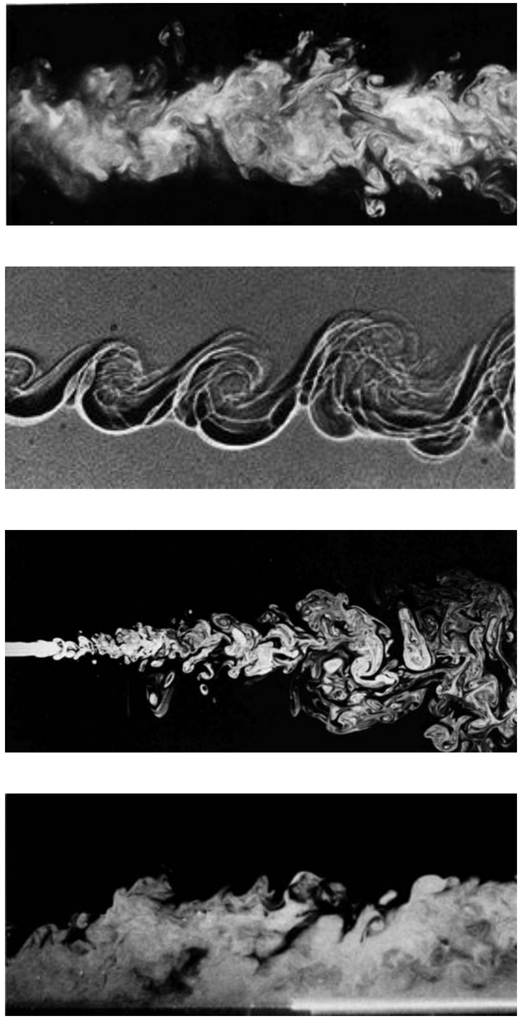
Data-driven modelling of turbulent flows under active flow control

PhD Student: Qihong Lorena Li-Hu
Supervisor/s: Prof. Andrea Ianiro & Prof. Stefano Discetti

PhD Aerospace Engineering UC3M
Doctoral Meetings 2026
2nd and 3rd June 2026

PHD

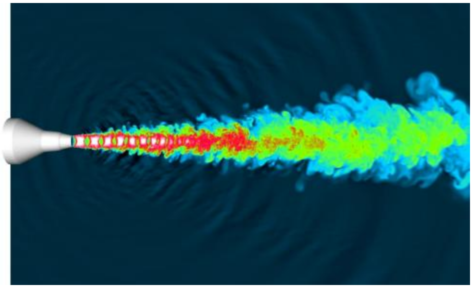
Motivation



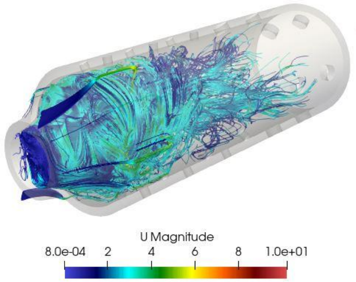
Milton Van Dyke

Andretti Autosport

Grupo One Air



Mendez S. et al (2020)



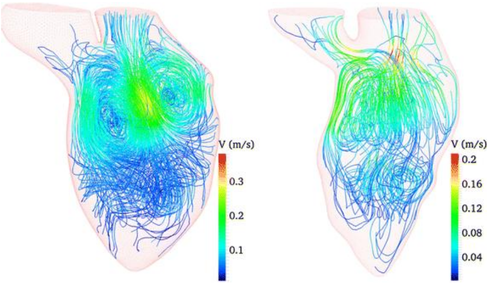
aeroengineering

ile
yw

Ryoh Ishihara

NASA

Airtecnics



Doost. S. N. et all (2016)

Eyink G. (2024)

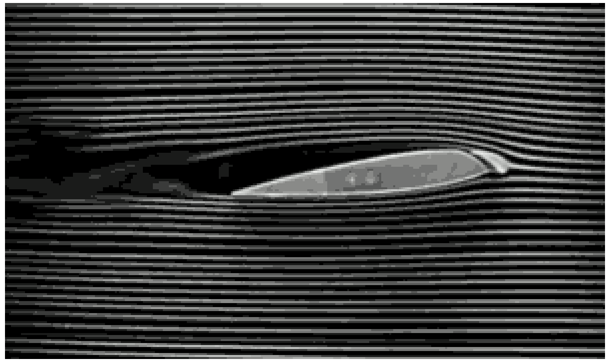
Motivation

Turbulent flows

Navier-Stokes Eqs

Dynamic models

Flow Control



Limitations

No analytical solution

Nonlinear dynamics

High-dimensionality

Technological limitations

Multi-scale problem

Literature review

uc3m PHD

Dynamic models

Data-driven models

White-box

For CFD people

Marten D. (2020)

Grey-box

Reduced Order Models & Physics

Black-box

Machine learning

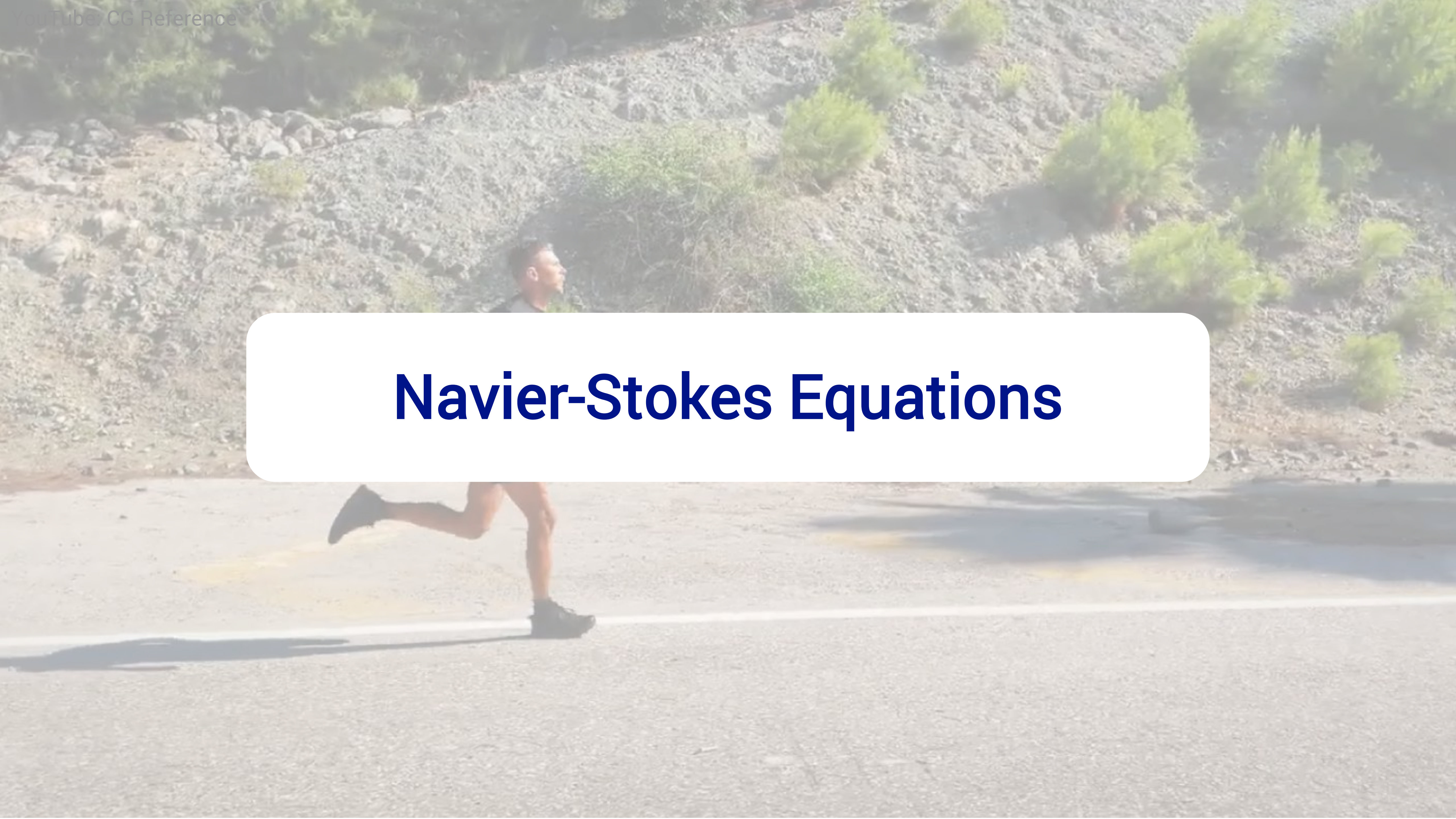
Dimensionality reduction

Data-driven modelling of turbulent flows under active flow control

Reduced-order data-driven models

- 2D → $O(10^4 - 10^5)\ DOF$

Navier-Stokes Equations



Time super-sampling from non-time-resolved fields

Li-Hu Q. L., García-Caspueñas, P., Ianiro, A., & Discetti, S. (2025). Model-based time super-sampling of turbulent flow field sequences. Physical Review Fluids, 10(8), 084901. Open code and datasets

Local dynamical analysis

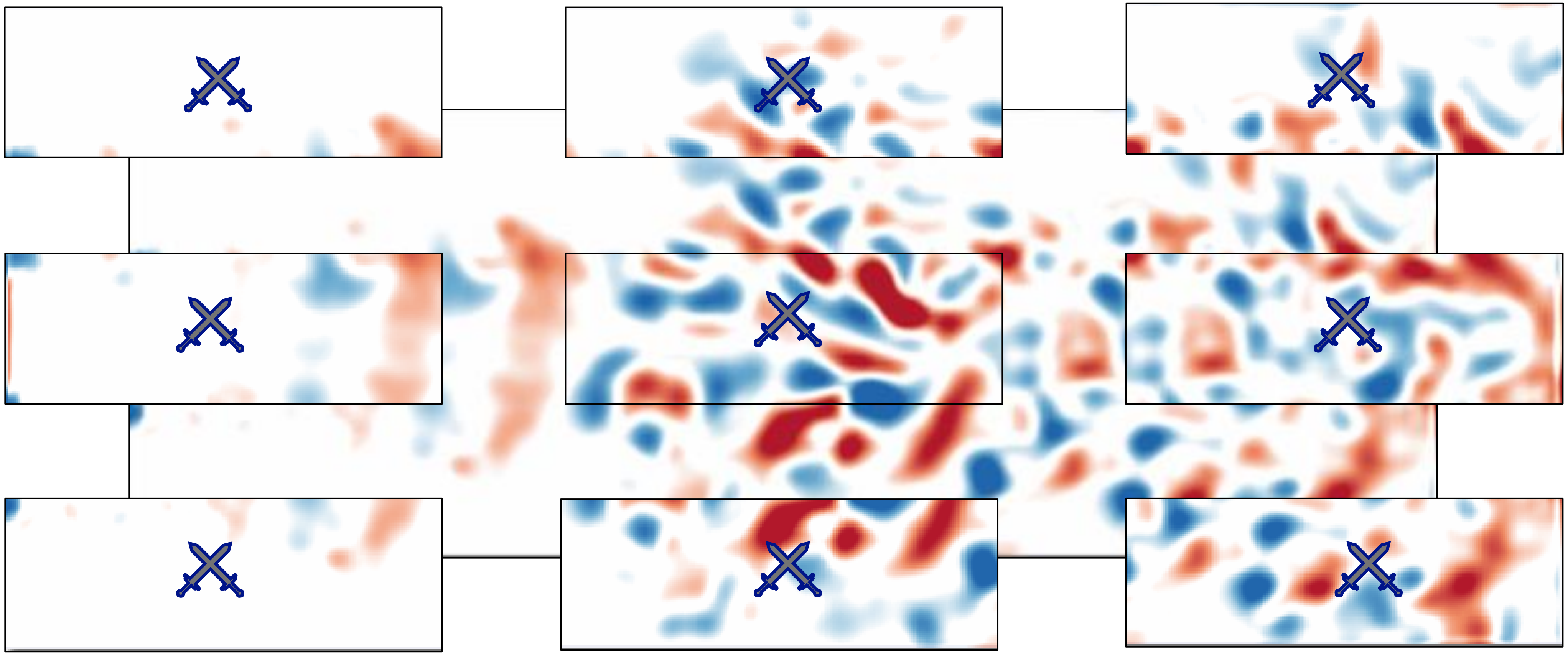
Local dynamical analysis

Reduced-order modelling

=

1. Divide

2. Conquer

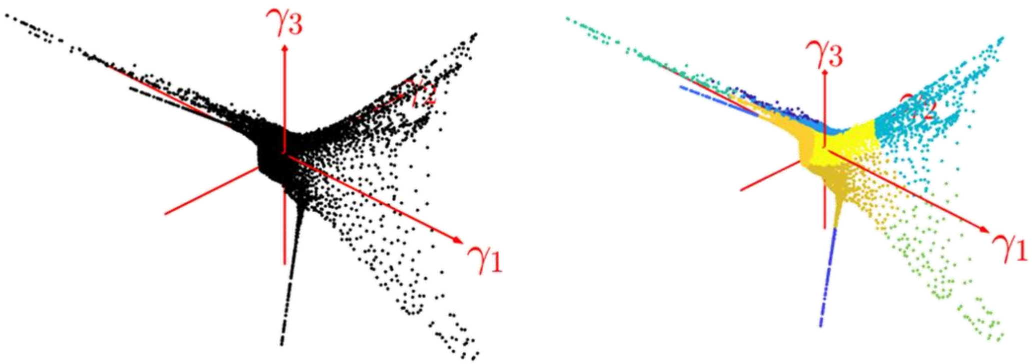


Local dynamical analysis

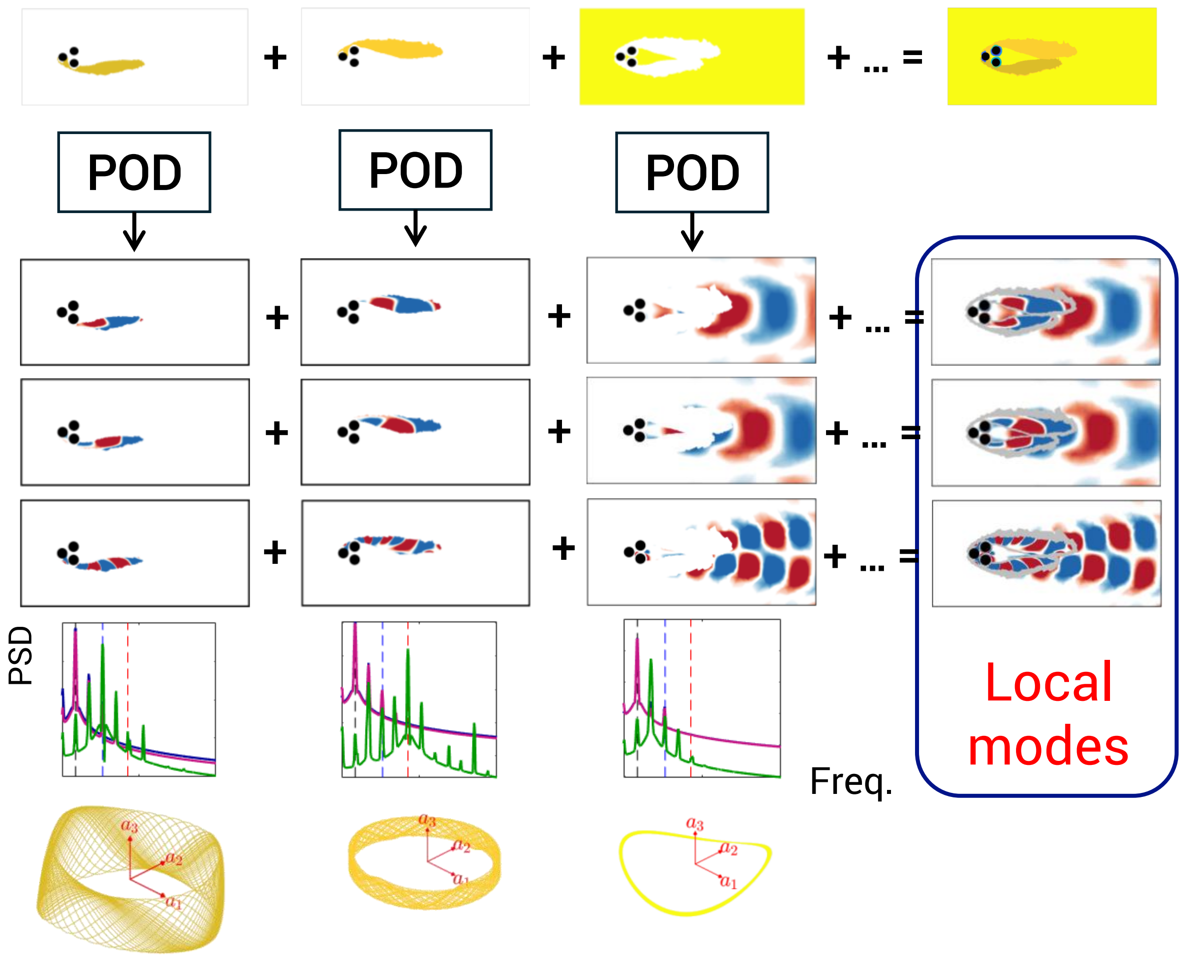
1. Spatial clustering

Similar kinematics
= close in the manifold

“Spatial” Manifold



2. Local dynamical analysis



3rd year

Galerkin model time super-sampling → Published in Physical Review Fluids
Open code and datasets

Divide and conquer complex flows → Multi-scale problem
Local dynamical models
Under review

Guy Y. Cornejo Maceda
Postdoctoral fellow UC3M

Estimation from sensors → Guide and steer flow predictions with sensor measurements

Research stay → September to December 2026

Prof. Dr. F. Scarano
TU Delft

Contributions

- Journal articles and code/datasets
  - ✓ Li-Hu Q. L., García-Caspueñas, P., Ianiro, A., & Discetti, S. (2025). Model-based time super-sampling of turbulent flow field sequences. *Physical Review Fluids, 10*(8), 084901. Open code and datasets
  - ✓ Li-Hu, Q. L., Maceda, G. Y. C., Ianiro, A., & Discetti, S. (2026). Divide and Conquer: Cluster and manifold-based interpretation of complex flows. *arXiv preprint arXiv:2601.05117.* Open code and datasets

- Conference contributions
  - ✓ **21st International Symposium on Applications of Laser and Imaging Techniques to Fluid Mechanics**, Jul 8 – 11, Lisbon, Portugal
  - ✓ **2nd International Conference on Mathematical Modelling in Mechanics and Engineering (ICME)**, Sep 12 – 14, 2024, Belgrade, Serbia
  - ✓ **APS Division of Fluid Dynamics Annual Meeting 2024**, Nov 22 – 24, 2024, Salt Lake City, UT, USA
  - ✓ **7th International Conference on Turbulence and Interactions**, Mar 30 – Apr 4, 2025, La Palma, Spain
  - ✓ **21st International Symposium on Flow Visualization (ISFV21)**, Jun 21 – 25, 2025, Tokyo, Japan
  - ✓ **16th International Symposium on Particle Image Velocimetry (ISPIV 2025)**, Jun 25 – 28, 2025, Tokyo, Japan

uc3m
Universidad Carlos III de Madrid
¿Qué es el sonido?
STWD RECORDS
RSEQ
UNIVERSIDADES MADRILEÑAS
NSEJERÍA DE EDUCACIÓN,
NCIA Y UNIVERSIDADES
PULSADOR DE ALARMA
EXTINTOR

Questions?

Data-driven modelling of turbulent flows under active flow control

PhD Student: Qihong Lorena Li-Hu
Supervisor/s: Prof. Andrea Ianiro & Prof. Stefano Discetti

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (grant agreement No 949085).

European Research Council
Established by the European Commission

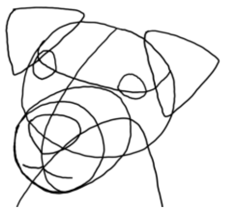
POD

- Dimensionality reduction

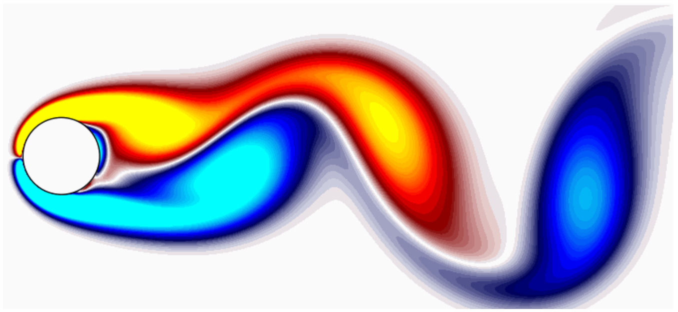
Proper Orthogonal Decomposition (POD)

$$U = \boxed{\Phi}\Sigma\Psi^*$$

Full order

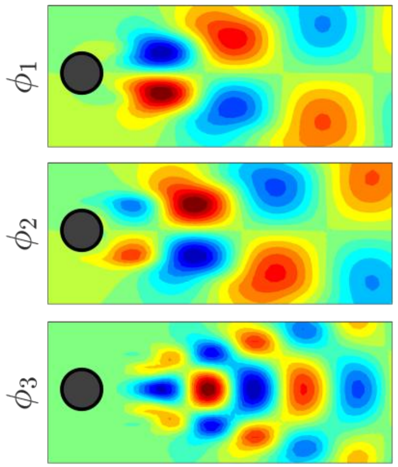


$n_r =$ 10 | 100 | 1000

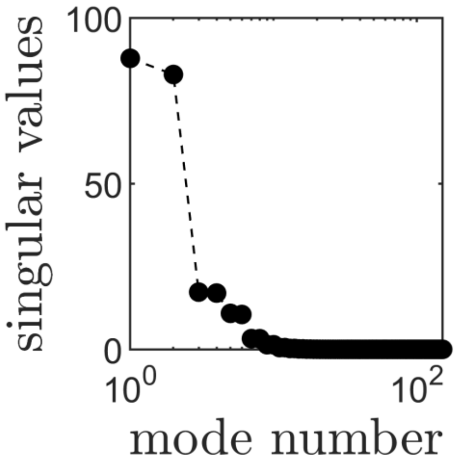


Medium

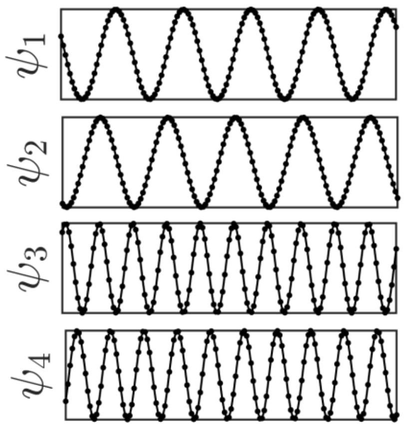
Spatial modes



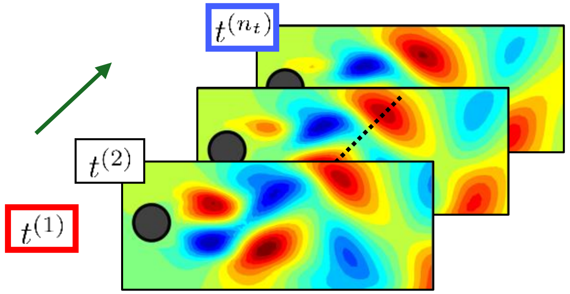
Singular values



Temporal modes



POD with snapshot methods



snapshots

rearrange

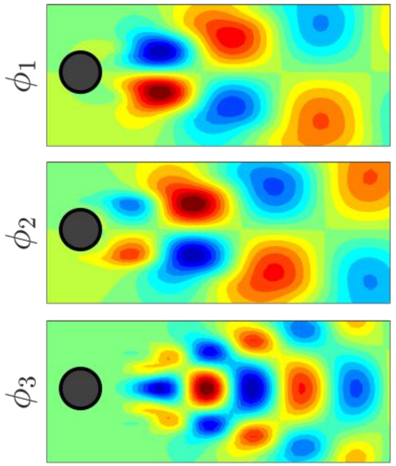
$t \in [1, n_t]$

$$\mathbf{U} = \begin{bmatrix} u'\left(x^{(1)}, t^{(1)}\right) & \cdots & u'\left(x^{(1)}, t^{(n_t)}\right) \\ u'\left(x^{(2)}, t^{(1)}\right) & \cdots & u'\left(x^{(2)}, t^{(n_t)}\right) \\ \vdots & \ddots & \vdots \\ u'\left(x^{(n_p)}, t^{(1)}\right) & \cdots & u'\left(x^{(n_p)}, t^{(n_t)}\right) \end{bmatrix} \in \mathbb{R}^{n_p \times n_t}$$

$n_t < n_p$

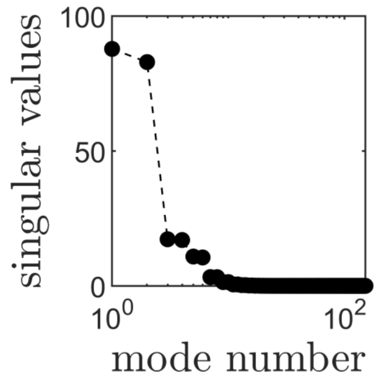
SVD

$$\Phi = \begin{bmatrix} | & | & & | \\ \phi_1 & \phi_2 & \cdots & \phi_{n_t} \\ | & | & & | \end{bmatrix}$$



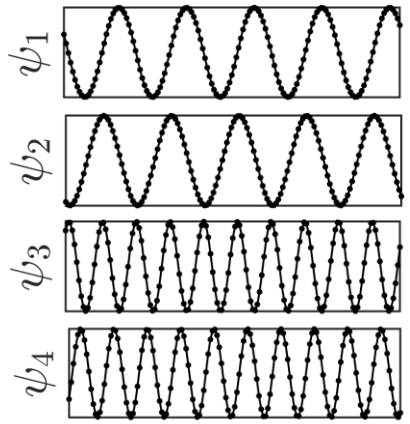
Spatial modes

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sigma_{n_t} \end{bmatrix}$$



Singular values

$$\Psi = \begin{bmatrix} | & | & & | \\ \psi_1 & \psi_2 & \cdots & \psi_{n_t} \\ | & | & & | \end{bmatrix}$$



Temporal modes

$$U = \boxed{\Phi}\boxed{\Sigma\Psi^*}$$

$$\boldsymbol{u}^{[r]}(\boldsymbol{x}, t) = \sum_{i=0}^{r} \boxed{a_i(t)}\boldsymbol{\phi}_i(\boldsymbol{x})$$

r: *number of retained modes*

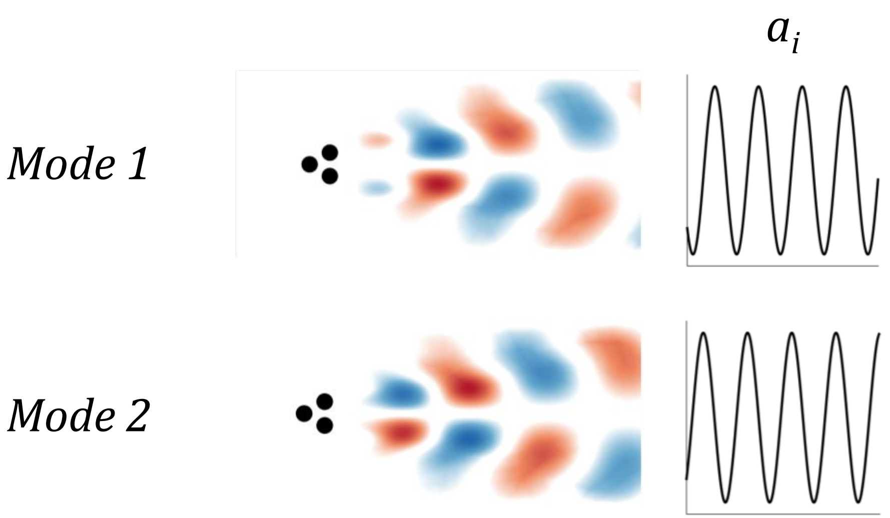
$r < n_t$

Dimensionality reduction

Linear methods

Proper Orthogonal Decomposition (POD)

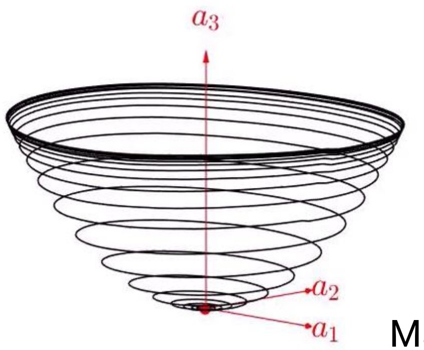
Lumley (*1967*)



Nonlinear methods

Isometric mapping (ISOMAP)

Tenenbaum *et al.* (*Science* 2000)

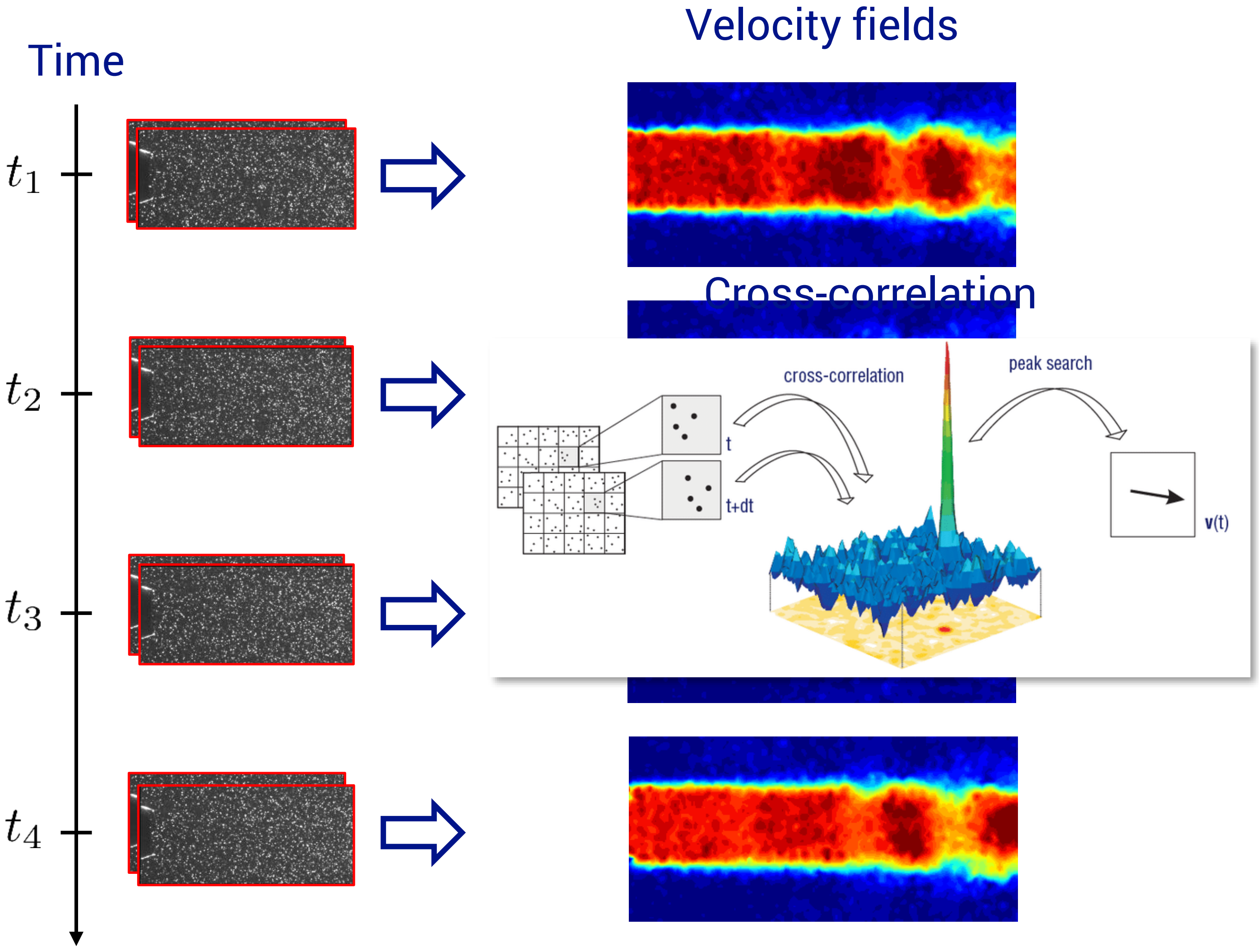


Marra et al. 2024b JFM
Farzamnik et al. 2023 JFM

Particle Image Velocimetry (PIV)

- **Technological barriers**
  - Maximum sampling
  - Pulsed light frequency

- **Time super-sampling of PIV**
  - Taylor's hypothesis
  - Vortex-in-cell



Galerkin projection for PIV fields

FOM representation

Navier Stokes equations

PDE

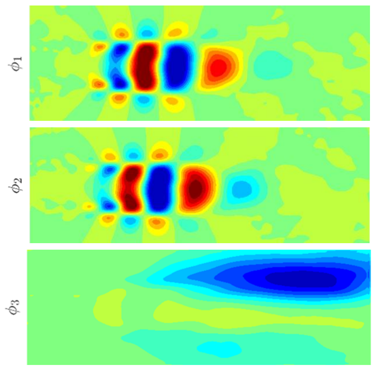
$$\begin{cases} \boldsymbol{\nabla} \cdot \boldsymbol{u} = \boldsymbol{0} \\ \partial_t \boldsymbol{u} + \boldsymbol{\nabla} \cdot (\boldsymbol{u}\,\boldsymbol{u}) = -\nabla p + \dfrac{1}{Re} \Delta \boldsymbol{u} \end{cases}$$

$\Rightarrow$

Projection on spatial modes

$$\boldsymbol{u}^{[r]}(\boldsymbol{x}, t) = \sum_{i=0}^{r} a_i(t) \boldsymbol{\phi}_i(\boldsymbol{x})$$

$$\begin{cases} (\boldsymbol{\nabla} \cdot \boldsymbol{u} = \boldsymbol{0}) \cdot \boldsymbol{\phi}_i \\ \left(\partial_t \boldsymbol{u} + \boldsymbol{\nabla} \cdot (\boldsymbol{u}\,\boldsymbol{u}) = -\nabla p + \dfrac{1}{Re} \Delta \boldsymbol{u}\right) \cdot \boldsymbol{\phi}_i \end{cases}$$



$\Downarrow$

$$\begin{cases} \boldsymbol{\nabla} \cdot \boldsymbol{\phi}_i = 0 \\ \dfrac{d}{dt} a_i \coloneqq \dfrac{1}{Re} \displaystyle\sum_{j=0}^{r} l_{ij} a_j + \sum_{j=0}^{r} \sum_{k=0}^{r} q_{ijk} a_j a_k + \boldsymbol{f}_{\boldsymbol{i}}^{\boldsymbol{\pi}}(\boldsymbol{a}) \end{cases} \qquad \begin{array}{c} for \\ i = 1,2, \dots, r \end{array}$$

$\Leftarrow$

ROM representation

ODEs

$$\frac{da_i}{dt} = f(a_i)$$

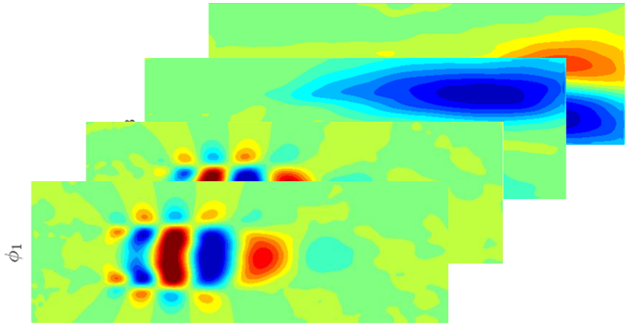
$$i = 1,2, \dots, r$$

Galerkin projection for PIV fields

Projection on spatial modes

$$\boldsymbol{u}^{[r]}(\boldsymbol{x},t) = \sum_{i=0}^{r} a_i(t)\boldsymbol{\phi}_i(\boldsymbol{x})$$

$$\begin{cases} (\boldsymbol{\nabla}\cdot\boldsymbol{u} = \boldsymbol{0})\cdot\boldsymbol{\phi}_i \\ \left(\partial_t\boldsymbol{u} + \boldsymbol{\nabla}\cdot(\boldsymbol{u}\,\boldsymbol{u}) = -\nabla p + \frac{1}{Re}\Delta\boldsymbol{u}\right)\cdot\boldsymbol{\phi}_i \end{cases}$$



$$\begin{cases} \boldsymbol{\nabla}\cdot\boldsymbol{\phi}_i = 0 \\ \frac{d}{dt}a_i \coloneqq \frac{1}{Re}\sum_{j=0}^{N} l_{ij}a_j + \sum_{j=0}^{N}\sum_{k=0}^{N} q_{ijk}a_ja_k + \boldsymbol{f}_i^{\pi}(\boldsymbol{a}) \end{cases}$$

for $i = 1,2,\dots,r$

VISCOUS TERM

$$l_{ij} \coloneqq \left(\boldsymbol{\phi}_i, \boldsymbol{\Delta\phi}_j\right)_\Omega$$

CONVECTIVE TERM

$$q_{ijk} \coloneqq \left(\boldsymbol{\phi}_i, \boldsymbol{\nabla}\cdot\left(\boldsymbol{\phi}_j\boldsymbol{\phi}_k\right)\right)_\Omega$$

PRESSURE TERM

$$\boldsymbol{f}_i^{\pi} \coloneqq (\boldsymbol{\phi}_i, -\boldsymbol{\nabla}p)_\Omega$$

Time marching integration for PIV supersampling

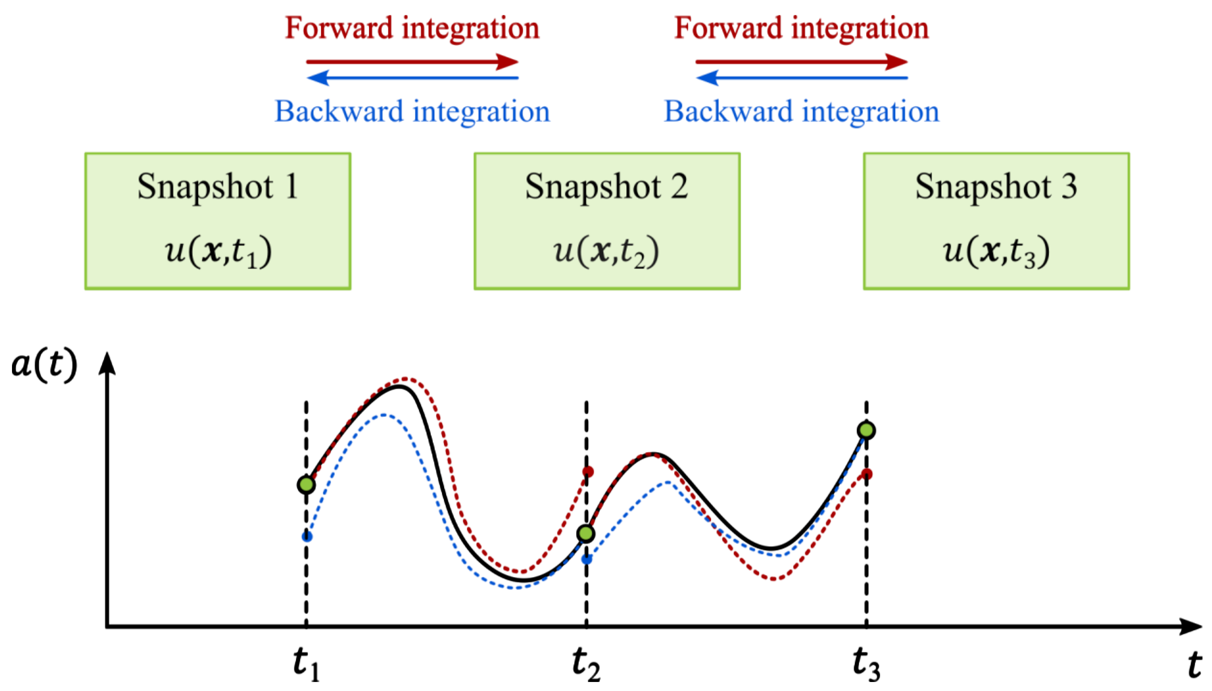
ROM representation

ODEs

$$\frac{da_i}{dt} = f(a_i)$$

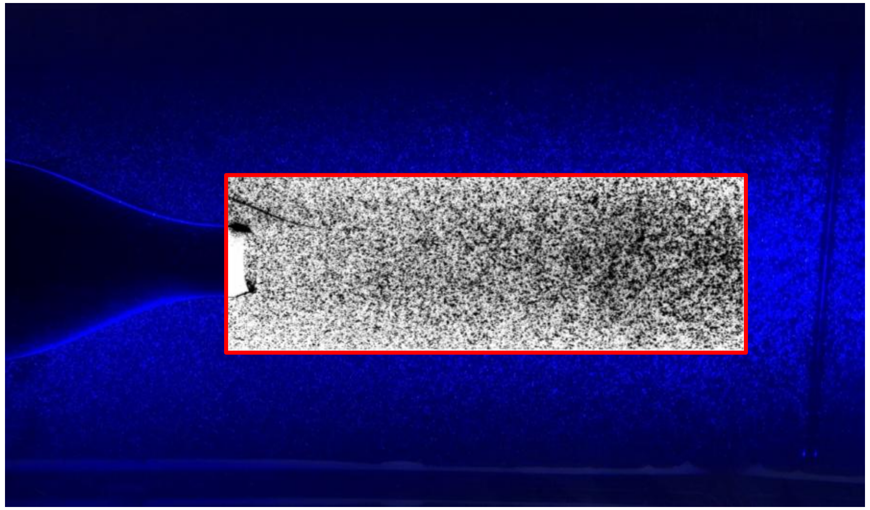
$$i = 1,2,\ldots,r$$

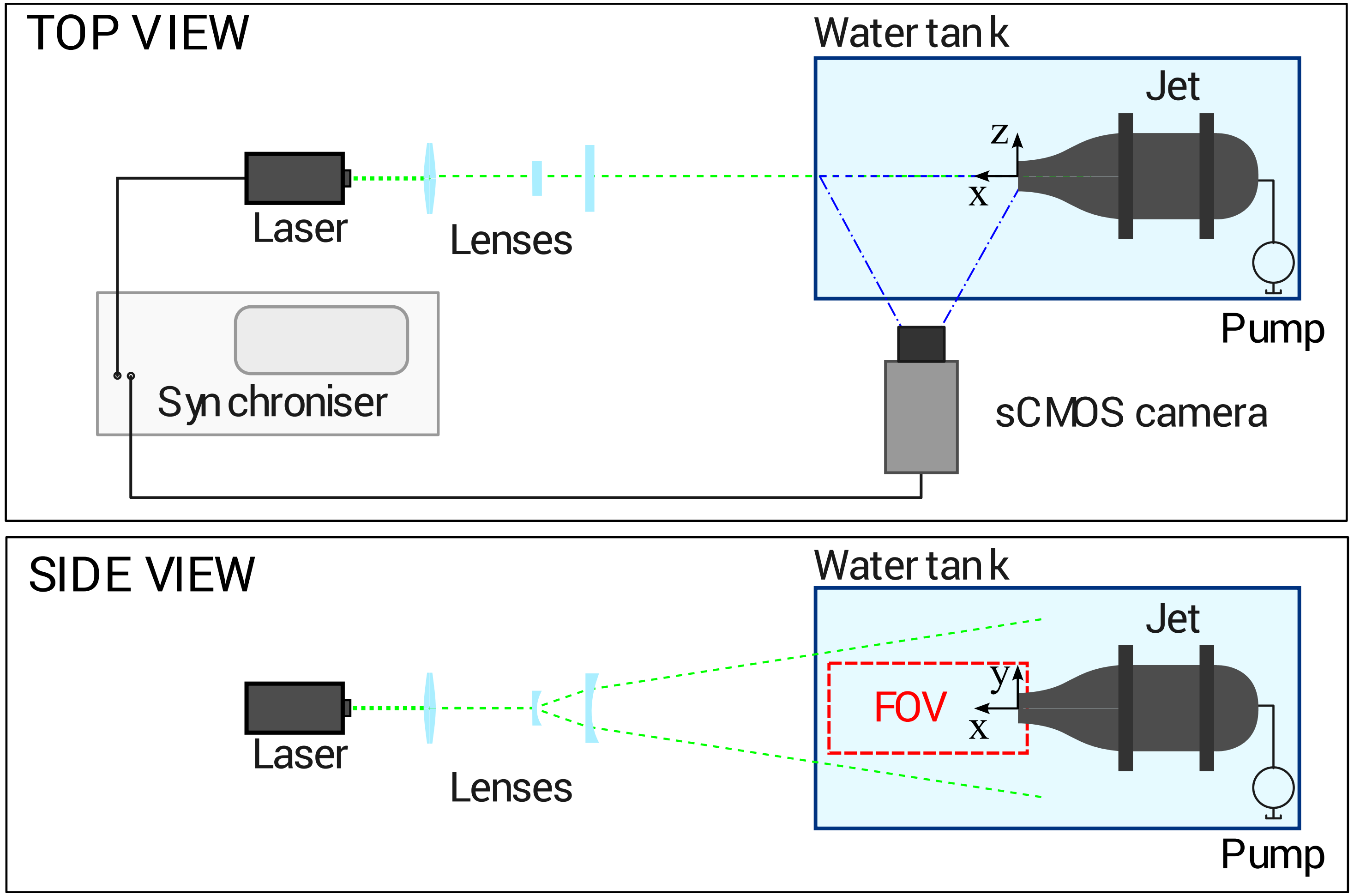
- Forward integration
- Backward Integration
- Weighted average



Experimental jet flow

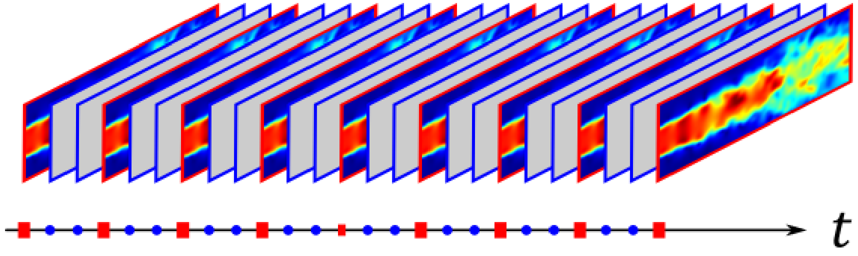
- Turbulent jet flow (water)
- $Re \approx 3300$
- $f_s = 90Hz$





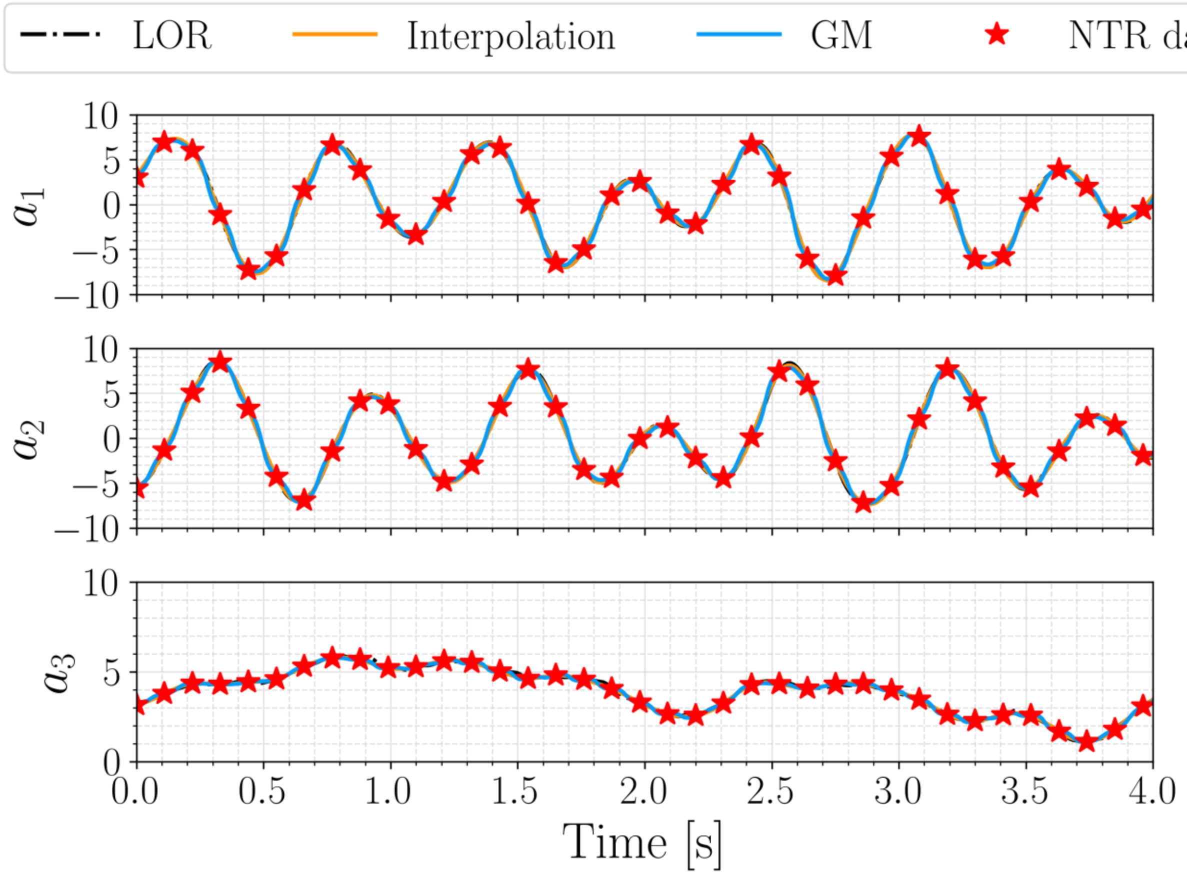
Experimental jet flow

$$S_r = 10$$



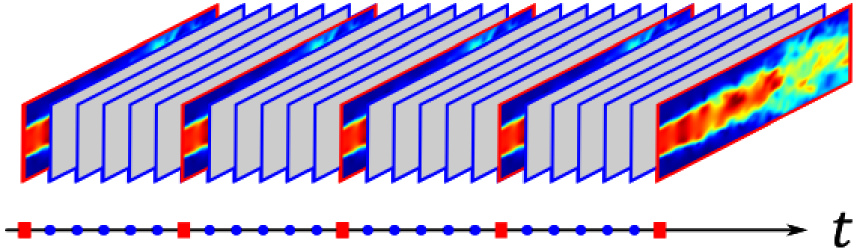
$$\delta_{MSE} = 0.02$$

$$\Delta t = 0.4 t_c$$



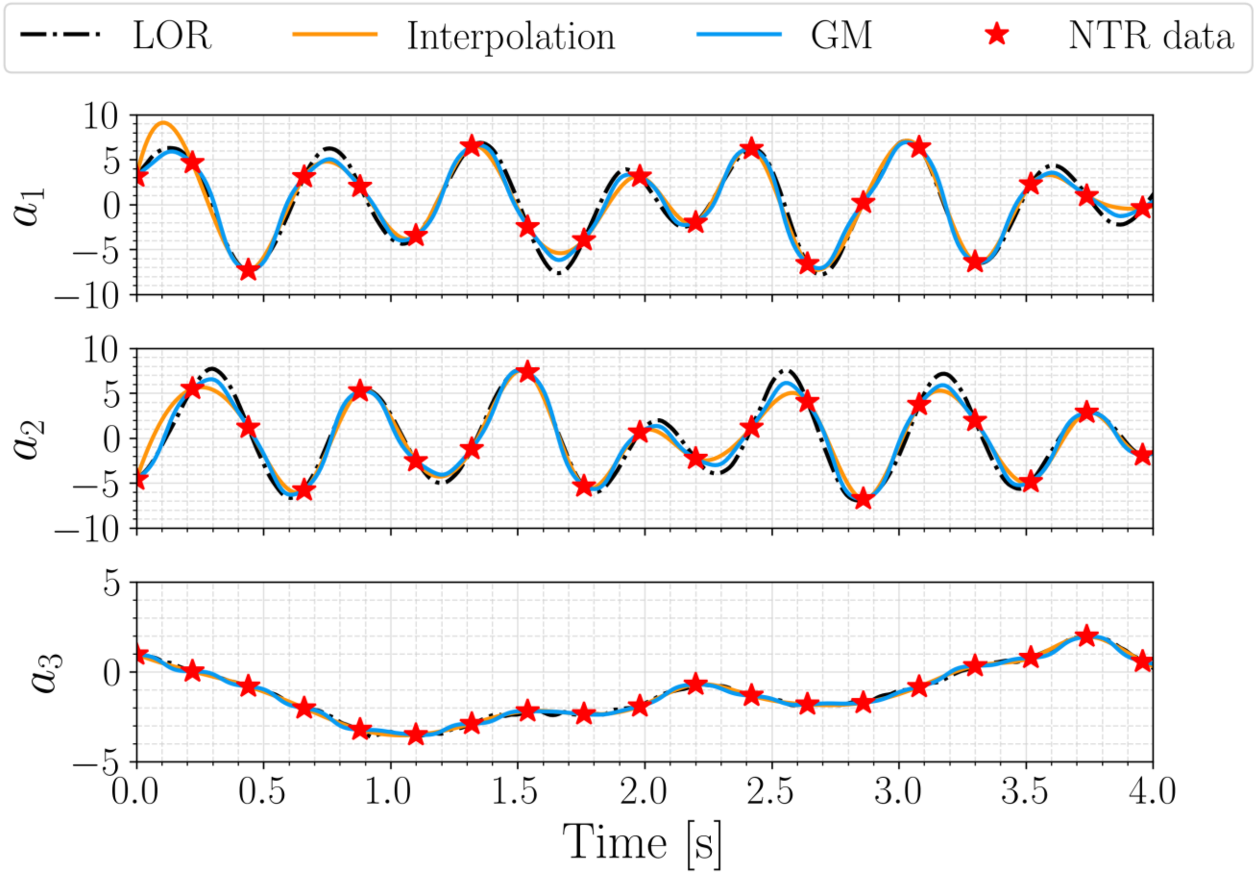
Experimental jet flow

$$S_r = 20$$

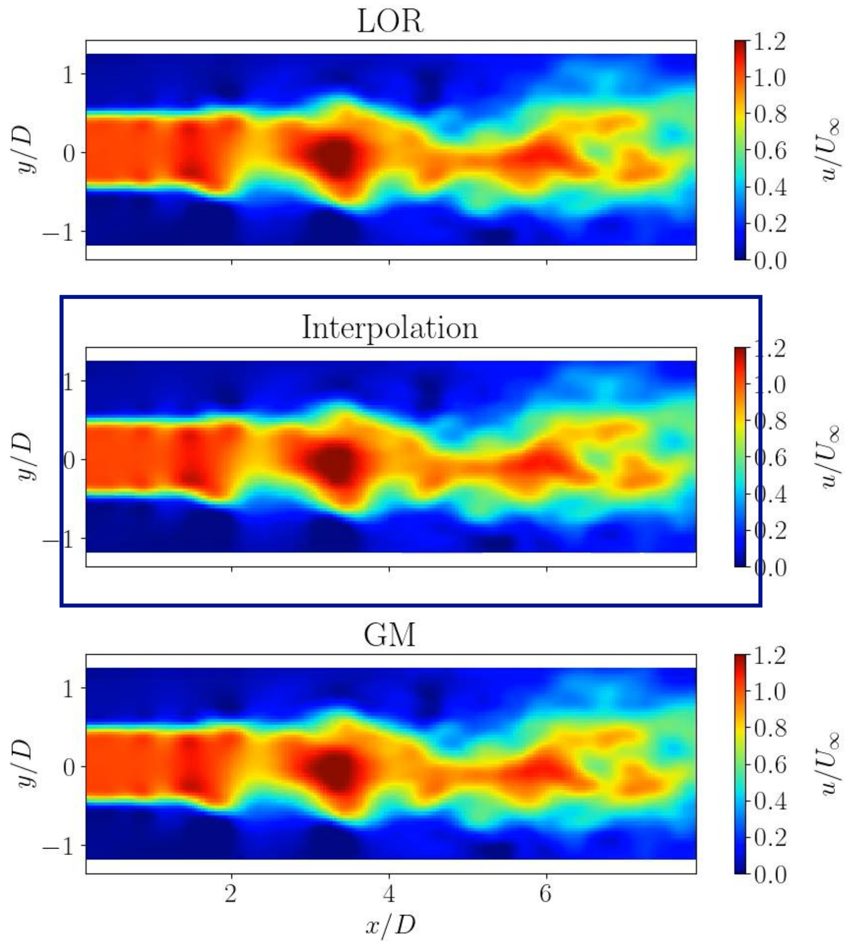


$$\delta_{MSE} = 0.07$$

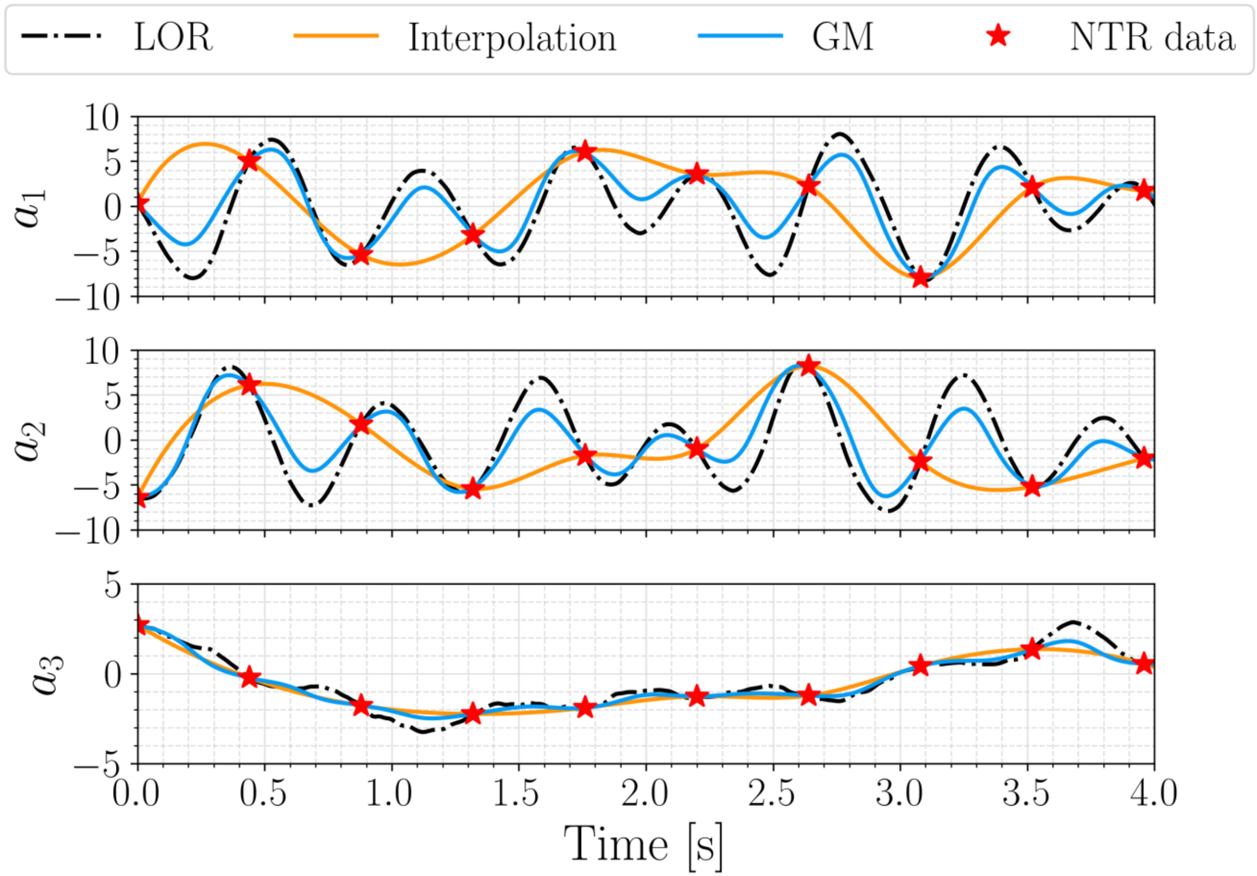
$$\Delta t = 0.8 t_c$$



Experimental jet flow



$$\Delta t = 1.6t_c$$



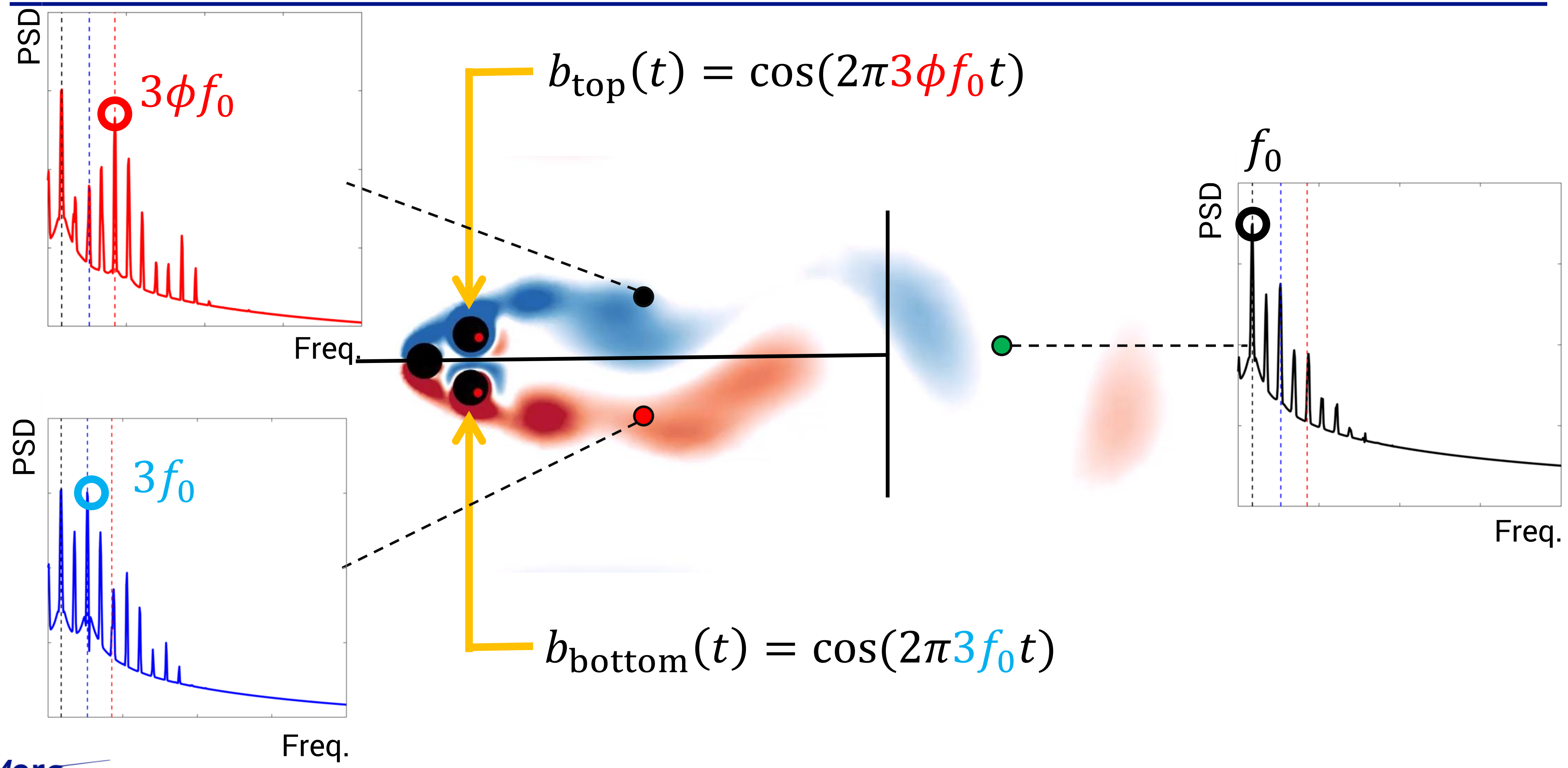
The fluidic pinball

<u>Fluidic pinball community:</u>

- **DRL control:** Feng *et al*. 2023 PoF
- **Model Predictive Control:** Bieker *et al.* 2020 TCFD, Marra *et al.* 2024a JFM
- **Actuation manifold learning:** Marra *et al* 2024b JFM
- **Experiments:** U Calgary, LISN/CNRS (Paris), UC3M (Madrid)

The fluidic pinball encapsulates key features of real flows

A multi-frequency flow

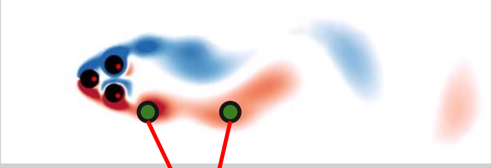


Divide and conquer complex flows

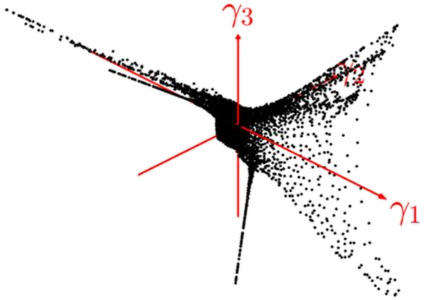
Cluster by dynamics

Clustering

"Spatial" Manifold



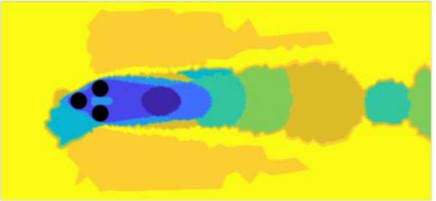
Similar dynamics
= close in the manifold



Which variable to cluster?

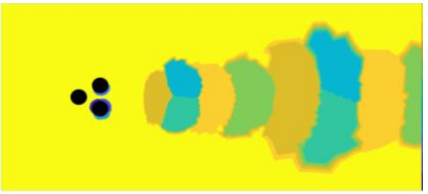
Velocity fields

u | v → K-means++ →



Fluctuations

u' | v' → K-means++ →



Vorticity

ω → K-means++ →

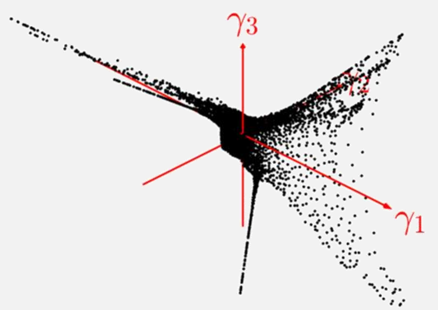
Manifolds of manifolds – Divide…

"Spatial" Manifold

Vorticity

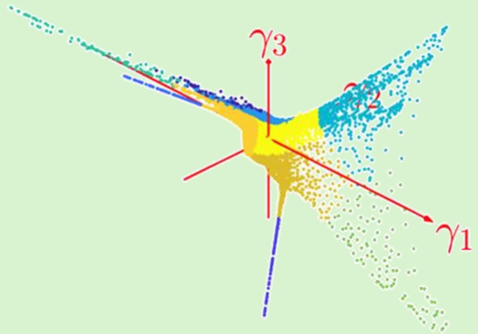
$X = \boxed{\omega}$

Isometric Mapping
Tenenbaum *et al.* 2000

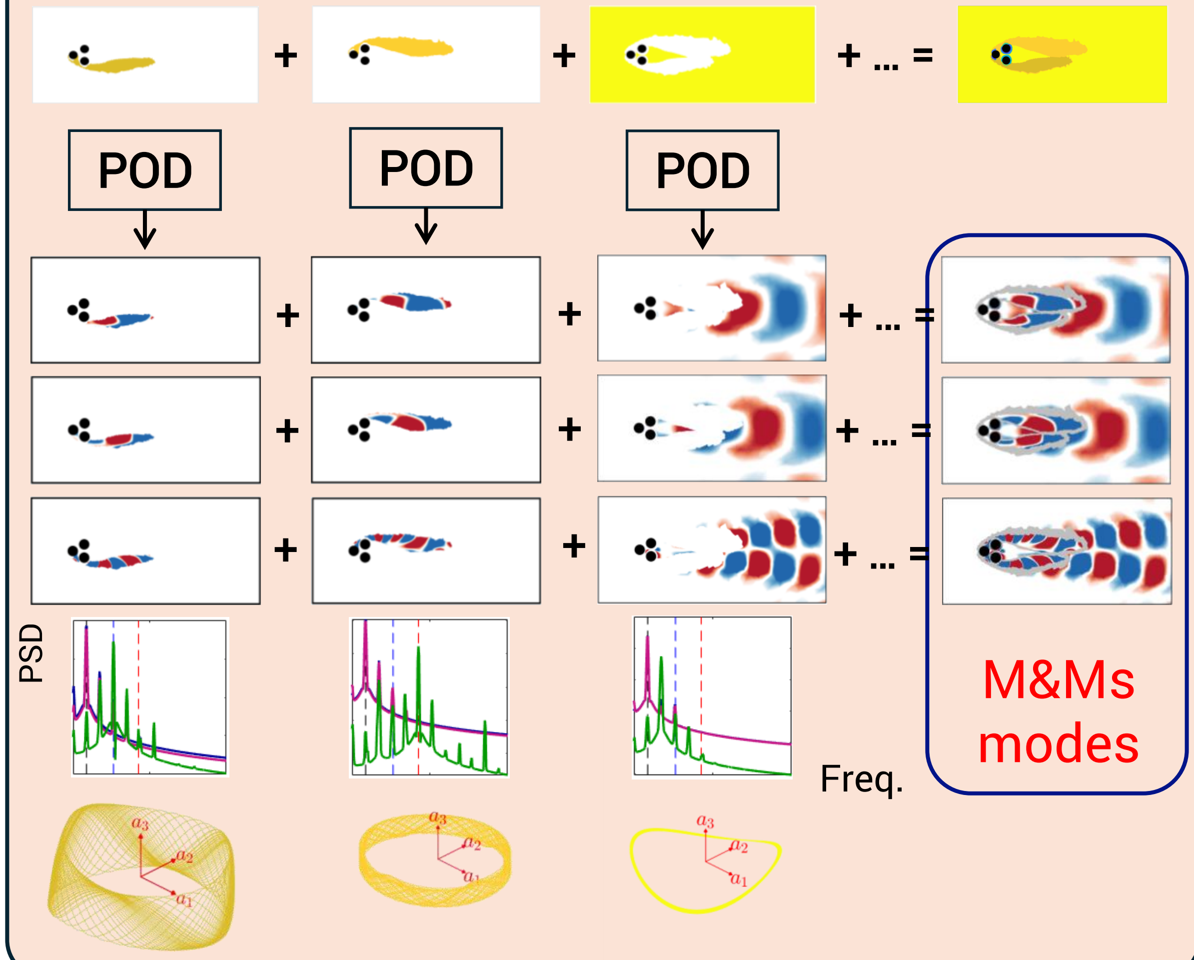


Clustering

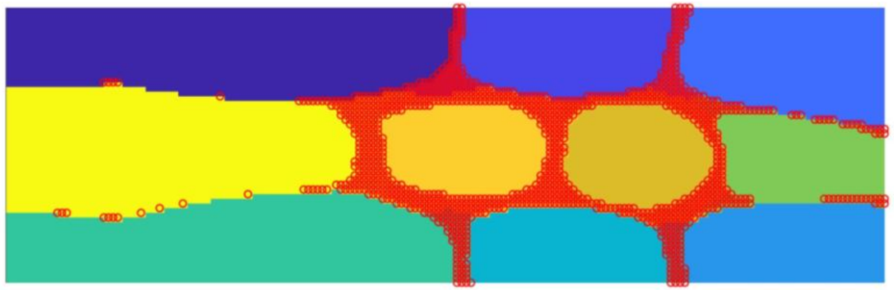
K-means++

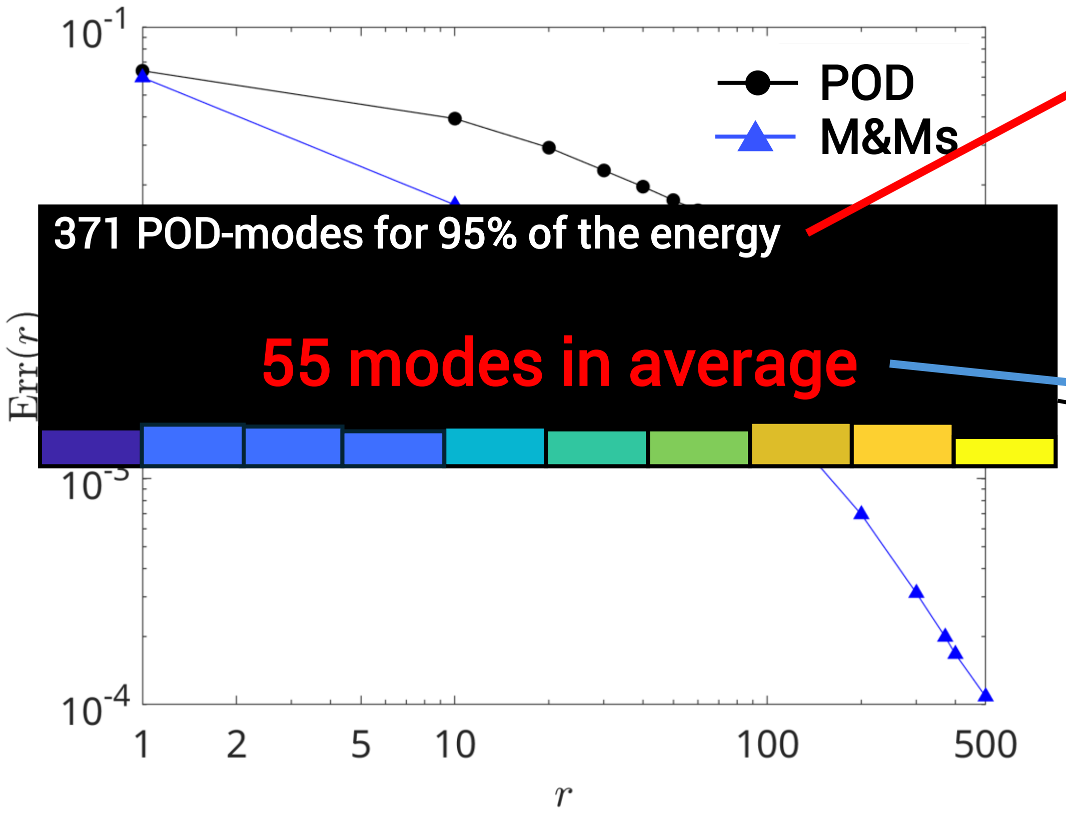


… of manifolds



Results – Jet flow





Galerkin model

1 system of 371 equations
- 138012 terms each

10 systems of ~55 equations
- 3080 terms each