf{j}_i$ → Current continuity
- ❑ $\mathbf{j}_e = \bar{\sigma}_e \cdot \left[-\nabla \phi + \frac{1}{en_e} \nabla(n_e T_e) - \frac{1}{en_e} \mathbf{F}_{ch} \right]$ → Ohm's law for electron current
- ❑ $\nabla \cdot \mathbf{P}_e'' = -\frac{3}{2} \frac{\partial(n_e T_e)}{\partial t} - \nabla \phi \cdot \mathbf{j}_e + P_{inel}'''$ → Electron energy equation
- ❑ $\mathbf{P}_e'' = \frac{5}{2} n_e T_e \mathbf{u}_e + \mathbf{q}_e$; $\mathbf{q}_e = -\bar{K}_e \cdot \nabla T_e$
- ❑ ϕ and T_e at MFAM cell centers and boundary faces centers
- ❑ Cathode at $\phi = 0$; anode wall at discharge voltage $\phi = V_d$
- ❑ Discharge current I_d collected at the anode is injected through the cathode

Hybrid particle-fluid plasma modeling of Hall effect thrusters and analysis of non-conventional configurations

The HYPHEN code (II)

S-module: coupling between quasineutral plasma and walls

(W)

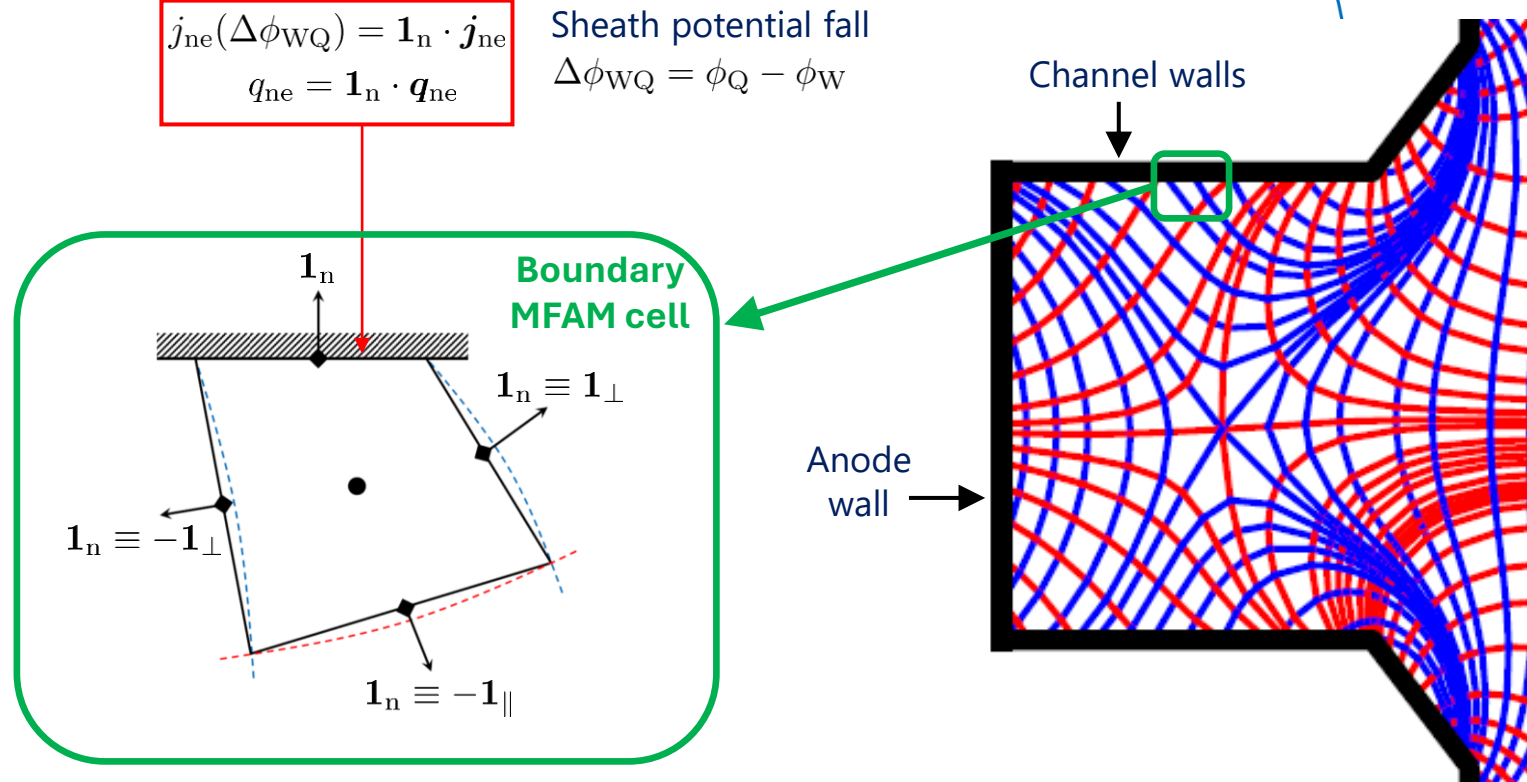
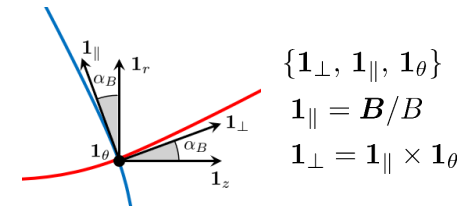
- Different types of walls (W): dielectric, conducting
- Appropriate BC for the E-module at quasineutral sheath edge (Q)
- Provides expresions for electron current and heat flux at Q:

$$\begin{aligned} j_{ne}(\Delta\phi_{WQ}) &= \mathbf{1}_n \cdot \mathbf{j}_{ne} \\ q_{ne} &= \mathbf{1}_n \cdot \mathbf{q}_{ne} \end{aligned}$$

Sheath potential fall

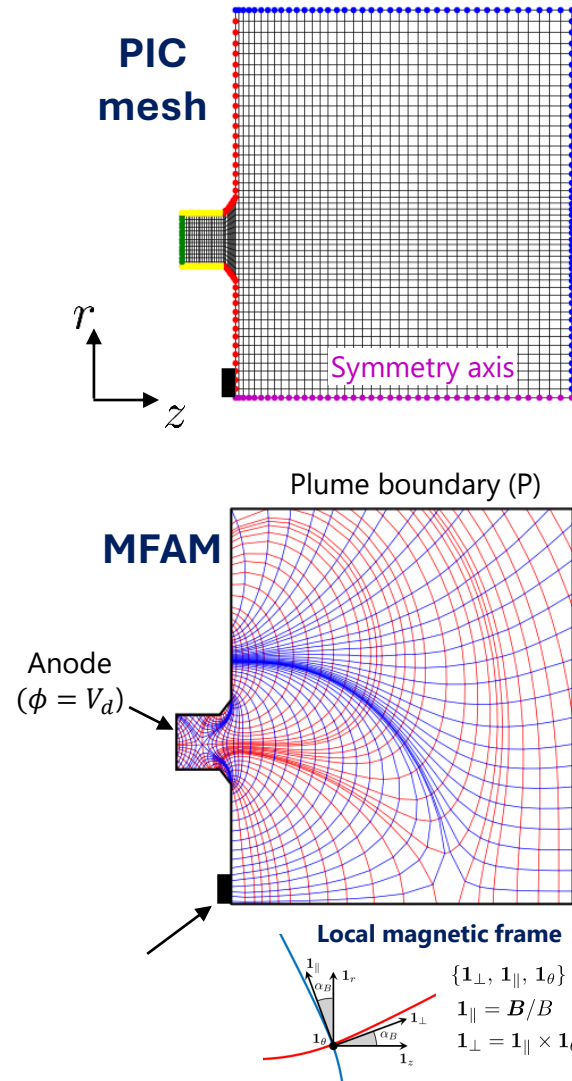
$$\Delta\phi_{WQ} = \phi_Q - \phi_W$$

Local magnetic frame



Hybrid particle-fluid plasma modeling of Hall effect
thrusters and analysis of non-conventional
configurations

The HYPHEN code (I)



I-module: PIC formulation for multiple heavy species

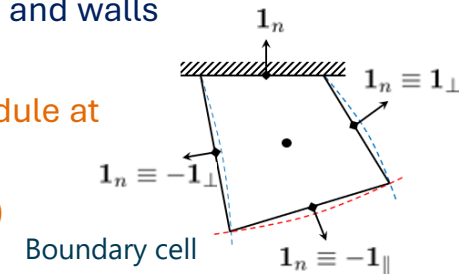
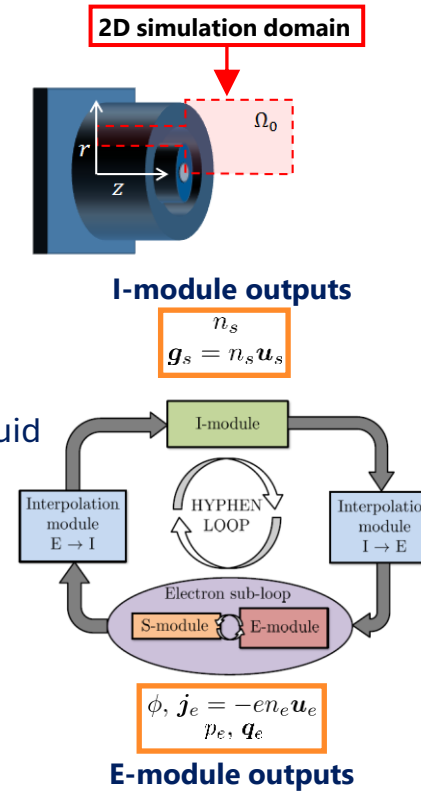
- Structured mesh
- Neutrals, singly and doubly-charged ions
- CEX slow ions and fast neutrals
- Noise reduction by population control
- Cylindrical corrections: important near the axis

E-module: magnetized drift-diffusive (inertialess) fluid model; applies quasineutrality

- Current continuity + Ohm's law for $\mathbf{j}_e$
- Electron energy + Fourier's law for $\mathbf{q}_e$
- Phenomenological model for turbulent transport
- FVM/GRM on unstructured MFAM

S-module: coupling between quasineutral plasma and walls (W)

- Appropriate boundary conditions for the E-module at quasineutral sheath edge (Q)
- Different types of walls (dielectric, conducting)



Hybrid particle-fluid plasma modeling of Hall effect thrusters and analysis of non-conventional configurations

The HYPHEN code (II)

➤ Massless electron fluid model (Green magnitudes are inputs from I-module)

$$\square \quad n_e = \sum Z_s n_s$$

$$\square \quad \nabla \cdot \mathbf{j}_e = -\nabla \cdot \mathbf{j}_i$$

→ Finite volume method (FVM)

$$\square \quad \mathbf{j}_e = \bar{\bar{\sigma}}_e \cdot \left[-\nabla \phi + \frac{1}{en_e} \nabla(n_e T_e) \right] + \mathbf{j}'_e$$

→ Gradient reconstruction (GRM)

$$\diamond \quad j_{\parallel e} = \sigma_{\parallel e} \left[-\frac{\partial \phi}{\partial 1_{\parallel}} + \frac{1}{en_e} \frac{\partial(n_e T_e)}{\partial 1_{\parallel}} \right] + j'_{\parallel e}$$

$$\diamond \quad j_{\perp e} = \sigma_{\perp e} \left[-\frac{\partial \phi}{\partial 1_{\perp}} + \frac{1}{en_e} \frac{\partial(n_e T_e)}{\partial 1_{\perp}} \right] + j'_{\perp e}$$

$$\square \quad \nabla \cdot \mathbf{q}_e = -\frac{3}{2} \frac{\partial(n_e T_e)}{\partial t} - \frac{5}{2} \nabla \cdot n_e T_e \mathbf{u}_e - \nabla \phi \cdot \mathbf{j}_e + P'_{inel} \rightarrow \text{Finite volume method (FVM)}$$

$$\square \quad \mathbf{q}_e = -\bar{\bar{K}}_e \cdot \nabla T_e \rightarrow \text{Gradient reconstruction (GRM)}$$

$$\diamond \quad q_{\parallel e} = -K_{\parallel e} \frac{\partial T_e}{\partial 1_{\parallel}}, \quad q_{\perp e} = -K_{\perp e} \frac{\partial T_e}{\partial 1_{\perp}}$$

➤ ϕ and T_e at MFAM cell centers and boundary faces centers

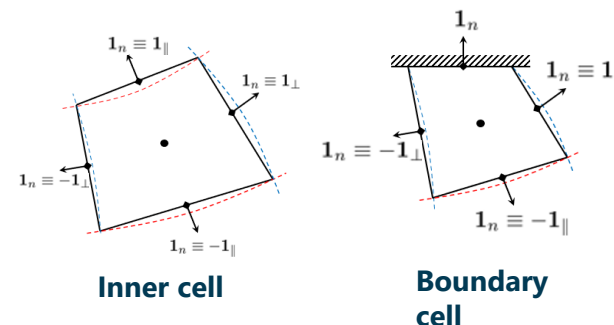
➤ Reference for the potential ($\phi=0$) at cathode wall

➤ S-module → BC at boundary faces (sheath edge Q):

$$\square \quad \text{Electron current: } j_{ne}(\phi_{WQ})$$

$$\square \quad \text{Electron energy flux: } P''_{ne}(\phi_{WQ}) = \frac{5}{2} n_e T_e u_{ne} + q_{ne}$$

$$\square \quad \text{Sheath potential drop } \phi_{WQ} = \phi_Q - \phi_W$$



The HYPHEN code (II)

➤ Base model for massless electrons

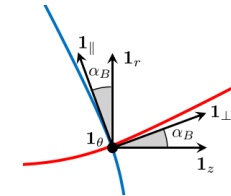
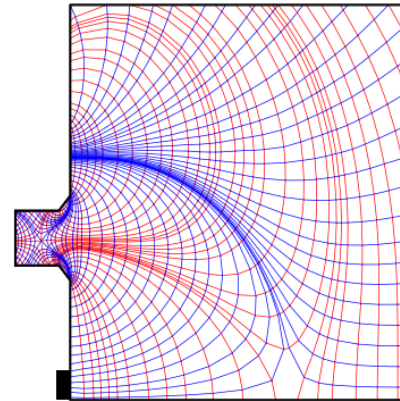
- $n_e = \sum Z_s n_s$
- $\nabla \cdot \mathbf{j}_e = -\nabla \cdot \mathbf{j}_i$
- $\mathbf{j}_e = \bar{\bar{\sigma}} \cdot \left[-\nabla \phi + \frac{1}{en_e} \nabla p_e \right] + \mathbf{j}'_e$
- $\nabla \cdot \mathbf{q}_e = -\frac{3}{2} \frac{\partial p_e}{\partial t} - \frac{5}{2} \nabla \cdot p_e \mathbf{u}_e - \nabla \phi \cdot \mathbf{j}_e + Q_{inel} +$
- $\mathbf{q}_e = -\bar{\bar{K}} \cdot \nabla T_e$

Input from Wave-module (only for EPTs)

Green magnitudes are input from I-module

Use of MFAM due to large anisotropy

$$\sigma_{\parallel e} \gg \sigma_{\perp e} \sim \sigma_{\parallel e} / \chi^2; \quad \chi = \omega_{ce} / \nu_e \sim 10^2;$$



$$\{ \mathbf{1}_{\perp}, \mathbf{1}_{\parallel}, \mathbf{1}_{\theta} \}$$

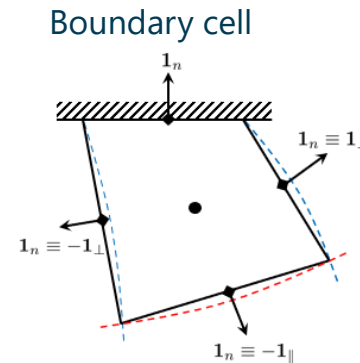
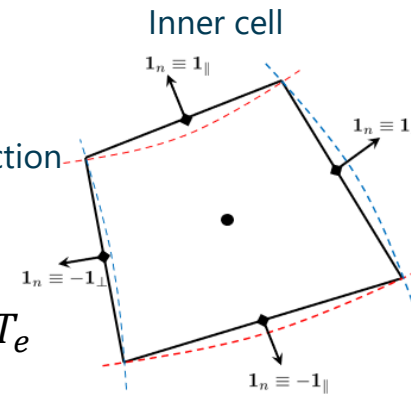
$$\mathbf{1}_{\parallel} = \mathbf{B} / B$$

$$\mathbf{1}_{\perp} = \mathbf{1}_{\parallel} \times \mathbf{1}_{\theta}$$

➤ Electron model in MFAM reference

- $\nabla \cdot \mathbf{j}_e = -\nabla \cdot \mathbf{j}_i \longrightarrow$ Finite Volume Method
- $j_{\parallel e} = \sigma_{\parallel e} \left[-\frac{\partial \phi}{\partial 1_{\parallel}} + \frac{1}{en_e} \frac{\partial (n_e T_e)}{\partial 1_{\parallel}} \right] + j'_{\parallel e}$
- $j_{\perp e} = \sigma_{\perp e} \left[-\frac{\partial \phi}{\partial 1_{\perp}} + \frac{1}{en_e} \frac{\partial (n_e T_e)}{\partial 1_{\perp}} \right] + j'_{\perp e}$
- Unknowns: $j_{\parallel e}$, $j_{\perp e}$, and ϕ
- Similar 3-equation structure for $q_{\parallel e}$, $q_{\perp e}$, and T_e

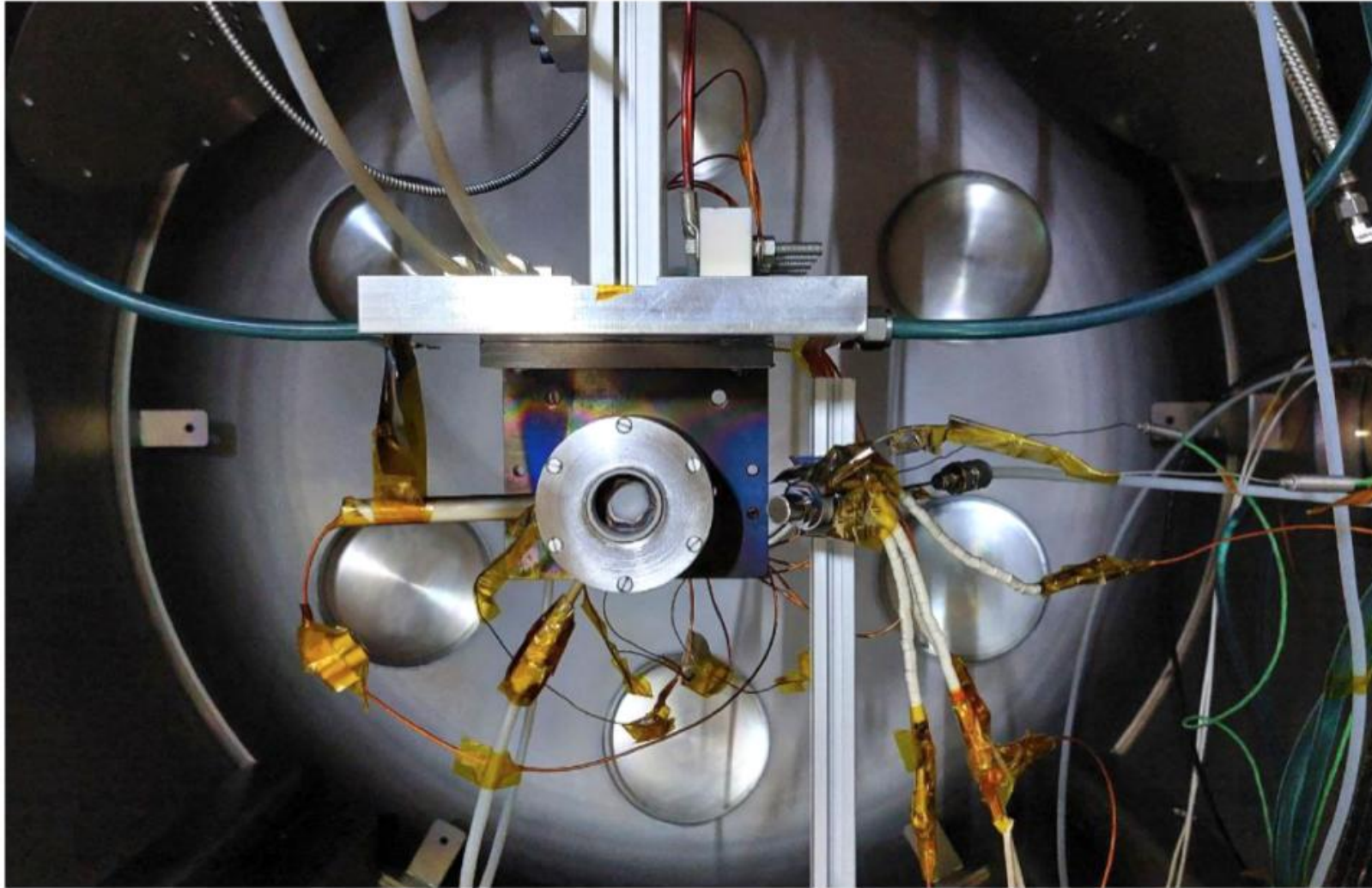
Gradient Reconstruction



The HYPHEN code (III)

- FVM/GRM yields matrix systems for ϕ and T_e at MFAM cell centers and boundary faces centers
- Reference for the potential ($\phi=0$) at cathode wall
- **The S-module provides boundary conditions:**
 - ❑ Sheath model yields $j_{ne} = f_{sh}(\phi_{WQ})$; $\phi_{WQ} = \phi_Q - \phi_W$
 - ❖ Includes SEE effects
 - ❖ Retains non-Maxwellian VDF features (partial replenishment of high energy tails)
 - ❑ Dielectric wall: Zero wall-collected current: $j_{ne} = -j_{ni}$
 - ❖ After solving for j_{ne} , the sheath model provides $\phi_{WQ} = f^{-1}(-j_{ni})$
 - ❑ Metallic wall at known voltage (e.g., HET anode wall):
 - ❖ $j_{ne} \equiv f_{sh}(\phi_{WQ}) = f_{ohm}(\phi_{WQ}) \rightarrow$ implicit equation for ϕ_{WQ}
 - ❖ Solved iteratively to satisfy current continuity

C. UC3M, MUSIC cathode, thrust, eletrostatic probes, discharge oscillations



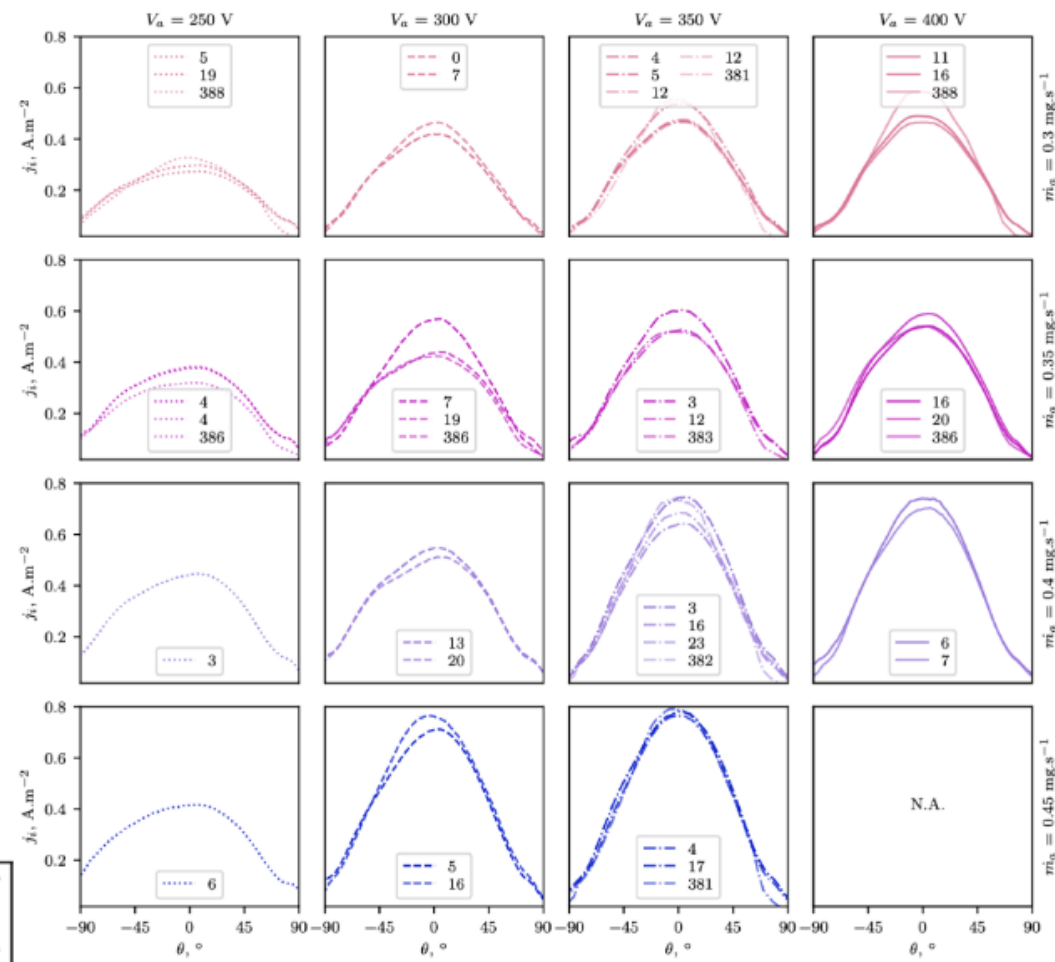
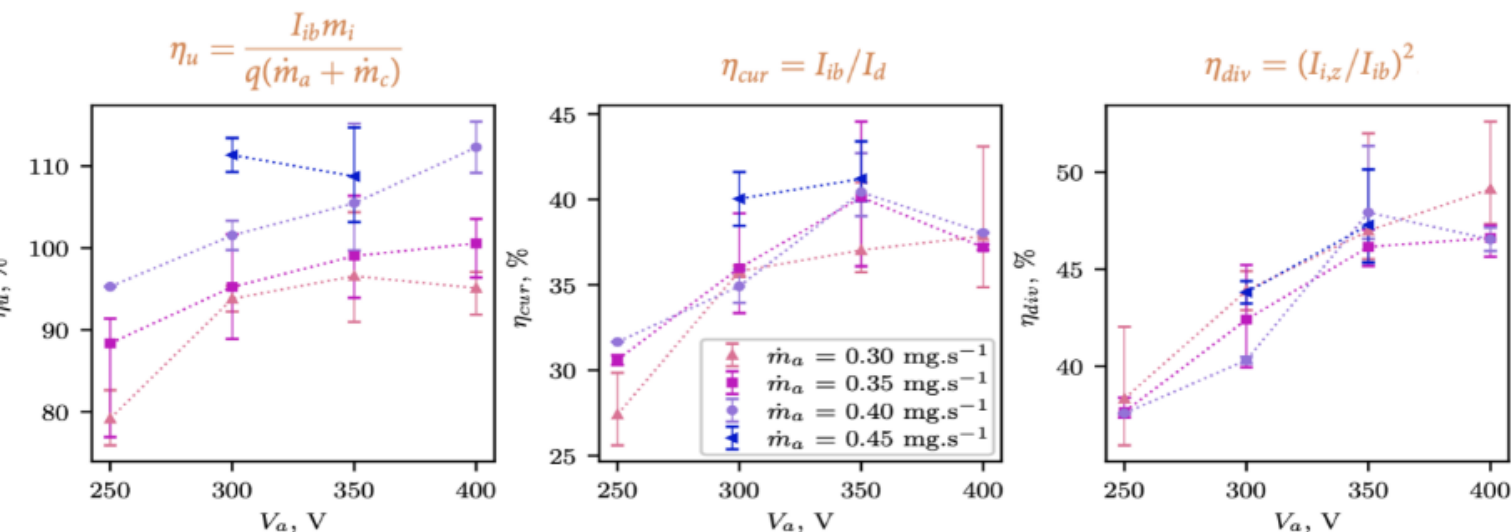
Tatiana Perrotin, Pablo Fajardo, Jaume Navarro-Cavallé, George-Cristian Potrivitu, Jian Wei Mark Lim; Direct thrust measurements of a 200 W Cylindrical Hall Thruster; 38th International Electric Propulsion Conference, Toulouse, France, 2024

Faraday cup

- Plume **asymmetries** from cathode, injector, coils, alignment
- **Dispersion** of I_d reflected in j_i
- Measurements scattered over > 1 year
- Partial efficiencies obtained by integrating beam current density
- Repetitions of $(\dot{m}_a, V_d)$ averaged

$$I_{ib} = \pi r^2 \int_{-\pi/2}^{\pi/2} j_i(\theta) |\sin(\theta)| d\theta,$$

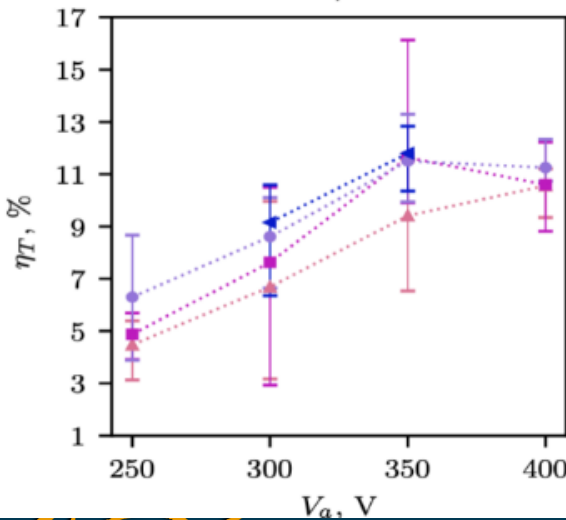
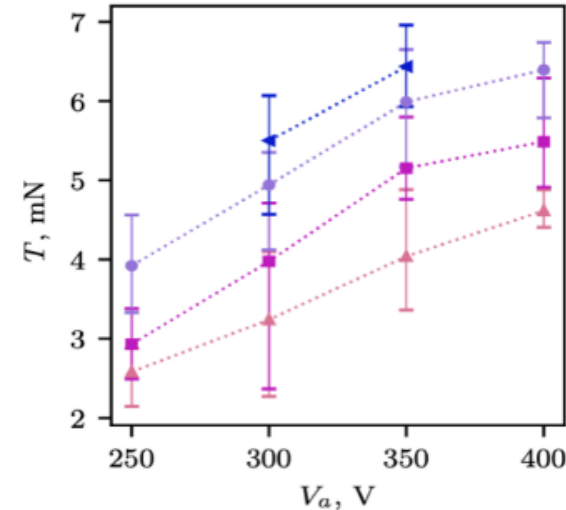
$$I_{i,z} = \pi r^2 \int_{-\pi/2}^{\pi/2} j_i(\theta) \cos(\theta) |\sin(\theta)| d\theta$$



- **Divergence & current utilization**
 - Strongly affected by V_d , not by $\dot{m}_a$
- **Propellant utilization**
 - Sensitive to V_d & $\dot{m}_a$
 - >1 → Xe²⁺, Xe³⁺

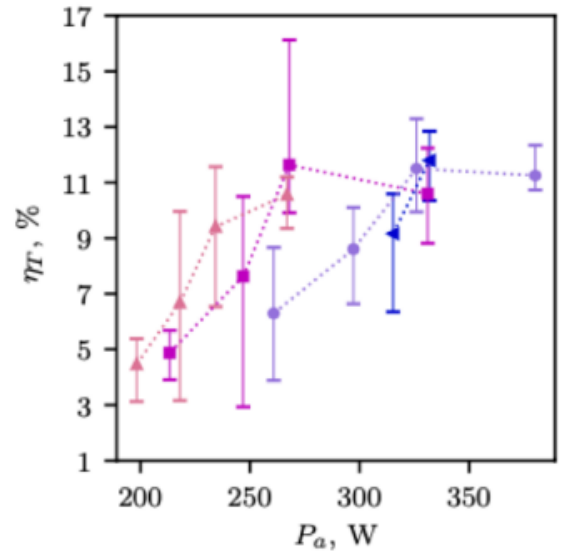
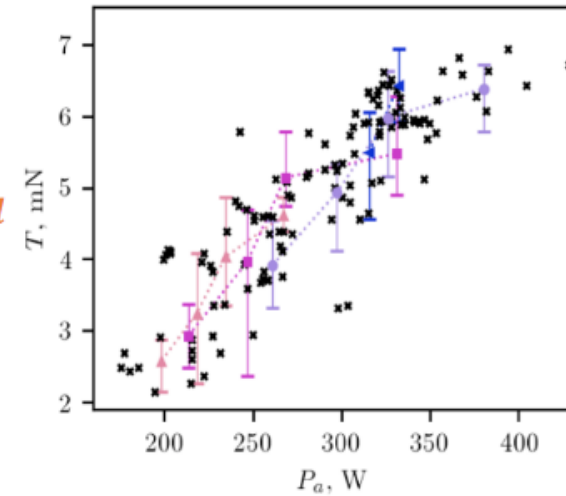
Thrust balance

Repetitions of $(\dot{m}_a, V_d)$ averaged



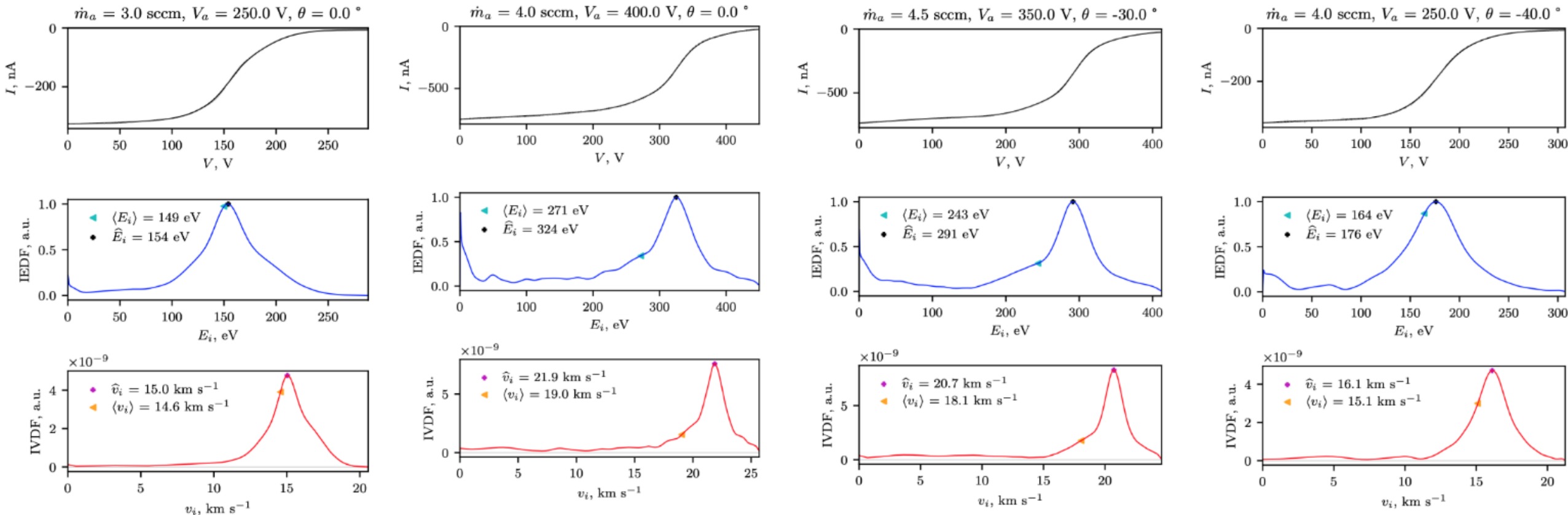
- Significant **dispersion**
- Comparing same $(\dot{m}_a, V_d)$ means $\neq P_d$
- Specific procedure to calibrate & measure thrust at shut-off
- **Thrust increases with $\dot{m}_a$ & V_d in covered range**
- **η_T mostly depends on V_d , maximum for 350 V, 0.4 mg s⁻¹, ~325 W**
 - **Saturates with P_d for all $\dot{m}_a$**
- Measured xenon cold thrust $\sim 198 \mu\text{N/mg s}^{-1}$
- η_T 3-2 times less than the best performing low-power CHTs
- No typical trend $\eta_T - P_d$ in other low-power CHT: very thruster-specific
- **B must be optimized** (tuning electromagnets)
 - maximize j_i and/or T
 - minimize divergence
- Increase **anode shielding** from electron current & increase η_{cur} (main CHT weak point)

$$\eta_T = \frac{T^2}{2P_d(\dot{m}_a + \dot{m}_c)}$$



Retarding Potential Analyzer

Some I-V trace, IEDF and IVDF examples



Note: E_i is energy per charge, v_i assumes singly-charged ions

Hybrid particle-fluid plasma modeling of Hall effect
thrusters and analysis of non-conventional
configurations

Langmuir probe measurements in CHT200 with MUSI

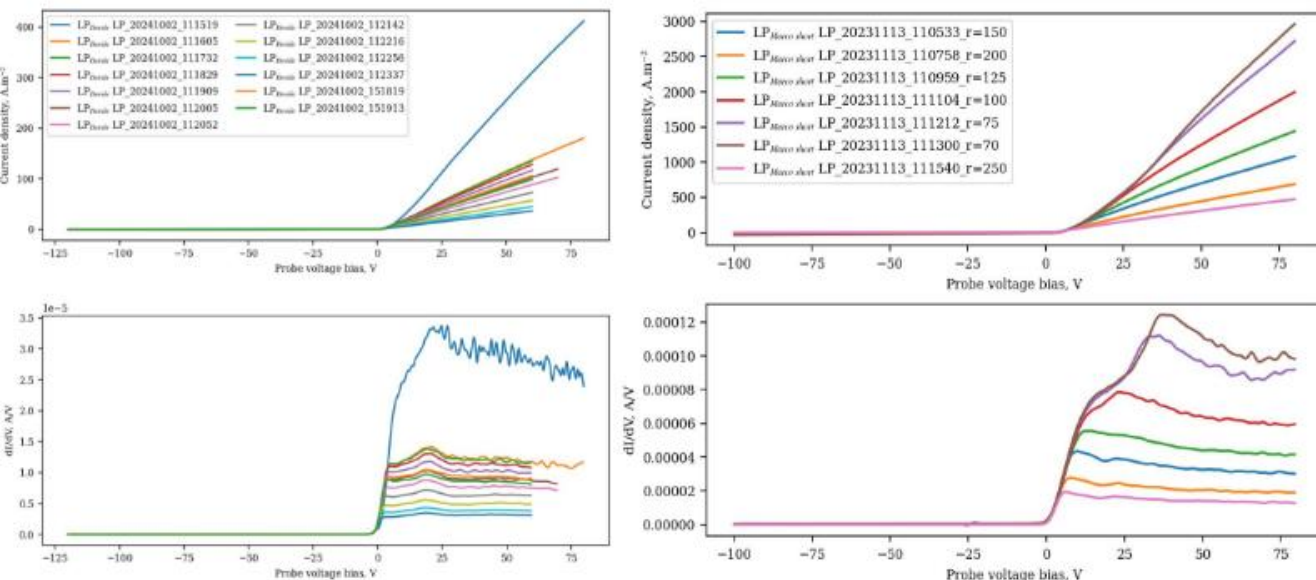
Langmuir probe (cylindrical):

- Measurements in near-field display double *peak/knee*:

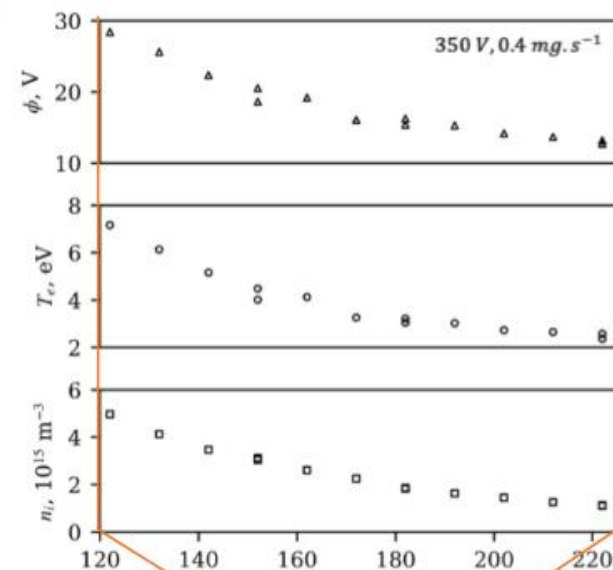
local perturbation - cathode beam - thruster mode – EEDF
ion flux - facility – SEE – high energy ions

probe size - probe build - sweep direction - operating
point probe orientation - transient - instrument

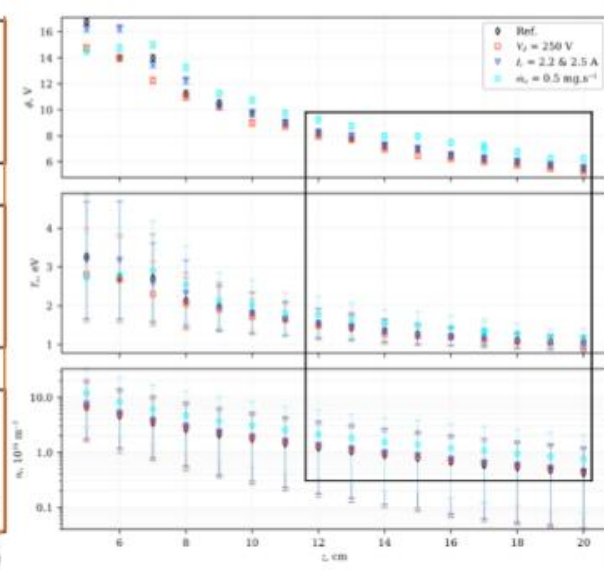
Seen by KAIST too, *avoided* with double LP, attributed to EEDF



MUSIC HC



MIREA



Not trivial to process for ϕ , T_e

