ASD Lab

uc3m | Universidad Carlos III de Madrid
Department of Aerospace Engineering

Development of a framework for coupled aerostructural optimisation of new generation wings

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Motivation

Sustainable aviation requires more efficient aircraft designs

Large Aspect Ratio Wings (LARW) and Truss-Braced Wings (TBW) are promising concepts to reduce induced drag and improve aerodynamic efficiency.

However, increased wing flexibility introduces:

1. stronger aeroelastic coupling
2. larger structural deformations
3. nonlinear structural effects
4. more complex optimisation constraints

Large aspect ratio / truss-braced concepts

Need for high-fidelity aerostructural optimisation frameworks involving aerodynamic and structural nonlinearities

Image source: INDIGO Sustainable Aviation project — indigo-sustainableaviation.eu

Motivation - From sustainable aviation to sensitivity analysis

Designing flexible, efficient wings requires optimisation of strongly coupled aerodynamic and structural models.

Sustainable aviation
Need for more efficient aircraft

Flexible wing concepts
LARW / TBW increase aeroelastic coupling

Aerostructural optimisation
Large number of design variables

Gradient-based methods
Need accurate derivatives

Sensitivity computation
Adjoint method and algorithmic differentiation (AD)

Key computational question: how can sensitivities be computed accurately and efficiently in a complex coupled solver?

State of the art and research gaps

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Research direction	Main achievement	Remaining issue
AD for structural sensitivities	Automated FE sensitivities and adjoint-based gradients (Haase et al., 2001, Bombardieri et al., 2021)	Runtime/memory AD overheads in large models is not characterised
Parallel AD differentiated solvers	Reverse-mode AD successfully deployed in large-scale parallel CFD adjoint solvers (Albring et al., 2015, 2016)	No comparable parallel nonlinear FE adjoint solvers. No scalability and implementation issues assessment.
High-fidelity aerostructural optimisation	Large-scale CFD–FEM adjoint optimisation of aircraft wings (Kenway et al., 2014)	Increasingly mature, but advanced structural constraints remain difficult to integrate.
Stability-aware optimisation	Buckling constraints increasingly included (Brooks et al., 2019; Gray & Martins, 2025)	Stress constraints often dominate; buckling constraints often simplified
Flexible-wing nonlinear modelling	Geometrically nonlinear aeroelastic optimisation increasingly considered (Gray & Martins, 2021; Verri et al., 2025)	Nonlinear equilibrium effects on modal/buckling behaviour are not yet systematically included

Thesis contributions and impact

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1 Structural sensitivity analysis

Contribution:

Reverse-mode AD + discrete adjoint sensitivities for structural size optimisation.

Stress, modal and buckling responses.

Output: AeroBest 2023 & 2025

2 Computational performance

Contribution:

Systematic characterisation of AD overheads.

Runtime, memory, MPI scalability, tangent consistency and selective tape recording.

Output: accepted journal article

3 Aerostructural optimisation

Contribution:

Integration into high-fidelity CFD–FEM workflows.

Stress, modal and buckling constraints evaluated on deformed configurations.

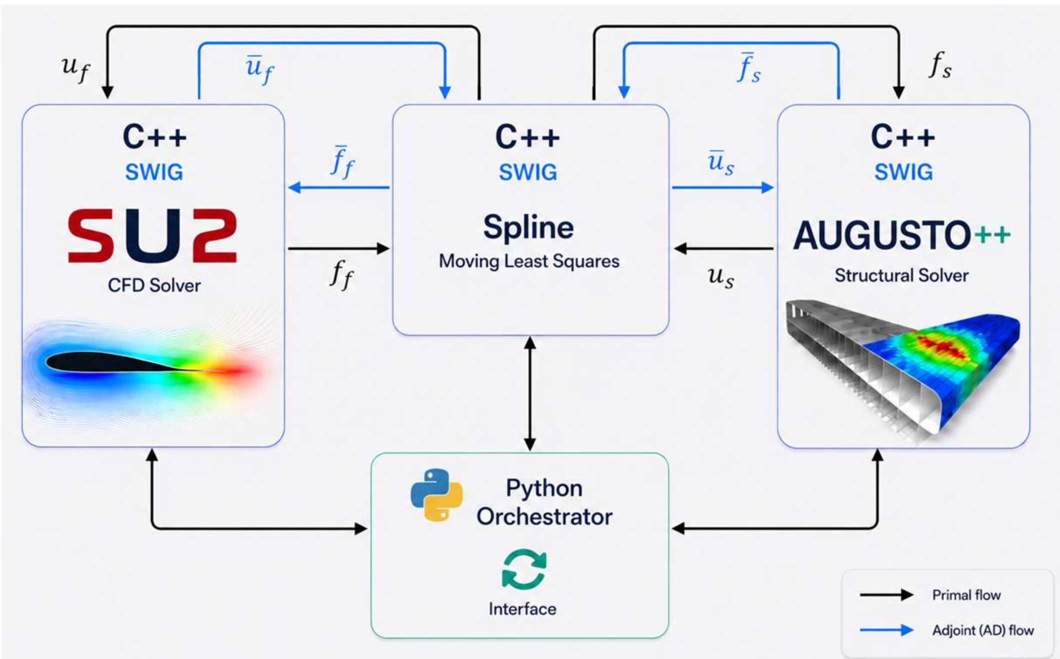
Output: next journal paper

Expected impact

Contribution to the high-fidelity design of large aspect-ratio and next-generation flexible wings

Framework Overview

Computational framework for high-fidelity nonlinear aerostructural optimisation of flexible wing



Main framework capabilities

1. **High-fidelity aerostructural model**
   CFD solver + spline-based load/displacement transfer + nonlinear shell-based structural solver.

2. **AD-differentiated solvers**
   Reverse-mode AD (via CodiPack) for discrete consistent coupled sensitivities.

3. **Large scale optimisation problems**
   Gradient-based + adjoint method + AD + MPI-parallel computations.

4. **Multidisciplinary design flexibility**
   Aerodynamic and structural design variables with aerodynamic, structural, or coupled objectives.

Methodology - The aerostructural optimisation problem

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Example problem: minimise structural mass with structural design variables. Constraint on stress and/or modal frequencies on deformed configuration.

Optimisation problem

$$\min_{\alpha_s} \quad m(\alpha_s)$$

subject to

$$G(y(\alpha_s), \alpha_s) = 0$$

(coupled field equations)

$$g(y(\alpha_s), \alpha_s) \geq 0$$

(optim. constraints)

Coupled aerostructural system G(y, α)

Aerodynamics	$\mathrm{F}(w, z) - w = 0$ $\mathrm{F_f}(w, z) - f_f = 0$	Fixed – Point form
Structure (static)	$\mathrm{S}(u_s, f_s, \alpha_s) - u_s = 0$	Fixed – Point form
Eigenproblem	$(K + \lambda_i M)\varphi_i = 0$ $\phi_i^T M \phi_i - 1 = 0$	
Mesh deformation	$K_{mesh}(u_{tot}) - z = 0$	
Load transfer	$H^{\mathrm{T}} f_f - f_s = 0$	
Displ. transfer	$H\, u_s - u_f = 0$	
Kinematics	$u_{tot} - u_f = 0$	

State and design variables

State vector y

- w: flow conservative variables
- z: volume mesh displacements
- u_s: structural displacements
- u_f: fluid contour displ.
- f_s: structural loads
- f_f: fluid loads
- u_{tot}: cumulative displacements of wing surface
- φ_i, λ_i: eigenpair

Design vector α_s

panels thicknesses

Methodological focus: compute reliable gradients for the coupled nonlinear problem using adjoint and AD.

Methodology - Discrete adjoint method

To compute the gradient of a response J, either an objective (*) or a constraint, the **Lagrangian** is introduced together with the adjoint variables $\bar{\boldsymbol{y}}$.

$$\mathcal{L}(y, \bar{y}, \alpha_s) = J(y, \alpha_s) + \bar{y}^T G(y, \alpha_s)$$

Adjoint eqs.

$$\frac{\partial \mathcal{L}}{\partial y} = 0$$

$$\frac{\partial J}{\partial \boldsymbol{w}} + \bar{\boldsymbol{w}}_n^{\,T} \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{w}} + \bar{\boldsymbol{f}}_f^{\,T} \frac{\partial \boldsymbol{F}_f}{\partial \boldsymbol{w}} = \bar{\boldsymbol{w}}_{n+1}^{\,T}$$

$$\frac{\partial J}{\partial \boldsymbol{z}} + \bar{\boldsymbol{w}}^T \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{z}} + \bar{\boldsymbol{f}}_f^{\,T} \frac{\partial \boldsymbol{F}_f}{\partial \boldsymbol{z}} + \bar{\boldsymbol{z}}^T = \boldsymbol{0}^T$$

$FSI_{adj}\ iter \mathrel{+}= 1$

$$\frac{\partial J}{\partial \boldsymbol{u}_s} + \bar{\boldsymbol{u}}_s^T \frac{\partial S}{\partial \boldsymbol{u}_s} + \bar{\boldsymbol{u}}_f^{\,T} \boldsymbol{H}_{MLS}^T + \bar{\boldsymbol{\phi}}_i^{\,T} \frac{\partial [\boldsymbol{K} - \lambda_i \boldsymbol{M}]}{\partial \boldsymbol{u}_s} \boldsymbol{\phi}_i = \bar{\boldsymbol{u}}_{s,n+1}^T$$

Sens. eqs.

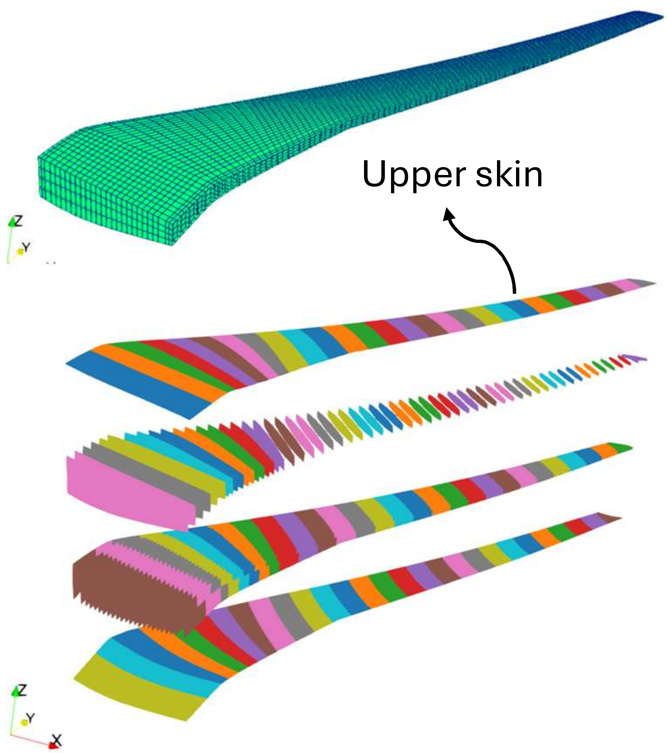
$$\frac{\partial \mathcal{L}}{\partial \alpha_s} = 0$$

$$\frac{dJ}{d\alpha_s}^T = \frac{\partial J}{\partial \alpha_s} + \frac{\partial S}{\partial \alpha_s}^T \bar{u}_s + \bar{\phi}_i^{\,T} \frac{\partial [K - \lambda_i M]\phi_i}{\partial \alpha_s}$$

Primal FSI equilibrium → Computational path recording (AD) → Adjoint cascade → Total sensitivity

(*) When $J = m$, response is state - independent and sensitivity evaluation simplifies

Aerostructural optimisation - NASA CRM test case



CRM wing model and structural sizing panels

Design variables

113 thickness variables

Optimisation constraints

1. Upper-skin stress aggregation

$$g_\sigma = KS(g_{\sigma,i}) = \frac{1}{\rho_{KS}} log \sum_{i=1}^{n} e^{\rho_{KS} \cdot g_{\sigma,i}} \leq 0 \quad \text{where} \quad g_{\sigma,i} = \frac{\sigma_i^{VM}}{\sigma_{ADM}} - 1$$

2. Frequency-spacing constraint on deformed configuration

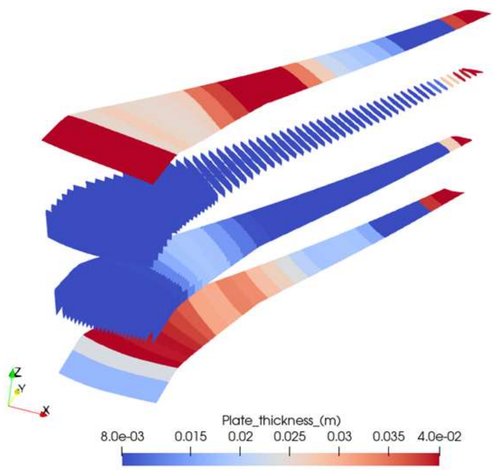
$$g_\omega = \rho_\omega \frac{(\omega_2 - \omega_1)_{init}}{\omega_2 - \omega_1} - 1 \leq 0$$

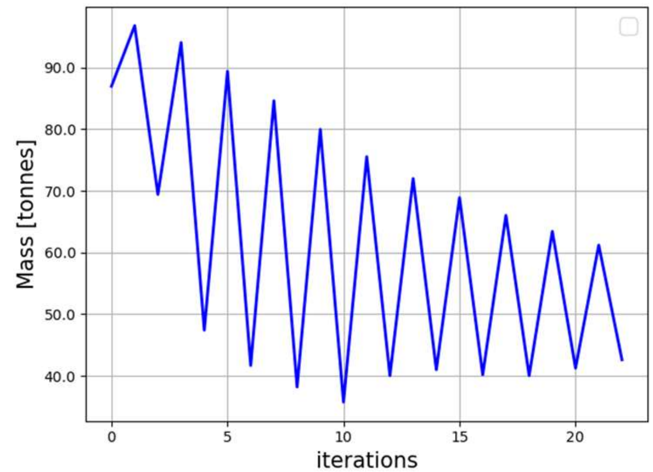
Flight condition

$M = 0.85 \quad C_L = 0.5$

Aerostructural optimisation – (Very) Preliminary results

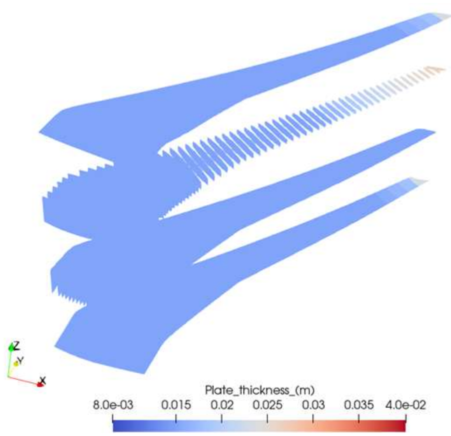
Test 1:
g_σ & g_ω

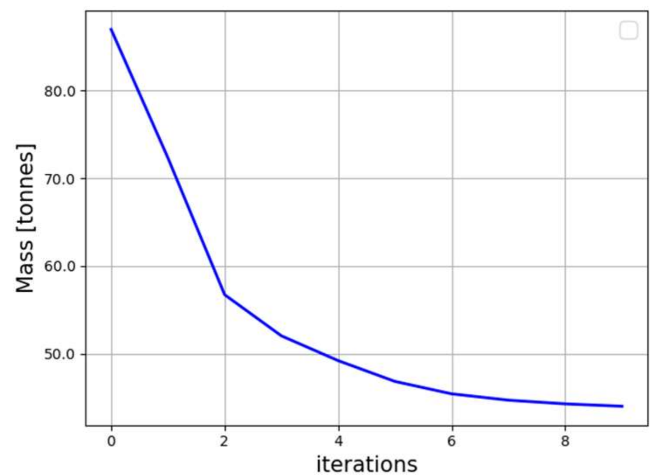




Quantity	Initial	Optimised
$Mass\ (Kg)$	86951.0	42633.0 (-51 %)
g_σ	0.371	0.390 (feasible)
g_ω	−0.102	−0.000738 (active)

Test 2:
g_σ





Quantity	Initial	Optimised
$Mass\ (Kg)$	86951.0	44004.0 (-49 %)
g_σ	0.371	0.0438 (feasible)

Achievements

Journal article

Accepted article: L. Scalia, R. Cavallaro, A. Cini. ***Structural sensitivity analysis assisted by Automatic Differentiation****. In Structural and Multidisciplinary Optimization.*

Conference papers

- L. Scalia, R. Cavallaro, A. Cini. ***Development of a FE code for adjoint-based coupled aerostructural optimisation****.*
  AeroBest 2023 – II ECCOMAS Thematic Conference on Multidisciplinary Design Optimization of Aerospace Systems, Instituto Superior Técnico, Lisbon, Portugal, 19–21/07/2023.

- L. Scalia, R. Cavallaro, A. Cini, N.R. Gauger, M. Sagebaum. ***Efficient gradient-based structural optimisation with modal and buckling constraints assisted by algorithmic differentiation.***
  AeroBest 2025 – II ECCOMAS Thematic Conference on Multidisciplinary Design Optimization of Aerospace Systems, Instituto Superior Técnico, Lisbon, Portugal, 22–24/04/2025.

Research stay

Research stay at the Chair of Scientific Computing (SciComp), University of Kaiserslautern-Landau, Germany. (Sept – Dec 2024)
Implementation of AD-based methods for sensitivity analysis of generalized eigenvalue problems, with application to modal and buckling optimisation constraints.

Thank you!
Questions?

Development of a framework for coupled aerostructural optimisation of new generation wings

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2. M. Pini, S. Vitale, P. Colonna, G. Gori, A. Guardone, T. Economon, J. Alonso, and F. Palacios. Su2: the open-source software for non-ideal compressible flows. Journal of Physics: Conference Series, 821(1):012013, 2017.

3. R. Cavallaro, A. Iannelli, L. Demasi, and A. M. Razon. Phenomenology of nonlinear aeroelastic responses of highly deformable Joined Wings. Advances in Aircraft and Spacecraft Science, 2(2):125–168, April 2015.

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5. M. Schwalbach, T. Verstraete, N.R. Gauger. Discrete adjoint gradient evaluations for linear stress and vibration analysis. Computing and Visualization in Science (2019) 21:23-31

6. L. Scalia, A. Cini, R. Cavallaro. Development of a FE code for adjoint-based coupled aerostructural optimisation. In AeroBest 2023 II ECCOMAS Thematic Conference on Multidisciplinary Design Optimisation of Aerospace Systems, Lisbon, 19-21 July 2023, Portugal

Spare slides

Methodology - Discrete adjoint method in Fixed - Point form

To compute the gradient of a response J, either an objective or a constraint, the **Lagrangian** is introduced together with the adjoint variables $\bar{y}$.

$$\mathcal{L}(y, \bar{y}, \alpha) = \boxed{J(y, \alpha_s) + \bar{y}^T G(y, \alpha_s)} =$$

$$= J + \bar{w}^T[F(w,z) - w] + \bar{f}_f^T[F_f(w,z) - f_f] + \bar{z}^T[K_{mesh}(u_{tot}) - z] +$$

$$+ \bar{f}_s^T[H^T f_f - f_s] + \bar{u}_s^T[S(u_s, f_s, \alpha_s) - u_s] + \bar{u}_f^T[H u_s - u_f] +$$

$$+ \bar{u}_{tot}^T[u_{tot} - u_f] + \bar{\varphi}_i^T[K(u_s)\varphi_i - \lambda_i M \varphi_i] + \bar{\lambda}_i[{\varphi_i}^T M \varphi_i - 1]$$

If $J = mass$ the response is state-independent and the sensitivity evaluation simplifies

The Karush–Kuhn–Tucker (KKT) conditions are imposed by taking the first derivative of the Lagrangian with respect to $\bar{\boldsymbol{y}}$, $\boldsymbol{y}$, and $\boldsymbol{\alpha}$

$\frac{\partial \mathcal{L}}{\partial \bar{y}} = 0$ → field equations

$\frac{\partial \mathcal{L}}{\partial y} = 0$ → adjoint equations

$\frac{\partial \mathcal{L}}{\partial \alpha} = 0$ → total sensitivities

Methodology - Discrete adjoint method in Fixed-Point form

$$\frac{\partial J}{\partial \boldsymbol{w}} + \bar{\boldsymbol{w}}_n{}^T \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{w}} + \bar{\boldsymbol{f}}_f{}^T \frac{\partial \boldsymbol{F}_f}{\partial \boldsymbol{w}} = \bar{\boldsymbol{w}}_{n+1}{}^T$$

$$\frac{\partial J}{\partial \boldsymbol{z}} + \bar{\boldsymbol{w}}^T \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{z}} + \bar{\boldsymbol{f}}_f{}^T \frac{\partial \boldsymbol{F}_f}{\partial \boldsymbol{z}} + \bar{\boldsymbol{z}}^T = \boldsymbol{0}^T$$

$$\frac{\partial J}{\partial \boldsymbol{u}_{tot}} + \bar{\boldsymbol{z}}^T \frac{\partial \boldsymbol{K}_{mesh}}{\partial \boldsymbol{u}_{tot}} + \bar{\boldsymbol{u}}_{tot}^T = \boldsymbol{0}^T$$

$$\frac{\partial J}{\partial \boldsymbol{u}_f} - \bar{\boldsymbol{u}}_f^T - \bar{\boldsymbol{u}}_{tot}^T = \boldsymbol{0}^T$$

$$\frac{\partial J}{\partial \boldsymbol{u}_s} + \bar{\boldsymbol{u}}_s^T \frac{\partial \boldsymbol{S}}{\partial \boldsymbol{u}_s} + \bar{\boldsymbol{u}}_f{}^T \boldsymbol{H}_{MLS}^T + \bar{\boldsymbol{\phi}}_i{}^T \frac{\partial [\boldsymbol{K} - \lambda_i \boldsymbol{M}]}{\partial \boldsymbol{u}_s} \boldsymbol{\phi}_i = \bar{\boldsymbol{u}}_{s,n+1}^T$$

$$\frac{\partial J}{\partial \boldsymbol{f}_s} + \bar{\boldsymbol{u}}_s{}^T \frac{\partial \boldsymbol{S}}{\partial \boldsymbol{f}_s} = \bar{\boldsymbol{f}}_s{}^T$$

$$\frac{\partial J}{\partial \boldsymbol{f}_f} - \bar{\boldsymbol{f}}_f{}^T + \bar{\boldsymbol{f}}_s{}^T \boldsymbol{H}_{MLS}^T = \boldsymbol{0}^T$$

$\boldsymbol{FSI_{adj}\ iter \mathrel{+}= 1}$

adjoint equations

sensitivity equations

$$\frac{dJ}{d\alpha_s}^T = \frac{\partial J}{\partial \alpha_s} + \frac{\partial S}{\partial \alpha_s}^T \bar{u}_s + \bar{\phi}_i{}^T \frac{\partial [K - \lambda_i M]}{\partial \alpha_s} \phi_i$$

red: input from previous block
green: output of current block

1. **Cascade solution —** For each adjoint iteration, module equations are solved sequentially.
2. **Fixed-point update —** Aerodynamic and structural adjoint equations are solved in fixed-point form.
3. **AD evaluation —** Products of the form $\bar{\boldsymbol{y}}\frac{\partial \boldsymbol{A}}{\partial \boldsymbol{x}}$ are computed with AD
4. **Module independence —** Each module applies AD independently, preserving modularity.

Methodology - Discrete adjoint method assisted by AD

The gradient of any response J (either an objective or a constraint) is obtained by combining a discrete adjoint formulation with modular reverse-mode AD.

Adjoint formulation

$$\mathcal{L}(y, \bar{y}, \alpha_s) = J(y, \alpha_s) + \bar{y}^T G(y, \alpha_s)$$

$\partial\mathcal{L} / \partial\bar{y} = 0$ → field equations

$\partial\mathcal{L} / \partial y = 0$ → adjoint equations

$\partial\mathcal{L} / \partial\alpha_s$ → total sensitivities

$$\frac{dJ}{d\alpha_s}^T = \frac{\partial J}{\partial \alpha_s} + \frac{\partial S}{\partial \alpha_s}^T \bar{u}_s + \bar{\phi}_i^T \frac{\partial [K - \lambda_i M]}{\partial \alpha_s}$$

For J = mass, the response is state - independent and sensitivity evaluation simplifies.

AD-assisted cascade solution

1. **Primal FSI solve**
   Obtain deformed equilibrium
2. **AD recording**
   Record one additional iteration of the primal FSI solver
3. **Adjoint cascade**
   Solve iteratively the coupled adjoint equations
4. **Sensitivity evaluation**
   Get $\frac{dJ}{d\alpha_s}$

AD computes terms of the form $\bar{\boldsymbol{y}}\frac{\partial \boldsymbol{A}}{\partial \boldsymbol{x}}$

Methodology - Gradients computation procedure

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Overall procedure for reverse-mode AD-assisted adjoint sensitivity analysis

AD tape recording

1. **Primal equilibrium**
   Solve the coupled primal problem and obtain the aerostructural equilibrium state y*.

2. **Start CoDiPack tape**
   Register independent variables / inputs for the recorded module.

3. **Record computational path**
   Perform one additional primal iteration at the converged state y*.

4. **Stop tape**
   Register outputs and finalize the reverse-mode trace.

5. **Adjoint cascade**
   Seed outputs and solve the module adjoint equations in cascade.

6. **Total sensitivities**
   Evaluate gradients through the sensitivity equations.

Why the extra primal iteration?

It records the computational path used by CoDiPack at the already-converged state, avoiding the need to tape the full primal convergence history.

Role of AD

AD evaluates local derivative products, while the discrete adjoint equations propagate sensitivities through the coupled framework.

Key idea: solve the primal once, record only the required operations, then use the adjoint cascade to compute many design sensitivities efficiently.

Aerostructural optimisation - Results: Test 1

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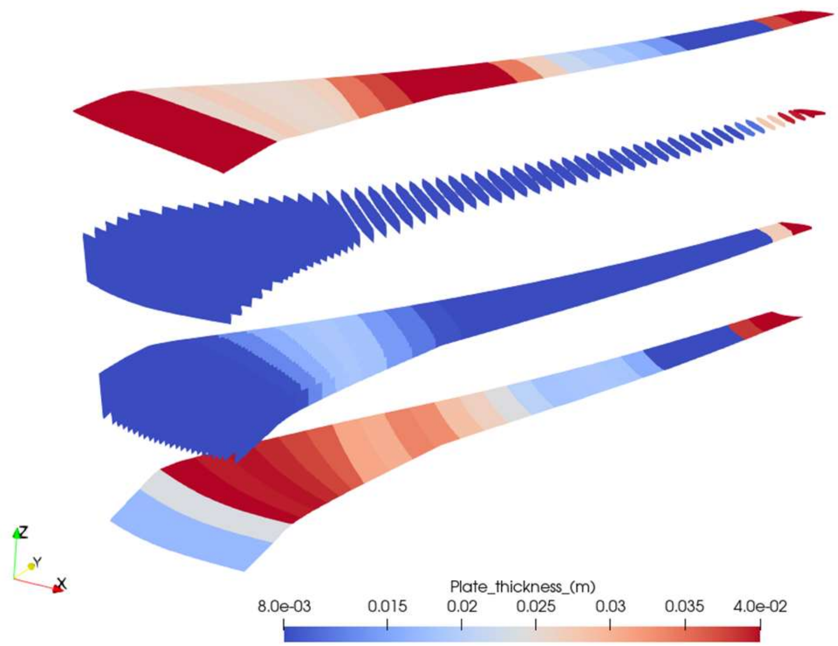
Mass reduction: −51.0% while satisfying stress and frequency-spacing constraints



Optimised thickness distribution

Key optimisation metrics

Quantity	Initial	Optimised	Change
Mass	86,951	42,633	**−51.0%**
Stress constraint g_σ	0.371	0.390	feasible
Freq. constraint g_ω	−0.102	−0.0007	**active**

Interpretation: the frequency-spacing constraint becomes active, while the stress constraint remains feasible.

Material redistribution driven by stress and modal-spacing requirements

The Structural Optimisation Problem

$$\min_{\boldsymbol{\alpha_s}} m$$

subject to $c_i \leq 0$
$(i = 1, \dots, n_c)$

Governing equations	$\boldsymbol{c_i \leq 0}$
$\boldsymbol{\mathcal{S}} = \boldsymbol{K_{el}}\boldsymbol{u_s} - \boldsymbol{f_s} = 0$	$J_\sigma = KS(g_i) = \frac{1}{\rho_{KS}} \log \sum_{i=1}^{n} e^{\rho_{KS} \cdot g_i} \leq 0$ $(where \;\; g_i = \frac{\sigma_i^{VM}}{\sigma_{ADM}} - 1)$
$(\boldsymbol{K_{el}} - \omega_k^2 \boldsymbol{M})\boldsymbol{\phi_k} = 0$	$J_\omega = \rho_\omega \frac{(\omega_j - \omega_i)_{init}}{\omega_j - \omega_i} - 1 \leq 0$
$(\boldsymbol{K_{el}} + \lambda_k^{cr} \boldsymbol{K_g})\boldsymbol{\phi_k^{cr}} = 0$	$J_{\lambda_{cr}} = \rho_{\lambda_{cr}} \frac{(\lambda_1^{cr})_{init}}{\lambda_1^{cr}} - 1 \leq 0$

m: structural mass
$\boldsymbol{u_s}$: structural displacements
$\boldsymbol{K_{el}}$: elastic part of the stiffness matrix
$\boldsymbol{\mathcal{S}}$: structural residual
$\boldsymbol{K_g}$: geometric part of the stiffness matrix
$\boldsymbol{f_s}$: applied external loads

$\boldsymbol{M}$: mass matrix
ω_k: k-th modal frequency
$\boldsymbol{\phi_k}$: k-th modal eigenvector
λ_k^{cr}: k-th critical eigenvalue
$\boldsymbol{\phi_k^{cr}}$: k-th critical eigenvector

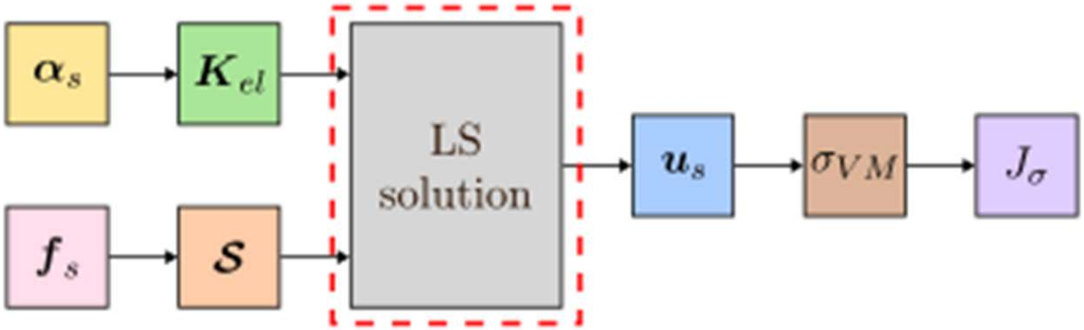
20

Computation of gradients

After recording with CodiPack the computer functions that returns either $m,\ J_\sigma,\ J_\omega\ or\ J_{\lambda_{cr}}$ (i.e., store the computational graph), compute the gradients in the following ways:

- **Mass gradient:** AD reverse mode (one run of reverse path)
- **Stress constraint gradient:** adjoint method + AD reverse mode

$$\boldsymbol{K}_{el}\bar{\boldsymbol{u}}_s = -\frac{\partial J_\sigma}{\partial \boldsymbol{u}_s}$$
$$\frac{dJ_\sigma}{d\boldsymbol{\alpha}_s} = \frac{\partial J_\sigma}{\partial \boldsymbol{\alpha}_s} + \bar{\boldsymbol{u}}_s^T \frac{d\boldsymbol{S}}{d\boldsymbol{\alpha}_s}$$



Two reverse path runs to solve:

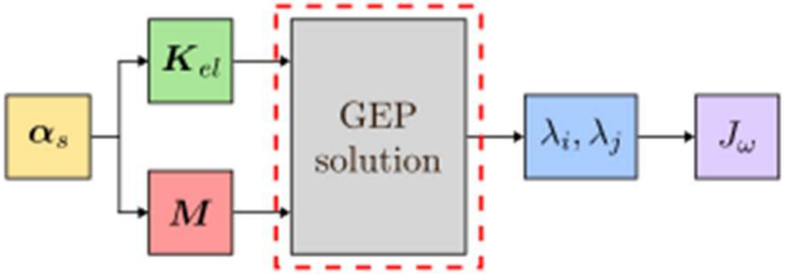
1) get $\bar{\boldsymbol{u}}_s$

2) get $\frac{dJ_\sigma}{d\boldsymbol{\alpha}_s}$

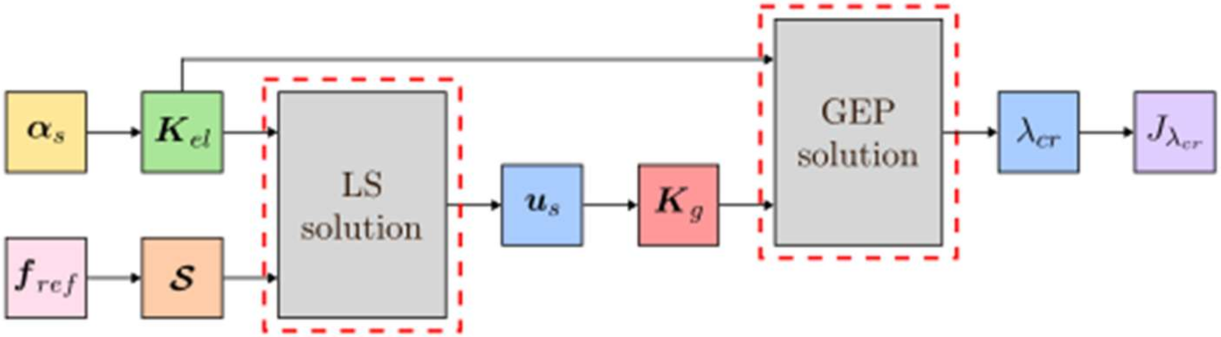
Computation of gradients

- **Frequency & Buckling constraints:** hand-differentiated expressions of J_ω and $J_{\lambda_{cr}}$ + AD reverse mode

$$\frac{dJ_\omega}{d\boldsymbol{\alpha_s}} = -\rho_\omega \frac{\omega_{j,init} - \omega_{i,init}}{\left(\omega_j - \omega_i\right)^2} \left(\frac{1}{2\omega_j} \frac{d\lambda_j}{d\boldsymbol{\alpha_s}} - \frac{1}{2\omega_i} \frac{d\lambda_i}{d\boldsymbol{\alpha_s}} \right)$$



$$\frac{dJ_{\lambda_{cr}}}{d\boldsymbol{\alpha_s}} = -\rho_{\lambda_{cr}} \frac{\lambda_{cr,init}}{\lambda_{cr}^2} \frac{d\lambda_{cr}}{d\alpha_s}$$



Each term $\frac{d\lambda_x}{d\boldsymbol{\alpha_s}}$ requires one reverse path run

Handling of Linear Systems and Eigenvalue Problems in AD

Differentiation of the solution of the linear system (LS) or generalized eigenvalue problem (GEP) is never performed:

- Efficiency issue (memory consumption)
- Impossible if solutions are handled by an external library

Procedure:

1. Interrupt computational graph recording in correspondence of LS and GEP solutions
2. Retrieve lost information by solving the adjoint statements related to the LS and GEP solutions

	Primal problem	Adjoint statement
Linear system	$\boldsymbol{K}_{el}\boldsymbol{u}_s = \boldsymbol{f}_s$	$\boldsymbol{K}_{el}^T\,\boldsymbol{s} = \frac{\partial J}{\partial \boldsymbol{u}_s}$ $\overline{\boldsymbol{K}}_{el} \mathrel{+}= -\boldsymbol{s}\cdot\boldsymbol{u}_s$ $\overline{\boldsymbol{S}} \mathrel{+}= \boldsymbol{s}$
Generalised eigenvalue problem	$\mathbf{A}\boldsymbol{\phi}_k = \lambda_k \boldsymbol{B}\boldsymbol{\phi}_k$	$\overline{\boldsymbol{A}} \mathrel{+}= \boldsymbol{\phi}_k\boldsymbol{\phi}_k^T$ $\overline{\boldsymbol{B}} \mathrel{+}= -\lambda_k\overline{\boldsymbol{A}}$

Schwalbach M. , Verstraete T. , N.R. Gauger . *Discrete adjoint gradient evaluations for linear stress and vibration analysis*. , Computing and Visualization in Science (2019) 21:23–31. https://doi.org/10.1007/s00791-019-00317-x

State of the Art — AD for Structural Sensitivity Analysis

Era / Research Direction	Main Achievement	Remaining Issue
Early AD structural applications	Automated sensitivities for truss/plate structures *Ozaki & Terano, 1993; Barthelemy & Hall, 1995*	Limited scalability and problem size
AD + adjoint methods	Efficient gradients for structural optimisation and dynamics *Haase et al., 2001; Tadjouddine et al., 2006*	Mainly moderate-size demonstrations
Parallel differentiable frameworks	MPI reverse-mode AD demonstrated mainly in CFD *Albring et al., 2015, 2016*	Limited structural FE applications
Current challenge	Geometrically nonlinear FE optimisation frameworks	AD overheads, scalability, and tangent matrix consistency remain insufficiently characterised

Gap: scalable reverse-mode AD integration in MPI-parallel nonlinear FE solvers remains insufficiently characterised.

State of the Art — Aerostructural Optimisation

Era / Research direction	Main achievement	Remaining issues
High-fidelity aerostructural optimisation	Large-scale CFD–FEM adjoint optimisation *Kenway et al., 2014*	Linear structural assumptions
AD-enabled coupled sensitivities	Exact coupled derivatives in MDO frameworks *Hwang & Martins, 2018*	Limited nonlinear stability integration
Flexible-wing aeroelastic optimisation	Nonlinear aeroelastic effects increasingly considered *Gray & Martins, 2021*	Equilibrium load redistribution remains challenging
Stability-aware optimisation	Buckling/modal constraints in optimisation loops *Gray & Martins, 2025*	Stress constraints dominate; buckling often simplified
Nonlinear structural modelling	Improved stress/deformation prediction in flexible wings *Verri et al., 2025*	Limited analyses on deformed equilibrium configurations

Gap: need for high-fidelity aerostructural optimisation with consistent modal and buckling analyses on deformed configurations.